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ON SOME CONTINUED FRACTIONS RELATED TO ${}_2\psi_2$ BASIC BILATERAL HYPERGEOMETRIC SERIES

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ABSTRACT

In this paper we obtain three continued fraction developments for the ratio of the basic bilateral hypergeometric series ${}_2\psi_2$ with some of its contiguous functions. Further, as special cases of these identities we generate a number of continued fraction developments including Euler continued fraction, Rogers—Ramanujan continued fraction.

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1. INTRODUCTION

Srinivasa Ramanujan has made some significant contributions to the theory of continued fraction expansions. The most beautiful and penetrating continued fraction expansions of Ramanujan can be found in chapter 12 of his second notebook [13] which is almost entirely devoted to the study of continued fraction expansions. The various q-continued fraction identities found in 'lost' notebooks of

Srinivasa Ramanujan [14] and his earlier notebooks [13] and their generalizations have been established by various authors including L. J. Rogers [15], L. Carlitz [8], G. E. Andrews [1], M. D. Hirschhorn [12], S. Bhargava and Chandrashekar Adiga [6], S. Bhargava, chandrashekar Adiga and D. D. Somashekara [7], R. Y. Denis [9] [10], Neera A. Bhagirathi [3] [5]. As usual for any complex number *a*

$$(a)_\infty = (a\,;q)_\infty = \prod_{n=0}^{\infty} (1 - aq^n), \qquad |q| < 1$$

and

$$(a)_n : = (a\,;q)_n : = \frac{(a\,;q)_\infty}{(aq^n\,;q)_\infty}, \quad n : \text{any integer.}$$

The basic bilateral hypergeometric series ${}_2\psi_2$ is defined by

$${}_2\psi_2\begin{bmatrix} a,\ b\ ;\ x \\ c,\ d \end{bmatrix} = \sum_{n=-\infty}^{\infty} \frac{(a)_n\,(b)_n}{(c)_n\,(d)_n}\,x^n, \qquad |q| < 1,\ \left|\frac{cd}{ab}\right| < |x| < 1.$$

In [4] N. A. Bhagirathi established the continued fraction identity

$$\frac{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ x \\ c,\ d \end{bmatrix}}{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ xq \\ c,\ d \end{bmatrix}} = \frac{d}{q\ +}\ \frac{x\alpha_0}{d(1-x)\,\beta_0/q\ +}\ \frac{\beta_0\gamma_0}{d/q\ +}\ \frac{x\alpha_1}{d(1-x)\beta_1/q\ +}\ \frac{\beta_1\gamma_1}{d/q\ +}\ \cdots \tag{1.1}$$

where for $i = 0, 1, 2, 3 \ldots$

$$\alpha_i = (1 - aq^i)\,(1 - bq^i),$$
$$\beta_i = (1 - aq^i)\,(1 - (1/bq^i))/((d/q) - aq^i)(1-(d/bq^{i+1}))$$

and

$$\gamma_i = abxq^{2i+1} - cdq^{i-1}$$

by making use of the following transformation due to W. N. Bailey [2] :

$$\sum_{n=-\infty}^{\infty} \frac{(a)_n\,(b)_n}{(c)_n\,(d)_n}\cdot x^n$$

$$= \frac{(bx)_\infty}{(x)_\infty} \frac{(c/b)_\infty}{(c)_\infty} \frac{(d/a)_\infty}{(q/a)_\infty} \frac{(cq/abx)_\infty}{(cd/abx)_\infty} \cdot \sum_{n=-\infty}^{\infty} \frac{(b)_n}{(bx)_n} \frac{\left(\frac{abx}{c}\right)_n}{(d)_n} \cdot \left(\frac{c}{b}\right)^n \tag{1.2}$$

In [11] R. Y. Denis established the continued fraction identity

$$\frac{{}_2\psi_2\left[\begin{matrix} a, bq ; x \\ cq, d \end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix} a, b ; x \\ c, d \end{matrix}\right]} = \frac{1}{A_0 +}\ \frac{xB_0}{C_0 +}\ \frac{xD_0}{A_1 +}\ \frac{xB_1}{C_1 +}\ \frac{xD_1}{A_2 +} \cdots \tag{1.3}$$

where for i = 0, 1, 2, 3, ...

$A_i = (1-bq^i)(cq^{2i+1} - d)/(1-cq^{2i})(bq^{i+1} - d)$,

$B_i = q^{i+1}(1-aq^i)(1-bq^i)(b-cq^i) / (1-cq^{2i+1})(1-cq^{2i})(bq^{i+1} - d)$,

$C_i = (1-aq^i)(cq^{2i+2} - d) / (1-cq^{2i+1})(aq^{i+1} - d)$

and

$D_i = q^{i+1}(1-bq^{i+1})(1-aq^i)(a-cq^{i+1}) / (1-cq^{2i+1})(1-cq^{2i+2})(aq^{i+1}-d)$.

The main purpose of this paper is to establish new continued fraction for the ratios appearing in the left hand side of (1.1) and (1.3). Also we obtain a new continued fraction for the ratio

$$\frac{{}_2\psi_2\left[\begin{matrix} a, bq ; x \\ c, d \end{matrix}\right]}{{}_2\psi_2\left[\begin{matrix} a, b ; x \\ c, d \end{matrix}\right]}.$$

In section 2 we obtain some functional relations for ${}_2\psi_2$. In section 3 we show how these functional relations lead to our continued fraction identities. In section 4 we discuss the convergence of our main continued fraction identities. In section 5 we consider several interesting special cases.

2. SOME FUNCTIONAL RELATIONS SATISFIED BY ${}_2\Psi_2$

Lemma. ${}_2\psi_2$ satisfies the following functional relations :

$$\frac{\left(\frac{d}{q}-a\right)}{(1-a)}\ {}_2\psi_2\begin{bmatrix}a,\ b\ ;\ x\\ c,\ d\end{bmatrix}-\frac{d}{q}\,{}_2\psi_2\begin{bmatrix}aq,\ b\ ;\ x\\ c,\ d\end{bmatrix}=\frac{ax(b-1)}{(1-c)}\ {}_2\psi_2\begin{bmatrix}aq,\ bq\ ;\ x\\ cd,\ d\end{bmatrix}, \tag{2.1}$$

$${}_2\psi_2\begin{bmatrix}a,\ b\ ;\ x\\ c,\ d\end{bmatrix}-\frac{(d-a)}{(q-a)}\ {}_2\psi_2\begin{bmatrix}a/q,\ b\ ;\ xq\\ c,\ d\end{bmatrix}=\frac{x\,(1-b)}{(1-c)}\ {}_2\psi_2\begin{bmatrix}a,\ bq\ ;\ x\\ cq,\ d\end{bmatrix}, \tag{2.2}$$

$$\frac{(bq-d)}{(1-b)}\ {}_2\psi_2\begin{bmatrix}a,\ b\ ;\ x\\ c,\ d\end{bmatrix}-\frac{(a-d)}{1-a/q}\ {}_2\psi_2\begin{bmatrix}a/q,\ bq\ ;\ x\\ c,\ d\end{bmatrix}=\frac{x\,(bq-a)}{(1-c)}\ {}_2\psi_2\begin{bmatrix}a,\ bq\ ;\ x\\ cq,\ d\end{bmatrix}, \tag{2.3}$$

$$\frac{(1-c)\,(bq-d)}{(1-b)}\ {}_2\psi_2\begin{bmatrix}a,\ b\ ;\ x\\ c,\ d\end{bmatrix}+(q-x)\ {}_2\psi_2\begin{bmatrix}a,\ bq\ ;\ x/q\\ cq,\ d\end{bmatrix}=(cq-ax)\ {}_2\psi_2\begin{bmatrix}a,\ bq\ ;\ x\\ cq,\ d\end{bmatrix}. \tag{2.4}$$

Proof : (2.1) follows easily from the following simple identity :

$$\frac{\left(\frac{d}{q}-a\right)}{(1-a)}\,(a)_n-\frac{d}{q}\,(aq)_n=-a\,(1-dq^{n-1})\,(aq)_{n-1}.$$

It is easy to see that

$$(a)_n - \frac{(d-a)}{(q-a)} \left(\frac{a}{q}\right)_n q^n = (a)_{n-1} (1-dq^{n-1}).$$

From this (2.2) follows. The functional relation (2.3) follows from the following :

$$(b-d)\,(a)_n\,(bq)_{n-1} - (a-d)\,(a)_{n-1}\,(bq)_n = (a)_{n-1}\,(bq)_{n-1}$$

Identity (2.4) follows immediately from (2.3) on using (1.2), then changing $\frac{abx}{c}$ to A, b to B, bx to C, d to D and $\frac{c}{b}$ to X ; and finally renaming A, B, C, D and X as a, b, c, d and x respectively.

Theorem 3.1.

$$\frac{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ xq \\ c,\ d \end{bmatrix}}{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ x \\ c,\ d \end{bmatrix}} = \frac{(1-x)}{((d/q)-bx)+}\ \frac{P_1}{Q_1+}\ \frac{P_2}{Q_2+}\ \cdots \tag{3.1}$$

where

$$P_n = x(cq^{n-1} - a)\,((d/q) - bq^{n-1}), \qquad n = 1, 2, 3, \ldots$$

and

$$Q_n = (d/q) + ax - q^{n-1}(bxq + c), \qquad n = 1, 2, 3, \ldots$$

Proof : Setting b to bq, c to cq in (2.2) and multiplying the resulting equation by (ax – cq) ; putting a to a/q, b to bq, c to cq in (2.1) and multiplying the resulting equation by (xq – q) ; putting a to a/q, x to xq in (2.4) and multiplying the resulting equation by {(d – a)/(q – a)} ; multiply (2.4) by (– 1) ; multiply (2.2) {(1 – c) (bq – d) / (1 – b)} now by adding all these equations we get,

$$(1-(x/q))\ {}_2\psi_2\begin{bmatrix} a,\ bq\ ;\ x/q \\ cq,\ d \end{bmatrix} = ((d/q)-bx)\ {}_2\psi_2\begin{bmatrix} a,\ bq\ ;\ x \\ cd,\ d \end{bmatrix}$$

$$+\ \frac{x(bq-1)\,(a-cq)}{q(1-cq)}\ {}_2\psi_2\begin{bmatrix} a,\ bq^2\ ;\ x \\ cq^2,\ d \end{bmatrix} \tag{3.2}$$

Setting x to xq in (3.2) and then multiply the resulting equation by (– q) ; change x to xq in (2.4) ; adding these two equations we get

$$\frac{(1-c)\,((d/q)-b)}{(1-b)}\ {}_2\psi_2\begin{bmatrix} a,\ b\ ;\ xq \\ c,\ d \end{bmatrix} = ((d/q)-c+ax-bxq)\ {}_2\psi_2\begin{bmatrix} a,\ bq\ ;\ xq \\ cq,\ d \end{bmatrix}$$

$$+\ \frac{x(bq-1)\,(a-cq)}{(1-cq)}\ {}_2\psi_2\begin{bmatrix} a,\ bq^2\ ;\ xq \\ cq^2,\ d \end{bmatrix} \tag{3.3}$$

Replacing b by b/q, c by c/q and x by xq, the equation (3.2) can be written as

$$\frac{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ xq \\ c,\ d \end{bmatrix}}{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ x \\ c,\ d \end{bmatrix}} = \frac{(1-x)}{((d/q)-bx)\ +}\ \frac{x(1-b)\,(c-a)}{(1-c)\ \dfrac{{}_2\psi_2\begin{bmatrix} a,\ b\ ;\ xq \\ c,\ d \end{bmatrix}}{{}_2\psi_2\begin{bmatrix} a,\ bq\ ;\ xq \\ cq,\ d \end{bmatrix}}} \tag{3.4}$$

Changing b to bq^{n-1}, c to cq^{n-1} in (3.3) we get,

$$Y_n = \frac{(1-cq^{n-1})\,((d/q)-bq^{n-1})}{(1-bq^{n-1})}\ \frac{{}_2\psi_2\begin{bmatrix} a,\ bq^{n-1}\ ;\ xq \\ cq^{n-1},\ d \end{bmatrix}}{{}_2\psi_2\begin{bmatrix} a,\ bq^{n}\ ;\ xq \\ cq^{n},\ d \end{bmatrix}}$$

$$= Q_n + \frac{P_{n+1}}{Y_{n+1}} \tag{3.5}$$

where P_n and Q_n are as in (3.1). Iterating (3.5) with n = 1, 2, 3, ... and on using (3.4) we get the required result.

Theorem 3.2. If $d = q^m$ (m = 1, 2, 3, ...) then

$$\frac{{}_2\Psi_2\left[\begin{matrix} a,\ bq\ ;\ x \\ c,\ d \end{matrix}\right]}{{}_2\Psi_2\left[\begin{matrix} a,\ b\ ;\ x \\ c,\ d \end{matrix}\right]} = \frac{\{((d/q)-b)\,(1-(c/bq))/1-b}{G_0 +}\ \frac{H_0}{G_1+}\ \frac{H_1}{G_2+}\cdots \tag{3.6}$$

where

$$G_n = (d + c + ax - bxq^{n+1})\, q^{n-1} - b^{-1}\, dcq^{-3}\, (q + 1)$$

and

$$H_n = b^{-2}\, q^{-4}\, (cd - abxq^{n+2})\, ((d/q) - bq^{n+1})\, (bq^{n+2} - c).$$

Proof : Putting a to aq, b to bq, c to cq, x to x/q in (2.2) and multiply the resulting equation by $\frac{d(a-1)\,(q-x)}{(d-aq)\,(1-c)}$; putting b to bq, c to cq and x to x/q in (2.1) and multiply the resulting equation by $\frac{(aq-q)\,(q-x)}{(d-aq)\,(1-a)}$; multiply (2.2) by $\frac{(d-bq)}{(1-b)}$; multiply (2.4) by $(1-c)^{-1}$; add all these resulting identities to get the following :

$$\frac{(1-c)\,(d-bq)\,(d-a)}{(1-b)\,(q-a)}\ {}_2\Psi_2\left[\begin{matrix} a/q,\ b\ ;\ xq \\ c,\ d \end{matrix}\right]$$

$$= \left[d(1-(x/q)) + x(bq-d) + (ax-cq)\right]\ {}_2\Psi_2\left[\begin{matrix} a,\ bq^2\ ;\ x \\ cq^2,\ d \end{matrix}\right]$$

$$+\frac{(1-a)(1-bq)(1-(x/q))}{(1-cq)}\;{}_2\Psi_2\begin{bmatrix}aq,\ bq^2\ ;\ x/q\\ cq^2,\ d\end{bmatrix} \tag{3.7}$$

Setting a to aq in (3.7) and applying [1.2] for each ${}_2\Psi_2$ appearing in the resulting equation and then setting abxq/c to A, b to B, bxq to C, d to D, c/b to X and finally replacing A, B, C, D and X by a, b, c, d and x respectively we get

$$\frac{((d/q)-b)\,(1-(c/bq))}{(1-b)}\;{}_2\Psi_2\begin{bmatrix}a,\ b\ ;\ x\\ c,\ d\end{bmatrix}$$

$$=\left[(1-(c/bq^2))(d/q)+(c/bq^2)(bq-d)+((ax/q-bx)\right]\;{}_2\Psi_2\begin{bmatrix}a,\ bq\ ;\ x\\ c,\ d\end{bmatrix}$$

$$+\frac{(1-bq)\,(cq-abxq^2)}{bq^3}\;{}_2\Psi_2\begin{bmatrix}a,\ bq^2\ ;\ x\\ c,\ d\end{bmatrix} \tag{3.8}$$

Now changing b to bq^n in (3.8) and multiplying the resulting identity by q^n we get

$$W_n=\frac{((d/q)-bq^n)\,(q^n-(c/bq))}{(1-bq^n)}\;\frac{{}_2\Psi_2\begin{bmatrix}a,\ bq^n\ ;\ x\\ c,\ d\end{bmatrix}}{{}_2\Psi_2\begin{bmatrix}a,\ bq^{n+1}\ ;\ x\\ c,\ d\end{bmatrix}}=G_n+\frac{H_n}{W_{n+1}} \tag{3.9}$$

where G_n and H_n are as in (3.6). Iterating (3.9) with n = 0, 1, 2, 3, ... and taking the reciprocal we get (3.6).

Theorem 3.3.

$$\frac{{}_2\Psi_2\begin{bmatrix}a,\ bq\ ;\ x\\ cq,\ d\end{bmatrix}}{{}_2\Psi_2\begin{bmatrix}a,\ b\ ;\ x\\ c,\ d\end{bmatrix}}=\frac{\{((d/q)-b)\,(1-c)/(1-b)\}}{J_1+}\;\frac{K_1}{J_2+}\;\frac{K_2}{J_3+\ \ldots} \tag{3.10}$$

where

$$J_n = \{(d/q) + (ax/q) - cq^{n-1} - bxq^{n-1}\} \qquad n = 1, 2, 3, \ldots$$

and

$$K_n = (x/q)(cq^n - a)((d/q) - bq^n) \qquad n = 1, 2, 3, \ldots$$

Proof : (3.10) follows easily from (3.3)

4. CONVERGENCE OF THE MAIN THEOREMS

It follows from the theorem 10.1 of H.S. Wall [16] that the continued fraction appearing in (3.1) converges uniformly in the region $|adxq/(d+axq)^2| < 1/8$ and $|q| < 1$. But left hand side of (3.1) converges if $|cd/ab| < |x| < 1$ and $|q| < 1$. Therefore theorem 3.1 is valid in the region common to $|adxq/(d+axq)^2| < 1/8$, $|cd/ab| < |x| < 1$ and $|q| < 1$.

In the same way one can discuss the convergence of theorem 3.2 and 3.3.

5. SOME SPECIAL CASES

If we set $d = q$, $c = 0$, $x = q/ab$ in theorem 3.1 or 3.2 or 3.3 and then letting $a \to \infty$ and $b \to \infty$ we get Rogers-Ramanujan continued fraction identity :

$$\frac{\sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q)_n}}{\sum_{n=0}^{\infty} \frac{q^{n^2}}{(q)_n}} = \frac{1}{1+}\,\frac{q}{1+}\,\frac{q^2}{1+}\,\frac{q^3}{1+}\cdots$$

It we set $d = q$, $c = -cq$, $x = xq/ab$ in theorem 3.1 and 3.2 and then letting $a \to \infty$ and $b \to \infty$ we get respectively

$$\frac{\displaystyle\sum_{n=0}^{\infty} \frac{q^{n(n+1)} x^n}{(-cq)_n (q)_n}}{\displaystyle\sum_{n=0}^{\infty} \frac{q^{n^2} x^n}{(-cq)_n (q)_n}} = \frac{1}{1+} \frac{xq}{(1+cq)+} \frac{xq^2}{(1+cq^2)+} \frac{xq^3}{(1+cq^3)+} \cdots \tag{5.1}$$

$$= \frac{1}{(1-c)+} \frac{c+xq}{(1-c)+} \frac{c+xq^2}{(1-c)+} \cdots \tag{5.2}$$

These continued fractions are due to Ramanujan and can be found in his 'lost' notebook [14]. In (5.1) or (5.2) if we set c= 0, $x = x^{-2} q^{-1/2}$, then changing q to q^2 and after some simplifications we get

$$\frac{\displaystyle\sum_{n=0}^{\infty} \frac{q^{n(2n+1)} x^{-2n}}{(q^2 ; q^2)_n}}{\displaystyle\sum_{n=0}^{\infty} \frac{q^{2n^2-n} x^{-2n}}{(q^2 ; q^2)_n}} = \frac{x}{x+} \frac{q}{x+} \frac{q^3}{x+} \frac{q^5}{x+} \cdots$$

In theorem 3.2 or 3.3 put d = q then b = 1, c = 0, x = xq/a and then letting a to ∞ we get

$$\sum_{n=0}^{\infty} (-x)^n q^{n(n+1)/2} = \frac{1}{(1+x)+} \frac{x(q^2-q)}{(1+x)q+} \frac{x(q^5-q^3)}{(1+x)q^2+} \cdots$$

If we set d = q and then $a = -\lambda/a$, b = c, c = –bq, x = –aq/c in theorem 3.2 then letting c to ∞ we get

$$\frac{\displaystyle\sum_{n=0}^{\infty} \frac{q^{n(n+1)/2} (-\lambda/a)_n (aq)^n}{(-bq)_n (q)_n}}{\displaystyle\sum_{n=0}^{\infty} \frac{q^{n(n+1)/2} (-\lambda/a)_n (a)^n}{(-bq)_n (q)_n}} = \frac{1}{M_0+} \frac{N_0}{M_1+} \frac{N_1}{M_2+} \cdots$$

where $M_n = (1 - b + aq^{n+1})$ and $N_n = b + \lambda q^{n+1}$.

If we set d = q and then b = 1, $a = q^{-1/3}$, $c = q^{-2/3}$ and $x = q^{2/3}$ in theorem 3.3 and then changing q to q^3 we get

$$\frac{(q^2\,;\,q^3)}{(q\,;\,q^3)} = \frac{1-q^{-2}}{J_1' +}\;\frac{K_1'}{J_2' +}\;\frac{K_2'}{J_3' + \cdots}$$

where $K_n' = q^{-2}(q^{3n-1}-1)(1-q^{3n})$ and $J_n' = 1+q^{-2}-q^{3n-5}-q^{3n-1}$

If we set d = q and then b = 1, a = q^a, c = q^c in theorem 3.3 and then letting q to 1 and replacing x by z/a letting a to ∞ we get

$$\sum_{k=0}^{\infty} \frac{z^n}{(c+1)_k} = \frac{-c}{J_1'' +}\;\frac{K_1''}{J_2'' +}\;\frac{K_2''}{J_3'' + \cdots}$$

where $K_n'' = nz$, $J_n'' = z-c-n+1$ and $(c)_k = c(c+1)(c+2)\ldots(c+k-1)$.

If we set $a = q^{-a}$, $b = q^b$, $c = q^c$, $d = q$ and $x = -x$ in theorem 3.3 and then letting $q \to 1$ we get Euler's continued fraction :

$$\frac{\displaystyle\sum_{k=0}^{\infty} \frac{(-a)_k\,(b+1)_k}{(c+1)_k\,(k!)}(-x)^k}{\displaystyle\sum_{k=0}^{\infty} \frac{(-a)_k\,(b)_k}{(c)_k\,(k!)}(-x)^k} = \frac{bx}{c-(a+b+1)x +}$$

$$\frac{(b+1)(a+c+1)x}{c+1-(a+b+2)x +}\;\frac{(b+2)(a+c+2)x}{(c+2)-(a+b+3)x + \cdots}$$

where $(a)_k = a(a+1)(a+2)\ldots(a+k-1)$.

In theorem 3.2 if we put d = q and then b = 1, then change a to q^a, c to q^{c+1} and then letting q to 1 and then changing x to x/q and further letting a to ∞ we get

$$\sum_{k=0}^{\infty} \frac{x^k}{(c+1)_k} = \frac{-c}{G_0 +}\;\frac{H_0}{G_1 +}\;\frac{H_1}{G_2 + \cdots}$$

where

$$G_n = 2n + 1 + x - c$$

$$H_n = -(n+1)(n+1-c)$$

and

$$(c+1)_k = (c+1)(c+2)(c+3)\ldots(c+k).$$

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