# FLOW AND HEAT TRANSFER OVER A STRETCHING SURFACE WITH POWER -LAW VELOCITY AND TEMPERATURE DUE TO A TRANSVERSE MAGNETIC FIELD

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## Abstract

**The boundary layer flow and heat transfer due to a plate stretching with a power-law velocity distribution in the presence of a transverse magnetic field is studied. The effect of Pr, Ec, n,on heat transfer has been studied numerically. The effect is quite prominent as evidenced from the results. The effect of Pr, shows an interesting behaviour on the Nusselt number. It is observed that the Nusselt number first decreases as Pr increases and then continuously increases as Pr -> ∞. Two important physical quantities, the skin friction coefficient and the Nusselt number are computed for a variety of combinations of the parameters.**

## 1. Introduction

The heat transfer from a stretching surface is of interest in polymer extrusion processes where the object, passing through a die, enters the fluid for cooling below a certain temperature. The rate at which such objects are cooled has an important bearing on the final product. The two -dimensional boundary layer flow caused by a linear stretching sheet in an ambient quiescent fluid was first discussed by Crane [1] who obtained a very closed form exponential solution. The solution of the associated linear heat conduction equation was also presented by Crane. Both the basic flow problem and the heat transfer problem have since been extended in various ways. Afzal and Varshney [2], Kuiken [3] and Banks [4] have considered the more general case of the sheet stretching with a power-law velocity, i.e.,U(x) ax$^m$, where a and m are constants. The solution have been studied for the range -2< $\beta$, where the Parameter $\beta = 2m/$

(1+m). It was found that no similarity solution was possible if $\beta = -2$. For $-2 < \beta < 2$, solution exist when a > 0 whereas, for $2 < \beta < \alpha$, a must be negative. In addition, apart from the case m = -1, all solutions exhibited exponential decay. The eigen solutions for this problem were further studied by Banks and Zaturska [5]. Recently, Afzal [6] presented the solution for the heat transfer from an arbitrarily stretching surface $U \alpha \ x^m$, for investigating the effects of non-uniform surface temperature. Several closed form solutions for the specific values of m including their numerical solution were also presented.

The linear stretching problem has been extended by Chakrabarti and Gupta[7] to include the effect of a constant transverse magnetic field. A closed form similarity solution was also found. Chiam [8] presented the similarity solution for the case of a micropolar fluid.

After generalizing the works of Afzal and Varshney [2] and Chakrabarti and Gupta [7], Chaim [9] recently studied the boundary layer flow of a Newtonian fluid caused by a stretching sheet according to a power-law velocity distribution in the presence of magnetic field. They have shown that similarity solutions are possible if the magnetic parameter are first derived. Then Crocco's transformation is used to obtain very accurate values for the Skin friction parameter especially for large magnetic field strength.

In this paper an attempt has been made to study the effect of heat transfer of a Newtonian fluid caused by a sheet stretching with power-law velocity distribution in the presence of a magnetic field B(x). The similarity variables and the special form of magnetic field proposed by Chaim [9] are used here for the solution of the similarity of the energy equation. A direct numerical solution of the similarity boundary value is obtained by using Runge -Kutta Shooting algorithm.

## 2. Mathematical Formulation of the Problem.

### 2.a.Flow Analysis:

Let us consider the flow of an electrically conducting incompressible fluid past a stretching sheet coinciding with the plane y=0. The uniform magnetic field B(x) is imposed along y-axis. The basic boundary layer equations for the steady two -dimensional flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B(x)^2}{\rho} u \tag{2}$$

where u, v are flow velocities in the x-and y-directions respectively, $\nu$ is the kinematic viscosity, $\rho$ the fluid density and $\sigma$ is the electrical conductivity. It is assumed that induced magnetic field is negligible, the external electric field is zero and the electric field due to polarization of charges is also negligible. The boundary conditions for the flow induced by stretching sheet moving with non-uniform surface speed U(x) in quiescent environment are:

$$y = 0, \ u = U(X), \ v = 0, \qquad y \to \alpha, \ u \to 0. \tag{3}$$

Let us introduce the similarity variables

$$\psi(x,y) = \sqrt{\frac{2\nu x\, U(x)}{(1+m)}}\, F(\eta), \tag{4}$$

$$\eta(x,y) = \sqrt{\frac{(1+m)\, U(x)}{2\nu x}}\, y. \tag{5}$$

The velocity component (u,v)is then

$$u = \frac{\partial \psi}{\partial y} = U(x)\, F'(\eta). \tag{6}$$

and $$v = -\frac{\partial \psi}{\partial x} - \left[\sqrt{\frac{(1+m)\, \nu U(x)}{2x}}\, F(\eta) + \frac{(m-1)\, U(x)}{2x}\, yF'(\eta)\right] \tag{7}$$

Similarity solution exist if we assume that $U(x)=ax^m$ and then magnetic field B(x) has the special form,

$$B(x) = B_o x^{(m-1)/2} \tag{8}$$

Using (6) and (7) it can be easily verified that the continuity equation (1) is identically satisfied, and using (8) the equation (2) gives the following equation

$$F''' + FF'' - \beta F'^2 - MF' = 0 \tag{9}$$

where $$\beta = \frac{2m}{1+m} \tag{10}$$

and $$M = \frac{2\sigma B_o^2}{\rho a(1+m)} \tag{11}$$

is the magnetic parameter. Prime denotes the differentiation with respect to $\eta$.

The boundary condition for F becomes

$$F(0) = 0,\ F'(0) = 1, F(\infty) = 0 \tag{12}$$

**2.b. Heat transfer:**

Consider the transfer of heat through stretching sheet. Due to transfer of heat, thermal boundary layer is developed around the sheet. By using boundary layer approximation, the energy equation for two dimensional constant pressure flow in the presence of magnetic field is given by

and $$\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 + \sigma B(x)^2 u^2 \tag{13}$$

where $\rho$, $c_p$, k, $\mu$, T, $\sigma$, B(x) are density, specific heat at constant pressure, thermal conductivity, viscosity, temperature, electrical conductivity and uniform magnetic field respectively.

The boundary conditions are

$$T = T_w \text{ at } y = 0,\ T \rightarrow T_\infty \text{ at } y \rightarrow \infty \qquad (14)$$

where $T_w$ and $T_\infty$ are constant temperature prescribed respectively at the sheet and a large distance from it.

Introducing the similarity variables defined in (4) and (5) and substituting

$$T = T_\infty + (T_w - T_\infty)\,\theta(\eta) \qquad (15)$$

$$U = ax^m,\ T_w = T_\infty + Cx^n$$

in (13) energy equation reduces to

$$\frac{1}{Pr}\theta'' + F\theta' - n(2-\beta)F'\theta + Ec\,F''^2 + M\,Ec\,F'^2 = 0 \qquad (16)$$

where and $Pr = \frac{\mu c_p}{k}$

and $Ec = \frac{U^2}{c_p(T_w - T_\infty)}$, are the Prandtl number and Eckert number respectively.

The boundary condition on θ became

$$\theta(0) = 1,\ \theta(\infty) = 0 \qquad (17)$$

The important physical quantity for this problem are the skin friction coefficient and Nusselt number, which are defined by

$$C_f = \frac{2\tau_w}{U^2},\qquad Nu = \frac{xq_w}{k(T_w - T_\infty)} \qquad (18)$$

where $\tau_w = u\left(\frac{\partial u}{\partial y}\right)_{y=0}$, and $q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$

Using (4),(5) and (15) the quantity (18) can be expressed as

$$Cf = \frac{2}{\sqrt{R(2-\beta)}} F''(0) \text{ and } Nu = -\sqrt{R/(2-\beta)}\,\theta'(0)$$

where $R = Ux / \nu$ is the Reynold's number.

## 3. Results and Discussion

The equations (9) and (13) together with the respective boundary conditions (12) and (14) are solved for various combinations of the parameters involve in the equations using an algorithm derived by Hazarika [10] based on the Shooting Method [11]. The convergence of the shooting method is established by comparing the results of the present problem to those of Chiam [9].

Table I and Table II represent the comparison of the results of chaim [9] and the present method. It is observed that our results are in good agreement to those of Chaim.

The third column of the table I and II represents the coefficient of skin friction [F'(0)]. It is observed that the coefficient of skin friction[ F"(0)] increases with the increase of the magnetic parameter M.

Tables III, IV, V represent the Nusselt number for various combinations of the parameters as indicted in the respective tables. From table III it is observed that $-\theta'(0)$ decreases with the increase of the Eckert number Ec. Hence the effect of viscous dissipation is to reduce the Nusselt number.

Table IV shows that $-\theta'(0)$ increases with the increase of uniform temperature distribution parameter n.

The variation of $\theta'(0)$ with Prandtl number Pr is presented in Table V. The behaviour exhibits by $\theta'(0)$ with the increase of Pr is in agreement with theory. It is observed that $\theta'(0)$ first decreases with increase of Pr showing a cooling effect at the beginning. For Pr > 10, $\theta'(0)$ increases continuously which is due to the generation of heat on account of very high Prandtl number.

Figures 1,2,3 depict the variation of temperature distribution with respect to Pr, Ec and n respectively. From these figures it is observed that temperature decreases with the increase of Pr and increases with increase of Ec and n. Thus the effects of all parameters are quite prominent.

Table 1. Comparison of values of F''(0) for β =1.5,5.0 and various values of M with *Chiam*

| ß | M | F''(0) (Chaim) | F''(0) (Present) |
|---|---|---|---|
| 1.5 | 0.0 | -1.14860 | -1.15027 |
| | 1.0 | -1.52527 | -1.52532 |
| | 5.0 | -2.51615 | -2.51616 |
| | 10.0 | -3.36631 | -3.36634 |
| | 50.0 | -7.16471 | -7.16499 |
| | 100.0 | -10.06640 | -10.06723 |
| 5.0 | 0.0 | -1.90253 | -1.90301 |
| | 1.0 | -2.15290 | -2.15295 |
| | 5.0 | -2.94144 | -2.94161 |
| | 10.0 | -3.69566 | -3.69599 |
| | 50.0 | -7.23561 | -7.32812 |
| | 100.0 | -10.18160 | -10.18830 |

Table 11. comparison of values of F''(0) for ß = -1.0, -1.5 and various values of M with *Chaim*

| ß | M | F''(0) (Chaim) | F''(0) (present) |
|---|---|---|---|
| -1.0 | 0.0 | 0.00000 | -0.00860 |
| | 0.5 | -0.52395 | -0.52508 |
| | 1.0 | -0.85111 | -0.85126 |

| | | | |
|---|---|---|---|
| | 5.0 | -2.16287 | -2.16284 |
| | 10.0 | -3.11003 | -3.10994 |
| | 50.0 | -7.04648 | -7.04652 |
| | 100.0 | -9.98335 | -9.98028 |
| -1.5 | 0.0 | 0.72725 | 0.71422 |
| | 0.1 | 0.45107 | 0.44102 |
| | 0.5 | -0.21922 | -0.22117 |
| | 1.0 | -0.65298 | -0.65320 |
| | 5.0 | -2.08524 | -2.08521 |
| | 10.0 | -3.05623 | -3.05613 |
| | 50.0 | -7.02249 | -7.02257 |
| | 100.0 | -9.96665 | -9.96284 |

***Estimated missing initial values of θ' (0) for various values of ß, Pr M, Ec and n***

Table 111

ß=1.5 Pr =.72 M =1 n=1

| Ec | θ' (0) |
|---|---|
| 0.0 | -0.547369 |
| 0.25 | -0.375050 |
| 0.5 | -0.202732 |
| 0.75 | -0.030413 |
| 1.0 | -0.141905 |
| 1.25 | -0.341223 |
| 1.5 | -0.486542 |
| 1.75 | -0.658860 |
| 2.0 | -0.831179 |

Table IV

ß=1.5 Pr =.72 Ec =1 M=1

| n | θ' (0) |
|---|---|
| -3 | 1.022593 |
| -2 | 0.761343 |
| -1 | 0.531694 |
| 0 | 0.326856 |
| 1 | 0.141905 |
| 2 | -0.026820 |
| 3 | -0.182101 |
| 4 | -0.326091 |

Table V

ß=1.5 Ec =.5 M=1 n=2

| Pr | θ' (0) |
|---|---|
| .5 | -0.295173 |
| .72 | -0.360639 |
| 2 | -0.585528 |
| 7 | -0.731850 |
| 10 | -0.694314 |
| 15 | -0.579516 |
| 20 | -0.440177 |
| 22 | -0.386330 |
| 30 | -0.100546 |
| 35 | -0.063692 |
| 40 | 0.227204 |
| 100 | 1.993231 |

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