FUZZY ALMOST $\alpha$-CONTINUOUS MAPPINGS

R. PRASAD
Department of Mathematics and Statistics
Dr. H. S. Gour Vishwavidyalaya
SAGAR (M. P.) 470003 INDIA

S. S. THAKUR
Department of Applied Mathematics
Government Engineering College
JABALPUR (M. P.) 482011 INDIA

And

R. K. SARAF
Department of Mathematics
Government Autonomous Girls Science College
DAMOH (M. P.) INDIA

ABSTRACT

The purpose of this paper is to introduce and study the concepts of fuzzy almost $\alpha$-continuous and fuzzy almost $\alpha$-open mappings in fuzzy topological spaces.

1. PRELIMINARIES

Let $X$ be a non empty set and $I$ the unit interval $[0, 1]$. A fuzzy set $\lambda$ in $X$ is a mapping from $X$ into $I$. The null fuzzy set $\phi$ is the mapping from $X$ into $I$ assumes only the value 0 and the whole fuzzy set 1 is the mapping from $X$ into $I$ which takes the value 1 only. The Union $\cup \lambda_k$ (resp. intersection $\cap \lambda_k$) of a family $\{\lambda_k : k \in \Lambda\}$, where $\Lambda$ is the index set of fuzzy sets of $X$, is defined to be the mapping $\cup \lambda_k$ (resp. $\inf \lambda_k$). A fuzzy set $\lambda$ is contained in a fuzzy set $\mu$ denoted by $\lambda \leq \mu$ iff $\lambda(x) \leq \mu(x)$ for each $x \in X$. The complement $\lambda^c$ of a fuzzy set $\lambda$ of $X$
is $1-\lambda$ defined by $(1-\lambda)(x) = 1-\lambda(x)$ for each $x \in X$. If $\lambda$ is a fuzzy set of $X$ and $\mu$ is fuzzy set of $Y$ then $\lambda \times \mu$ is a fuzzy set of $X \times Y$, defined by $(\lambda \times \mu)(x, y) = \min (\lambda(x), \mu(y))$, for each $(x, y) \in X \times Y$ [1]. A fuzzy point $x_\beta$ in $X$ is a fuzzy set in $X$ defined by,

$$x_\beta(y) = \begin{cases} 
\beta (\beta \in (0, 1]) & \text{for } y = x \\
0 & \text{otherwise},
\end{cases}$$

for $y \in X$. $x$ and $\beta$ are respectively called the support and value of $x_\beta$. A fuzzy point $x_\beta$ is said to belonging to fuzzy set $\lambda$ iff $\beta \leq \lambda(x)$. A fuzzy set $\lambda$ is the union of all fuzzy points which belonging to $\lambda$. A fuzzy set $\lambda$ in $X$ is said to be quasi-cocncident [8] with a fuzzy set $\mu$ of $X$ denoted by $\lambda_\cap \mu$ if there exists $x \in X$ such that $\lambda(x) + \mu(y) > 1$. For any two fuzzy sets $\lambda$ and $\mu$ of $X$, $\lambda \mu$ iff $|\lambda_\cap \mu_\cap|$. Let $f : X \rightarrow Y$ be a mapping. If $\lambda$ is fuzzy set of $X$, then $f(\lambda)$ is a fuzzy set of $Y$, defined by,

$$f(\lambda)(y) = \begin{cases} 
\sup \lambda(x) & \text{if } f^{-1}(y) \neq \emptyset \\
x \in f^{-1}(y) & \text{otherwise},
\end{cases}$$

(y \in Y).

If $\mu$ is fuzzy set of $Y$, then $f^{-1}(\mu)$ is a fuzzy set of $X$ defined by $f^{-1}(\mu(x)) = \mu(f(x))$, for each $x \in X$.

A family $\gamma$ of fuzzy sets of $X$, is called fuzzy topology on $X$ [4] if,

(i) \hspace{1cm} 0, 1 \hspace{1cm} \text{belong to } \gamma.

(ii) \hspace{1cm} Any union of members of $\gamma$ is in $\gamma$, and

(iii) \hspace{1cm} A finite intersection of members of $\gamma$ is in $\gamma$.

Members of $\gamma$ are called fuzzy open sets of $X$ and their complements fuzzy closed sets. For a fuzzy set $\lambda$ of $X$, the closure and interior are defined respectively, as

$$\text{Cl } \lambda = \inf \{u : u \geq \lambda, \ u \in \gamma\}, \text{ and}$$

$$\text{Int } \lambda = \sup \{u : u \leq \lambda, \ u \in \gamma\}.$$
Definition 1.1 : A fuzzy set $\lambda$ of a fuzzy topological space $X$ is called:

(a) Fuzzy regular open [1] if $\lambda = \text{Int Cl } \lambda$.
(b) Fuzzy regular closed [1] if $\lambda = \text{Cl Int } \lambda$.
(c) Fuzzy $\alpha$-open [2] if $\lambda \leq \text{Int Cl Int } \lambda$.
(d) Fuzzy $\alpha$-closed [2] if $\lambda \geq \text{Cl Int Cl } \lambda$.

Every fuzzy regular open (resp. fuzzy regular closed) set is fuzzy open (resp. fuzzy closed) and every fuzzy open (resp. fuzzy closed) set is fuzzy $\alpha$-open (resp. fuzzy $\alpha$-closed) but the converse may not be true. A fuzzy set $\lambda$ is fuzzy regular closed (resp. fuzzy $\alpha$-closed) iff $\lambda^c$ is fuzzy regular open (resp. fuzzy $\alpha$-closed). The intersection of all fuzzy $\alpha$-closed set which contains a fuzzy set $\lambda$ of a fuzzy topological space is called $\alpha$-closure [7] (denoted by $\alpha\text{Cl}$) of $\lambda$. The union of all fuzzy $\alpha$-open subsets of a fuzzy set $\lambda$ is called $\alpha$-interior [7] (denoted by $\alpha\text{Int}$) of $\lambda$.

Definition 1.2 : A mapping $f$ from a fuzzy topological space $X$ to a fuzzy topological space $Y$ is called.

(a) Fuzzy continuous [4] (resp. fuzzy $\alpha$-continuous [10] or fuzzy strongly semicontinuous [2]) if $f^{-1}(\lambda)$ is fuzzy open (resp. Fuzzy $\alpha$-open) in $X$ for every fuzzy open set $\lambda$ of $Y$.
(b) Fuzzy almost continuous [1] if $f^{-1}(\lambda)$ is fuzzy open in $X$ for every fuzzy regular open set $\lambda$ of $Y$.

Definition 1.3 : A mapping $f$ from a fuzzy topological space $X$ to a fuzzy topological space $Y$ is called.

(a) Fuzzy open (resp. fuzzy $\alpha$-open [10]) if $f(\lambda)$ is fuzzy open (resp. fuzzy $\alpha$-open) in $Y$ for every fuzzy open set $\lambda$ of $X$.
(b) Fuzzy almost open [1] if $f(\lambda)$ is fuzzy open in $Y$ for every fuzzy regular open set $\lambda$ of $X$. 
2. FUZZY ALMOST $\alpha$-CONTINUOUS MAPPINGS

Definition 2.1: A mapping $f : X \rightarrow Y$ is termed fuzzy almost $\alpha$-continuous if the inverse image of every fuzzy regular open set of $Y$ is fuzzy $\alpha$-open in $X$.

Remark 2.1: It is clear from the definition 1.2 and definition 2.1 that the following implications are true:

Fuzzy Almost Continuous $\iff$ Fuzzy Continuous $\iff$ Fuzzy Almost $\alpha$-continuous $\iff$ Fuzzy $\alpha$-continuous

The converse of above implications may not be true. For,

Example 2.1: Let $X = \{a, b\}$, $Y = \{x, y\}$ and $\lambda_1, \lambda_2, \mu$ be defined as,

$\lambda_1(a) = 0.3 \quad \lambda_1(b) = 0.4 \quad \lambda_2(a) = 0.4$

$\lambda_2(b) = 0.5 \quad \mu(x) = 0.4 \quad \mu(y) = 0.4$

Let $\gamma = \{1, \lambda_1, \lambda_2, 0\}$ and $\Gamma = \{1, \mu, 0\}$. Then the mapping $f : (X, \gamma) \rightarrow (Y, \Gamma)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy almost $\alpha$-continuous but not fuzzy almost continuous.

Example 2.2: Let $X = \{a, b\}$, $Y = \{x, y\}$ and $\lambda, \mu$ be defined as,

$\lambda(a) = 0.3 \quad \lambda(b) = 0.4 \quad \mu(x) = 0.6 \quad \mu(y) = 0.7$

Let $\gamma = \{0, 1, \lambda\}$ and $\Gamma = \{0, 1, \mu\}$. Then the mapping $f : (X, \gamma) \rightarrow (Y, \Gamma)$ defined by $f(a) = x$, $f(b) = y$ is fuzzy almost $\alpha$-continuous but not fuzzy $\alpha$-continuous.

Theorem 2.1: Let $f : X \rightarrow Y$. Then the following conditions are equivalent

(a) $f$ is fuzzy almost $\alpha$-continuous.
(b) $f^{-1}(\mu)$ is fuzzy $\alpha$-closed set of $X$ for every fuzzy regular closed set $\mu$ of $Y$.
(c) $f^{-1}(\lambda) \subseteq \alpha\text{Int} f^{-1}(\text{Int Cl} \lambda)$ for every fuzzy open set $\lambda$ of $Y$. 
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(d) $\alpha \text{Cl } f^{-1}(\text{Cl \ Int } \mu) \leq f^{-1}(\mu)$ for every fuzzy closed set $\mu$ of $Y$.

(e) For each fuzzy point $x_\beta$ of $X$ and each fuzzy regular open set $\mu$ of $Y$ containing $f(x_\beta)$, there exists a fuzzy $\alpha$-open set $\lambda$ of $X$ such that $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$.

(f) For each fuzzy point $x_\beta$ of $X$ and each fuzzy regular open set $\mu$ of $Y$ containing $f(x_\beta)$, there exists a fuzzy $\alpha$-open set $\lambda$ in $X$ such that $x_\beta \in \lambda$ and $f(\lambda) \leq \mu$.

(g) For each fuzzy point $x_\beta$ of $X$ and every fuzzy regular open set $\mu$ of $Y$ such that $f(x_\beta) \in \mu$, there exists a fuzzy $\alpha$-open set $\lambda$ in $X$ such that $x_\beta \in \lambda$ and $f(\lambda) \leq \mu$.

(h) For every fuzzy point $x_\beta$ of $X$ and every fuzzy regular open set $\mu$ of $Y$ such that $f(x_\beta) \in \mu$, there exist a fuzzy $\alpha$-open set $\lambda$ in $X$ such that $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$.

Proof:

(a) $\leftrightarrow$ (b) Since $f^{-1}(\lambda^c) = (f^{-1}(\lambda))^c$ for every fuzzy set $\lambda$ of $Y$. It follows from theorem 5.2 [1].

(a) $\rightarrow$ (e) Since $\lambda$ is fuzzy open set of $Y$, $\lambda \leq \text{Int } \text{Cl } \lambda$ and hence $f^{-1}(\lambda) \leq f^{-1}(\text{Int } \text{Cl } \lambda)$. By theorem 5.6(b) [1].

$\text{Int } \text{Cl } \lambda$ is a fuzzy regular open set of $Y$. Hence

$\lambda \leq f^{-1}(\text{Int } \text{Cl } \lambda)$ is a fuzzy $\alpha$-open set of $X$. Thus

$f^{-1}(\lambda) \leq f^{-1}(\text{Int } \text{Cl } \lambda) = \alpha \text{Int } f^{-1}(\text{Int } \text{Cl } \lambda)$.

(e) $\rightarrow$ (a) Let $\lambda$ be a fuzzy regular open set of $Y$, then we have $f^{-1}(\lambda) \leq \alpha \text{Int } f^{-1}(\text{Int } \text{Cl } \lambda)$. Thus $f^{-1}(\lambda) = \alpha \text{Int } f^{-1}(\lambda)$ shows that $f^{-1}(\lambda)$ is a fuzzy $\alpha$-open set of $X$.

(b) $\rightarrow$ (d) Since $\mu$ is fuzzy closed set of $Y$, $\text{Cl } \text{Int } \mu \leq \mu$ and hence $f^{-1}(\text{Cl } \text{Int } \mu) \leq f^{-1}(\mu)$. By theorem 5.6(a) [1]. $\text{Cl } \text{Int } \mu$ is fuzzy regular closed set of $Y$, hence $f^{-1}(\text{Cl } \text{Int } \mu)$ is fuzzy $\alpha$-closed set of $X$. Thus $\alpha \text{Cl } f^{-1}(\text{Cl } \text{Int } \mu) = f^{-1}(\text{Cl } \text{Int } \mu) \leq f^{-1}(\mu)$. 


(d) $\Rightarrow$ (b) Let $\mu$ be a fuzzy regular closed set of $Y$, then we have $\alpha\text{Cl} f^{-1}(\mu) = \alpha\text{Cl} f^{-1}(\text{Cl Int } \mu) \leq f^{-1}(\mu)$ Thus $\alpha\text{Cl} f^{-1}(\mu) f^{-1}(\mu)$, shows that $f^{-1}(\mu)$ is fuzzy $\alpha$-closed in $X$.

(a) $\Rightarrow$ (e) Let $x_\beta$ be a fuzzy point of $X$ and $\mu$ be a fuzzy regular open set of $Y$ such that $f(x_\beta) \in \mu$. Put $\lambda = f^{-1}(\mu)$. Then by (a) $\lambda$ is fuzzy $\alpha$-open, $x_\beta \in \lambda$ and $\lambda \leq f^{-1}(\mu)$.

(e) $\Rightarrow$ (f) Let $x_\beta$ be a fuzzy point of $X$ and $\mu$ be a fuzzy regular open set containing $f(x_\beta)$. By (e) there exists a fuzzy $\alpha$-open set $\lambda$ such that $x_\beta \in \lambda$, $f(\lambda) \leq f(f^{-1}(\mu)) \leq \mu$.

(f) $\Rightarrow$ (a) Let $\mu$ be a fuzzy regular open set of $Y$ and $x_\beta$ be a fuzzy point of $X$ such that $x_\beta \in f^{-1}(\mu)$. Then $f(x_\beta) \in f(f^{-1}(\mu)) \leq \mu$. By (f), there exists a fuzzy $\alpha$-open set $\lambda$ such that $x_\beta \in \lambda$ and $f(\lambda) \leq \mu$. This shows that $x_\beta \in \lambda \leq f^{-1}(\mu)$. By theorem 5 [7], it follows that $f^{-1}(\mu)$ is fuzzy $\alpha$-open and hence $f^{-1}$ is fuzzy almost $\alpha$-continuous.

(a) $\Rightarrow$ (g) Let $x_\beta$ be a fuzzy point of $X$ and $\mu$ be a fuzzy regular open set of $Y$ such that $f(x_\beta) \notin \mu$. Then $f^{-1}(\mu)$ is fuzzy $\alpha$-open in $X$ and $x_\beta \notin f^{-1}(\mu)$ (by proposition 4.2 [12]). If we take $\lambda = f^{-1}(\mu)$ then $x_\beta \notin \lambda$ and $f(\lambda) = f(f^{-1}(\mu)) \leq \mu$.

(g) $\Rightarrow$ (h) Let $x_\beta$ be a fuzzy point of $X$ and $\mu$ be a fuzzy regular open set of $Y$ such that $f(x_\beta) \notin \mu$. Then by (g) there exists a fuzzy $\alpha$-open set $\lambda$ of $X$ such that $x_\beta \notin \lambda$ and $f(\lambda) \leq \mu$. Hence we have $x_\beta \notin \lambda$ and $\lambda \leq f^{-1}(f(\lambda)) \leq f^{-1}(\mu)$.

(h) $\Rightarrow$ (a) Let $\lambda$ be a fuzzy regular open set of $Y$ and $x_\beta$ be a fuzzy point of $X$ such that $x_\beta \in f^{-1}(\mu)$. Then $f(x_\beta) \in \mu$. Choose the fuzzy point $x_\beta^c(x) = 1 - x_\beta(x)$. Then $f(x_\beta^c) \notin \mu$. And so by (h) there exists a fuzzy $\alpha$-open set $\lambda$ in $X$ such that $x_\beta \notin \lambda$ and $f(\lambda) \leq \mu$. Now $x_\beta^c \notin \lambda$ implies $x_\beta^c(x) + \lambda(x) = 1 - x_\beta(x) + \lambda(x) > 1$. It follows that $x_\beta \notin \lambda$. Thus $x_\beta \in \lambda \leq f^{-1}(\mu)$. Hence by theorem 5 [7] $f^{-1}(\mu)$ is fuzzy $\alpha$-open in $X$.

**Definition 2.2:** A fuzzy topological space $(X, \gamma)$ is said to be fuzzy semiregular if for each fuzzy open set $\lambda$ and each fuzzy point $x_\alpha \notin \lambda$ there exists a fuzzy open set $\mu$ such that $x_\alpha \notin \mu$ and $\mu \leq \text{Int } \text{Cl } \mu \leq \lambda$. [5].
Theorem 2.2: Let \( f : X \rightarrow Y \) be a mapping from a fuzzy topological space \( X \) to a fuzzy semiregular space \( Y \). Then \( f \) is fuzzy almost \( \alpha \)-continuous if and only if \( f \) is fuzzy \( \alpha \)-continuous.

Proof: Necessity: Let \( x_\alpha \) be a fuzzy point in \( X \) and \( \lambda \) be a fuzzy open set in \( Y \) such that \( f(x_\alpha) \subseteq \lambda \). Since \( Y \) is fuzzy semiregular there exists a fuzzy open set \( \mu \) in \( Y \) such that \( f(x_\alpha) \subseteq \mu \) and \( \mu \leq \text{Int} \ Cl \mu \leq \lambda \). Since \( \text{Int} Cl \mu \) is fuzzy regular open in \( Y \) and \( f \) is fuzzy almost \( \alpha \)-continuous by theorem 2.1 (g), there exists a fuzzy \( \alpha \)-open set \( \mu_1 \) in \( X \) such that \( x_\alpha \subseteq \mu_1 \) and \( f(\mu_1) \subseteq \text{Int} Cl \mu \). Thus \( \mu_1 \) is fuzzy \( \alpha \)-open set such that \( x_\alpha \subseteq \mu_1 \) and \( f(\mu_1) \subseteq \lambda \). Hence by theorem 3.3 [3], \( f \) is fuzzy \( \alpha \)-continuous.

Sufficiency: Obvious.

Theorem 2.3: Let \( f : X \rightarrow Y \) be a mapping from a fuzzy topological space \( X \) to another fuzzy topological space \( Y \). If the graph mapping \( g : X \rightarrow X \times Y \) of \( f \) is fuzzy almost \( \alpha \)-continuous, then \( f \) is fuzzy almost \( \alpha \)-continuous.

Proof: Let \( \mu \) be a fuzzy regular open set in \( Y \), then by theorem 2.4 [1] \( f^{-1} (\mu) = 1 \cap g^{-1} (1 \times \mu) \). Now \( 1 \times \mu = 1 \times \text{Int} Cl \mu = \text{Int} (1 \times \text{Cl} \mu) = \text{Int} Cl (1 \times \mu) \), \( 1 \times \mu \) is fuzzy regular open in \( X \times Y \). Since \( g \) is fuzzy almost \( \alpha \)-continuous, \( f^{-1} (\mu) = g^{-1} (1 \times \mu) \) is fuzzy \( \alpha \)-open in \( X \). Hence \( f \) is fuzzy almost \( \alpha \)-continuous.

Theorem 2.4: Let \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) be two mappings satisfying either of the following conditions.

(i) \( f \) is fuzzy \( \alpha \)-continuous and \( g \) is fuzzy almost continuous.

(ii) \( f \) is fuzzy \( \alpha \)-irresolute and \( g \) is fuzzy almost \( \alpha \)-continuous.

Then \( gof : X \rightarrow Z \) is fuzzy almost \( \alpha \)-continuous.

Theorem 2.5: Let \( f : X \rightarrow Y \) be a fuzzy almost \( \alpha \)-continuous mapping and \( A \subseteq X \). Then \( f|A \) is fuzzy almost \( \alpha \)-continuous if \( A \) is fuzzy preopen in \( X \).

Proof: Let \( \lambda \) be a fuzzy regular open set in \( Y \) then \( f^{-1} (\lambda) \) is fuzzy \( \alpha \)-open in \( X \). Since \( A \) is fuzzy preopen in \( X \) by Lemma 4.5 [10]. \( A \cap f^{-1} (\lambda) = (f|A)^{-1} (\lambda) \) is fuzzy \( \alpha \)-open in \( A \). Therefore \( f|A \) is fuzzy almost \( \alpha \)-continuous.
Theorem 2.6: Let \( f : X \rightarrow Y \) be a mapping and \( \{ A_\beta: \beta \in \Lambda \} \) be a cover of \( X \) such that each \( A_\beta \) is fuzzy \( \alpha \)-open for each \( \beta \in \Lambda \) then \( f \) is fuzzy almost \( \alpha \)-continuous, if the restriction \((f|A_\beta): A_\beta \rightarrow Y \) is fuzzy almost \( \alpha \)-continuous for each \( \beta \in \Lambda \).

Proof: Let \( \lambda \) be a fuzzy regular open set in \( Y \) then for each \( \beta \in \Lambda \), \((f|A_\beta)^{-1}(\lambda) \) is fuzzy \( \alpha \)-open in \( A_\beta \). Since each \( A_\beta \) is fuzzy \( \alpha \)-open in \( X \) by lemma 4.7 [10], \((f|A_\beta)^{-1}(\lambda) \) is fuzzy \( \alpha \)-open in \( X \), for each \( \beta \in \Lambda \). But \( f^{-1}(\lambda) = \bigcup_{\beta \in \Lambda} (f|A_\beta)^{-1}(\lambda) \). Then \( f^{-1}(\lambda) \) is fuzzy \( \alpha \)-open in \( X \). Hence \( f \) is fuzzy almost \( \alpha \)-continuous.

3. FUZZY ALMOST \( \alpha \)-OPEN MAPPING

Definition 3.1: A mapping \( f : X \rightarrow Y \) is said to be fuzzy almost \( \alpha \)-open if for each fuzzy regular open set \( \lambda \) in \( X \), \( f(\lambda) \) is fuzzy \( \alpha \)-open in \( Y \).

Remark 3.1: It is clear from the definition 1.3 and definition 3.1 that the following implications are true:

\[
\text{Fuzzy Almost Open} \leftrightarrow \text{Fuzzy Open} \leftrightarrow \text{Fuzzy Almost } \alpha\text{-open} \leftrightarrow \text{Fuzzy } \alpha\text{-open}
\]

The converse of above implications may not be true. For.

Example 3.1: Let \( X = \{ a, b \} \) \( Y = \{ x, y \} \) and \( \lambda, \mu_1, \mu_2 \) be defined as,

\[
\begin{align*}
\lambda(a) &= 0.4 & \lambda(b) &= 0.4 & \mu_1(x) &= 0.3 \\
\mu_1(y) &= 0.4 & \mu_2(x) &= 0.4 & \mu_2(y) &= 0.5
\end{align*}
\]

Let \( \gamma = \{ 1, 0, \lambda \} \) and \( \Gamma = \{ 1, 0, \mu_1, \mu_2 \} \).
Then the mapping \( f : (X, \gamma) \rightarrow (Y, \Gamma) \) defined by \( f(a) = x, f(b) = y \) is fuzzy almost \( \alpha \)-open but not fuzzy almost \( \alpha \)-open.

**Example 3.2**: Let \( X = \{ a, b \} \) and \( \lambda, \mu \) be defined as,
\[
\lambda(a) = 0.7 \quad \lambda(b) = 0.8 \quad \mu(x) = 0.4 \quad \mu(y) = 0.3
\]
Let \( \gamma = \{ 1, 0, \lambda \} \) and \( \Gamma = \{ 1, 0, \mu \} \). Then the mapping \( f : (X, \gamma) \rightarrow (Y, \Gamma) \) defined by \( f(a) = x, f(b) = y \) is fuzzy almost \( \alpha \)-open but not fuzzy \( \alpha \)-open.

**Theorem 3.1**: Let \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) be two mappings. If \( f \) is fuzzy almost open and \( g \) is fuzzy \( \alpha \)-open. Then the mapping \( g \circ f : X \rightarrow Z \) is fuzzy almost \( \alpha \)-open.

**Proof**: Let \( \lambda \) be fuzzy regular open in \( X \). Then \( f(\lambda) \) is fuzzy open in \( Y \) because \( f \) is fuzzy almost open. Therefore \( g(f(\lambda)) \) is fuzzy \( \alpha \)-open in \( Z \). Because \( g \) is fuzzy \( \alpha \)-open. Since \( (g \circ f)(\lambda) = g(f(\lambda)) \), it follows that the mapping \( g \circ f \) is fuzzy almost \( \alpha \)-open.

**Theorem 3.2**: Let \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) be two mappings such that \( g \circ f : X \rightarrow Z \) is fuzzy almost \( \alpha \)-open and \( g \) is fuzzy \( \alpha \)-irresolute and injective then \( f \) is fuzzy almost \( \alpha \)-open.

**Proof**: Suppose \( \lambda \) is fuzzy regular open set in \( X \). Then \( (g \circ f)(\lambda) \) is fuzzy \( \alpha \)-open in \( Z \) because \( g \circ f \) is fuzzy almost \( \alpha \)-open. Since \( g \) is injective, we have \( g^{-1}(g(f(\lambda))) = f(\lambda) \). Therefore \( f(\lambda) \) is fuzzy \( \alpha \)-open in \( Y \), because \( g \) is fuzzy \( \alpha \)-irresolute. This implies \( f \) is fuzzy almost \( \alpha \)-open.

**Theorem 3.3**: Let \( f : X \rightarrow Y \) be fuzzy almost \( \alpha \)-open mapping. If \( \mu \) is fuzzy set of \( Y \) and \( \lambda \) is fuzzy regular closed set of \( X \) containing \( f^{-1}(\mu) \) then there is a fuzzy \( \alpha \)-closed set \( \upsilon \) of \( Y \) containing \( \mu \) such that \( f^{-1}(\upsilon) \leq \lambda \).

**Proof**: Let \( \upsilon = 1 - f(1 - \lambda) \). Since \( f^{-1}(\mu) \leq \lambda \) we have \( f(1 - \lambda) \leq 1 - \mu \). Since \( f \) is fuzzy almost \( \alpha \)-open then \( \upsilon \) is fuzzy \( \alpha \)-closed set of \( Y \) and \( f^{-1}(\upsilon) = 1 - f^{-1}(f(1 - \lambda)) \leq 1 - (1 - \lambda) = \lambda \). Thus \( f^{-1}(\upsilon) \leq \lambda \).

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REFERENCES


