INTUITIONISTIC FUZZY SUBGROUPS

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Abstract

In this paper we define intuitionistic fuzzy subgroups, and study its properties.

1. Introduction

The notion of intuitionistic fuzzy sets (IFSs) was introduced by Atanassov [2] as a generalization of the notion of Zadeh's fuzzy sets[7]. Atanassov in [2] justified with an example that fuzzy sets can be regarded as IFSs but the converse is not true. In [2-6] he defined various operations on IFSs and these are different from the similar operations on fuzzy sets.

Rosenfeld [1] defined fuzzy subgroups and stated some properties. In [8], we defined anti fuzzy subgroups and studied a relation between fuzzy subgroups and anti fuzzy subgroups. In the present paper we define intuitionistic fuzzy subgroups and make some characterizations.

2. Preliminaries

We give below some basic preliminaries.

Definition 2.1

If E is any set, a mapping

$\mu_A : E \longrightarrow [0,1]$

is called a fuzzy subset of E.

Definition 2.2

Let G be a group. A fuzzy set A of G is called a fuzzy subgroup of G if $\forall$ x, y ε G

i) $\mu_A(xy) \geq \min\{\mu_A(x).\ \mu_A(y)\}$
ii) $\mu_A(x^{-1}) \geq \mu_A(x)$.

Proposition 2.1

A fuzzy subset A of a group G is a fuzzy subgroup of G iff $\forall$ x,y ε G

$$\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\}$$

Definition 2.3

Let A be a fuzzy subset of a set E. Then complement of A is A^c with membership function μ_A defined by

$$\mu_{A^c}(x) = 1 - \mu_A(x), \forall x \varepsilon E.$$

Definition 2.4

Let G be a group. A fuzzy subset A of G is called an anti fuzzy subgroup of G if $\forall$ x,y, ε G

i) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$
ii) $\mu_A(x^{-1}) = \mu_A(x)$

Proposition 2.2

A fuzzy subset A of a group G is an anti Fuzzy subgroup of G iff $\forall$ x,y, ε G

$$\mu_A(xy^{-1}) \leq \max\{\mu_A(x), \mu_A(y)\}$$

Proposition 2.3

A is an anti fuzzy subgroup of a group G iff A^c is a fuzzy subgroup of G.

Definition 2.5

Let a set E be fixed. An IFS A in E is an object having the form

$$A^* = \{ < x, \mu_A(x), \gamma_A(x) > \mid x \varepsilon E\}$$

where the functions $\mu_A : E \rightarrow [0,1]$ and $\gamma_A : E \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \varepsilon E$ to the set A, which is a subset of E, respectively, and for every $x \varepsilon E$:

$$0 \leq \mu_A(x) + \gamma_A(x) \leq 1.$$

Definition 2.6

$A \subset B$ iff $(\forall x \varepsilon E)\ (\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x))$,

$A \supset B$ iff $B \subset A$,

$A = B$ iff $(\forall x \varepsilon E)\ (\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x))$.

$\ddot{A} = \{ < x, \gamma_A(x)\ \mu_A(x) > \mid x \varepsilon E \}$,

$A \cap B = \{ < x, \max(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) >) \mid x \varepsilon E\}$,

$A \cup B = \{ < x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x)) >) \mid x \varepsilon E\}$,

$A + B = \{ < x, \mu_A(x) + \mu_B(x) - \mu_A(x).\ \mu_B(x), \gamma_A(x).\ \gamma_B(x) > \mid x \varepsilon E\}$,

$A . B = \{ < x, \mu_A(x) . \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x).\ \gamma_B(x) > \mid x \varepsilon E\}$,

$\square\ A = \{ < x, \mu_A(x), 1 - \mu_A(x) > \mid x \varepsilon E \}$,

$\diamond\ A = \{ < x, 1 - \gamma_A(x), \gamma_A(x) > \mid x \varepsilon E \}$.

obviously, every fuzzy set has the form

$$\{ < x, \mu_A(x), \mu_{A^c}(x) > : x \varepsilon E \}$$

In [2], Atanssov gave an example of IFS which is not a fuzzy set. From now onwards in this paper, by an IFS A we shall mean the IFS (A, μ_A, ν_A),where the meaning is obvious .

3. Intuitionistic Fuzzy Subgroups

We know that all intuitionistic fuzzy sets (IFSs) are not fuzzy sets. Keeping this view in mind , we define below intuitionistic fuzzy subgroups (rather, say intuitionistic fuzzy groups, in short, IFGs).

Definition 3.1

Let G be a group and $A = (A, \mu_A, \nu_A)$ be an IFG of G. Then A is called an IFG of G if he following are satisfied $\forall$ x, y ε G :

i) $\mu_A(xy) \geq \min\{ \mu_A(x), \mu_A(y)\}$

ii) $\mu_A(x^{-1}) = \mu_A(x)$
iii) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$
iv) $\nu_A(x^{-1}) = \nu_A(x)$

Example

Consider the group $G = \{1, a\}$ when $a^2=1$. Consider μ_A and ν_A in such a way that

$$\mu_A(1) = .9,\ \mu_A(a) = .3,\ \nu_A(a) = .4,\ \nu_A(1) = .06.$$

Then the IFS (A, μ_A, ν_A) is an IFG of G .

The following proposition are straightforward.

Proposition 3.1

If $A = (A, \mu_A, \nu_A)$ is an IFG of group G, then $\forall x \in G$

i) $\mu_A(e) \geq \mu_A(x)$
ii) $\nu_A(e) \leq \nu_A(x)$

where e is the identity element of G.

Proposition 3.2

If $A = (A, \mu_A, \nu_A)$ is an IFG of a group G , then

i) μ_A is a fuzzy subgroup of G, and
i) ν_A is an anti fuzzy subgroup of G.

Definition 3.2

Let A be IFS of a set E. Atanassov [2] defined Ä, complement of A by

$$\ddot{A} = \{ < x, \nu_A(x), \mu_A(x) > : x \in E \}.$$

We , define another type of complement of A, denoted by A^c as follows:

$$A^c = \{ < x, \mu^c_A(x), \nu^c_A(x) > : x \in E \}.$$

Proposition3.3

If A and B are two IFSs of E, then

i) $(A^c)^c = A$
ii) $(\ddot{A})^c = (\ddot{A}^c)$
iii) $(A \cup B)^c = A^c \cap B^c$
iv) $(A \cap B)^c = A^c \cup B^c$

Proof: We prove only the (iv)

$$A^c \cup B^c = (< x, \mu^c_A(x), \nu^c_A(x) > \} \cup \{ < x, \mu^c_B(x), \nu^c_B(x) > \}$$
$$= (< x, \mu^c_A(x) \vee \mu^c_B(x), \nu^c_A(x) \Delta \nu^c_B(x) > \}$$
$$= (< x, \mu^c_{(A\cap B)}(x), \nu^c_{(A\cup B)}(x) > \}$$
$$= (< x, \mu_{(A\cap B)}(x), \nu_{(A\cup B)}(x) > \}^c$$
$$= (A \cap B)^c.$$ Hence proved.

The following propositions are straightforward.

Proposition 3.4

If A is an IFG of a group G, then (A^c) is also an IFG of G.

Proposition 3.5

A necessary and sufficient condition for an IFS A of a group G to be an IFG of G is that $\forall$ x, y ε G,

i) $\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\}$

and ii) $\nu_A(xy^{-1}) \leq \max\{\nu_A(x), \nu_A(y)\}$

Proposition 3.6

A necessary and sufficient condition for an IFS A of a group G to be an IFG of G is that μ_A is a fuzzy subgroup and ν_A is an anti fuzzy subgroup of G.

Proposition 3.7

If A and B are two IFGs of a group G, then $A \cap B$ is also an IFG of G

Proof : Clearly, μ_A and μ_B are fuzzy subgroups of G.

$\Rightarrow \mu_A \cap \mu_B$ is also so.

Again ν_A and ν_B are anti fuzzy subgroups of G.

$\Rightarrow \nu_A \cap \nu_B$ is also so.

=> $(A \cap B, \mu_A \cap \mu_B, \nu_A \cup \nu_B)$ Is an IFG of G .

Hence proved

The following proposition are also straightforward.

Proposition 3.8

If A is an IFS of a set E, then

i) $(\overline{\Box A})^c = \Box A = \overline{((\Box A)^c)}$.

ii) $(\overline{\Diamond A})^c = \Diamond A = \overline{((\Diamond A)^c)}$.

Proposition 3.9

If A i s an IFG of a group G, then

(i) $\Box$ A is an IFG of G.

(ii) $\Diamond$ A is an IFG of G.

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