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NULL CONTROLLABILITY OF NONLINEAR LARGE-SCALE NEUTRAL SYSTEMS

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ABSTRACT

Sufficient conditions for the null controllability of nonlinear uncertain large-scale neutral system are developed. Namely, if the uncontrolled system is uniformly asymptotically stable and if the linear systems is controllable, then the nonlinear large-scale neutral delay system is null controllable. An application is provided to explain the result.

Key Words : Null controllability, large-scale neutral system, fixed point theorem.

AMS (MOS) Subject Classification : 93B05

1. INTRODUCTION

A differential equation in which the expression for $\dot{x}(t)$ involves $\dot{x}(g(t))$ for some $g(t) < t$ is said to be of neutral type. For an arbitrary function D(t, x(g(t)) if the differential equation contains d/dt[x(t)–D(t, x(g(t))], it is said to be uncertain nuetral system. Instead of x(t) and g(t) if it is $x_i(t)$ and the changeable delays

$g_i(t) < t$ for $i = 1, 2, \ldots N$, the differential equation is said to be uncertain large-scale neutral system. If x is a function on some set which includes [t–r, t], the new function x_t on [-r, 0] is defined by $x_t(s) = x(t+s)$ for $-r \leq s \leq 0$ (see [4]). By using the new function, Chukwu [2] showed that if the linear delay system

$$\dot{x}(t) = L(t, x_t) \tag{1}$$

is uniformly asymptotically stable and

$$\dot{x}(t) = L(t, x_t) + B(t)u(t) \tag{2}$$

is proper, then

$$\dot{x}(t) = L(t, x_t) + B(t)u(t) + f(t, x_t, u(t)) \tag{3}$$

is Euclidean null controllable provided f satisfies certain growth and continuity conditions and $x(t) = \phi(t)$ on $t \in [-r, 0]$. He also developed conditions in [3] that if (1) is uniformly asymptotically stable and (2) is function space controllable, then (3) is function space controllable with constraints. Underwood and Young [13] proved that if the linear approximation (2) to the system

$$\dot{x}(t) = f(t, x_t, u(t)) \tag{4}$$

is function space controllable, then (4) is function space locally null controllable under certain conditions on f.

Sinha [12] developed sufficient conditions for the null controllability of the infinite delay system

$$\dot{x}(t) = L(t, x_t) + B(t)u(t) + \int_{-\infty}^{0} A(s)x(t+s)\, ds + f(t, x(\cdot), u(\cdot))$$

$$x(t) = \phi(t), \qquad t \in (-\infty, 0]$$

where $L(t, \phi)$ is continuous in t, linear in ϕ with constant delays $r_k \geq 0$ and given by

$$L(t, \phi) = \sum_{k=1}^{N} A_k(t)\, \phi(-r_k)$$

Khambadkone [8] derived sufficient conditions for Euclidean null controllability of linear systems with distributed delays in control. Recently Balachandran and Dauer [1] extended this to the nonlinear infinite delay systems with distributed delays in control. In this paper, we obtain several sufficient conditions for the controllability of the following nonlinear large-scale neutral delay systems. The considered system has been encountered as a mathematical model of the transmission time in triode oscillator, compartmental models and many other physical phenomena for more details see [5, 6].

2. PRELIMINARIES

Definition : 2.1 **[9]** : The transition matrix $\Phi_i(t, s)$ of the system $\dot{x}_i(t) = A_i(t)x_i(t)$ is defined for each t, s in $(-\infty, \infty)$ and has the following properties

(i) $\Phi_i(t, t) = I$, an identity matrix of appropriate dimension of A_i

(ii) $\Phi_i^{-1}(t, s) = \Phi_i(s, t)$

(iii) $\Phi_i(t_0, t_1)\,\Phi_i(t_1, t_2) = \Phi_i(t_0, t_2)$ for $t_0, t_1, t_2 \in (-\infty, \infty)$

(iv) $\partial/\partial t\,[\Phi_i(t, s)] = A_i(t)\Phi_i(t, s)$.

Definition : 2.2 **[11]** : Let A be a Banach algebra and $x \in A$, the spectrum $\sigma(x)$ of x is the set of all complex numbers $\gamma \in C$ such that $(\gamma I - x)$ is not invertible. The complement of $\sigma(x)$ is the resolvent set of x it consists of all γ for which $(\gamma I - x)^{-1}$ exists. The spectral radius of x is the number

$$\rho(x) = \sup\{\,|\gamma| : \gamma \in \sigma(x)\,\}.$$

It is the radius of the smallest closed circular disc in C, with center at 0 which contains $\sigma(x)$.

Consider the nonlinear large-scale neutral system of the form

$$\frac{d}{dt}[x_i(t) - D_i(t, x_{it})] = A_i(t)x_i(t) + B_j(t)u_j(t) + f_i(t, x_{it}, u_j(t)), \quad t \in J = [0, T] \tag{5}$$

$$x_i(t) = \phi(t), \qquad t \in [-r, 0]$$

where the function x_{it} on $[-r, 0]$ is defined by $x_{it}(s) = x_i(t+s)$ for $-r \le s \le 0$, $x_i(t) \in R^{n_i}$ $(i = 1, 2, \ldots, N)$ such that $\sum_{i=1}^{N} n_i = n$, $u_j(t) \in R^{m_j}$ $(j = 1, 2, \ldots, M)$ such that $\sum_{j=1}^{M} m_j = m$ and $A_i(t)$, $B_j(t)$ are respectively $n_i \times n_i$, $n_i \times m_j$ continuous matrix valued functions, $D_i : R \times C^{n_i} \to R^{n_i}$ and $f_i : R \times C^{n_i} \times R^{m_j} \to R^{n_i}$ are sufficiently smooth functions. Then for any constant initial function $\phi(t) = \phi$ defined for $t \in [-r, 0]$ and the transition matrix $\Phi_i(t, s)$ for $t, s \in J$ equation (5) has unique continuous solution (see [14])

$$x_i(t) = D_i(t, x_{it}) + \Phi_i(t, t-t_1)[x_i(t-t_1) - D_i(t-t_1, x_{it-t_1})]$$

$$+ \int_{t-t_1}^{t} \Phi_i(t, s)[A_i(s) D_i(s, x_{is}) + B_j(s)u_j(s) + f_i(s, x_{is}, u_j(s))]\, ds, \; t \in J \tag{6}$$

$$x_i(t) = \phi(t), \qquad t \in [-r, 0]$$

Here $C^{n_i} = C([-r, 0], R^{n_i})$ denotes the Banach space of continuous functions mapping from the interval $[-r, 0]$ to R^{n_i}. For $\phi \in C^{n_i}$, we define

$$\| \phi \|_r = \max_{-r \leq \tau \leq 0} \| \phi(\tau) \|$$

where $\| \cdot \|$ is any convenient norm in R^{n_i}, and x_{it} is an element of C^{n_i} and $\| x_{it} \|_r = \max_{-r \leq \tau \leq 0} \| x_i(t + \tau) \|$. Note that

$$\int_{t-t_1}^{t} d(t, s)ds = \int_{0}^{t_1} d(t, t-s)ds.$$

Let $L_2(J, U)$ be a Banach space of squared integrable functions from an interval J into U equipped with the L_2 norm where U is a compact convex subset of E^{m_j}.

Definition 2.3 : The system (5) is said to be **null controllable** on the interval J if, for every continuous function $\phi \in C^{n_i}$ there exists a control $u_j \in L_2(J, U)$, U a compact convex subset of E^{m_j} such that the solution $x_i(t)$ of (5) satisfies $x_i(0) = \phi$, $x_i(T) = 0$.

Theorem 2.1 : The linear system

$$\frac{d}{dt}[x_i(t) - D_i(t, x_t)] = A_i(t)\, x_i(t) + B_j(t)\, u_j(t) \tag{7}$$

is null controllable on J if and only if the controllability Grammian

$$W(t_1, T) = \int_{T-t_1}^{T} \Phi_i(T, s)B_j(s)B_j^*(s)\, \Phi_i^*(T, s)\, ds$$

is positive definite for every $T \geq t_1$.

Proof : Assume that the controllability Grammian matrix W is positive definite, so that W is nonsingular (see [10]), hence W^{-1} exists and using it define the control function u_j on J such that

$$u_j(t) = -B_j^*(t)\,\Phi_i^*(T, t)\,W^{-1}\Big[D_i(T, x_{iT}) + \Phi_i(T, T-t_1)\,\{x_i(T-t_1) - D_i(T-t_1, x_{iT-t_1})\} + \int_{T-t_1}^{T} \Phi_i(T, s)\,A_i(s)\,D_i(s, x_{is})\,ds\Big]. \tag{8}$$

The solution of (7) is given by

$$x_i(t) = D_i(t, x_{it}) + \Phi_i(t, t-t_1)\,[x_i(t-t_1) - D_i(t-t_1, x_{it-t_1})] + \int_{t-t_1}^{t} \Phi_i(t, s)\,[A_i(s)\,D_i(s, x_{is}) + B_j(s)\,u_j(s)]ds, \quad t \in J$$

$$x_i(t) = \phi(t), \qquad t \in [-r, 0].$$

Using the control (8) in the above solution, it is seen that $x_i(T) = 0$, $t \in J$ and $x_i(0) = \phi$, the system (7) is null controllable on J.

Conversely, assume that the system (7) is null controllable on J, we must prove that W is positive definite. To prove this, it is enough to prove if W is not positive definite then we have a contradiction. Hence let W is not positive definite then there exists a vector $y \neq 0$ such that $y^*Wy = 0$. If follows that

$$\int_{T-t_1}^{T} y^*\Phi_i(T, s)\,B_j(s)\,B_j^*(s)\,\Phi_i^*(T, s)y\,ds = 0$$

that is

$$\int_{T-t_1}^{T} y^*\Phi_i(T, s)\,B_j(s)\,\big(y^*\Phi_i(T, s)\,B_j(s)\big)^*\,ds = 0$$

Therefore $y^*\Phi_i(T, s)\,B_j(s) = 0$ for $s \in J$ and $T \geq t_1$. (9)

But the system is null controllable, there exists a control u_j such that $x_i(T) = 0$.

That is

$$D_i(T, x_{iT}) + \Phi_i(T, T-t_1)\,[x_i(T-t_1) - D_i(T-t_1, x_{iT-t_1})]$$

$$+ \int_{T-t_1}^{T} \Phi_i(T, s)\,[A_i(s)\,D_i(s, x_{is}) + B_j(s)u_j(s)]\,ds = 0$$

But D_i, Φ_i, A_i, B_j are nonzero, we have from the above equation that the first and third terms are non zero. Therefore for $y^* \neq 0$ we have

$$y^*\,\Phi_i(T, T-t_1)\,x_i(T-t_1) + \int_{T-t_1}^{T} y^*\,\Phi_i(T, s)[A_i(s)\,D_i(s, x_{is}) + B_j(s)u_j(s)\,]\,ds = 0$$

using (9) and the nonzero Φ_i, A_i, D_i the only possibility is

$$y^*\,\Phi_i(T, T-t_1)\,x_i(T-t_1) = 0,$$

which implies $y^* = 0$, which is a contradiction to the fact that $y \neq 0$. Hence W must be positive definite.

In this paper, we shall study the null controllability of nonlinear large-scale neutral system (5). For that let us assume the following conditions

(i) the free system

$$\frac{d}{dt}[x_i(t) - D_i(t, x_{it})] = A_i(t)\,x_i(t)$$

is asymptotically stable, so that the following are satisfied (see from Xu [14]). The transition matrix $\Phi_i(t, s)$ for $t, s \in J$ of the system

$$\frac{d}{dt}x_i(t) = A_i(t)\,x_i(t)$$

satisfies

$$\|\,\Phi_i(t, s)\,\| \leq H_i \exp(-\alpha_i \times [t-s]),\ i = 1, 2, \ldots N, \tag{10}$$

where H_i, α_i are positive constants, and

$$\|\,D_i(t, x_{it})\,\| \leq \sum_{k=1}^{N} c_{ik}\,\|\,x_{kt}\,\|_r \tag{11}$$

$$\| A_i(t)D_i(t, x_{it}) \| \leq \sum_{k=1}^{N} d_{ik} \| x_{kt} \|_r \tag{12}$$

where c_{ik} and d_{ik} are nonnegative constants and

$$\rho(c_{ik} + d_{ik} \int_0^{\infty} \| \Phi_i(t, t-s) \| ds) < 1,$$

$\rho(\eta_{ik})$ denotes the spectral radius of $\eta_{ik} = c_{ik} + d_{ik} \int_0^{\infty} \| \Phi_i(t, t-s) \| ds.$

ii) the linear system (7) is controllable

iii) the continuous function f satisfies

$|f_i(t, x_{it}, u_j(t))| \leq \exp(-\beta_i t)\pi(x_i(\cdot), u_j(\cdot))$ for all $(t, x_{it}, u_j(t)) \in J \times C^{n_i} \times L_2(J, U)$

where $\int_0^{\infty} \pi(x_i(\cdot), u_j(\cdot))ds \leq K_{il} < \infty$ and $\beta_i \geq 0$.

3. MAIN RESULT

Theorem 3. 1 : Suppose that the conditions (i) - (iii) are satisfied. Then (5) is null controllable on J.

Proof : Since the system (7) is null controllable on J, $W^{-1}(t_1, T)$ exists for each $T \geq t_1$. Suppose the pair of functions x_i, u_j form a solution pair to the set of integral equations

$$u_j(t) = -B_j^*(t)\, \Phi_i^*(T, t)\, W^{-1} \Big[D_i(T, x_{iT}) + \Phi_i(T, T-t_1) \left\{ x_i(T-t_1) - D_i(T-t_1, x_{iT-t_1}) \right\} + \int_{T-t_1}^{T} \Phi_i(T, s) \left\{ A_i(s)\, D_i(s, x_{is}) + f(s, x_{is}, u_j(s)) \right\} ds \Big]. \tag{13}$$

$$x_i(t) = D_i(t, x_{it}) + \Phi_i(t, t-t_1) \left[x_i(t-t_1) - D_i(t-t_1, x_{it-t_1}) \right] + \int_{t-t_1}^{t} \Phi_i(t, s) \left[A_i(s)\, D_i(s, x_{is}) + B_j(s)u_j(s) + f_i(s, x_{is}, u_j(s)) \right] ds, \quad t \in J$$

$$x_i(t) = \phi(t), \qquad t \in [-r, 0] \tag{14}$$

Hence using the control (13) into (14) for $t \geq t_1$ we have $x_i(T) = 0$, $t \in J$ and $x_i(0) = \phi$. We now show that $u_j : J \to U$ is an aribtrary compact constraint subset of E^{m_j} that is $\| u_j \| \leq a$ for some constant $a > 0$. For that let

$$\| B_j(t)\, \Phi_i(T, t)\, W^{-1} \| \leq K_{j2}$$

then

$$\| u_j(t) \| \leq K_{j2} \Big[\| D_i(T, x_{iT}) + \Phi_i(T, T-t_1) \{ x_i(T-t_1) - D_i(T-t_1, x_{iT-t_1}) \} \|$$

$$+ \int_{T-t_1}^{T} \| \Phi_i(T, s) \{ A_i(s) D_i(s, x_{is}) + f(s, x_{is}, u_j(s)) \} \| ds \Big]$$

The first and fourth terms are combined and using (11) and (12) the terms are

$$K_{j2} \Big[\| D_i(T, x_{iT}) \| + \int_{T-t_1}^{T} \| \Phi_i(T, s) A_i(s) D_i(s, x_{is}) \| ds \Big]$$

$$\leq k_{j2} \sum_{k=1}^{N} \Big[c_{ik} + d_{ik} \int_0^{t_1} \| \Phi_i(T, T-s) \| ds \Big] \| x_{kT} \|_r$$

Further using the Kronecker delta function δ_{ik}, the above inequality and

$$\max \{ \| x_{kT-t_1} \|_r, \| x_{kT} \|_r \} = \| x_{kT} \|_{t_1 + r}, \text{ we get}$$

$$\| u_j(t) \| \leq K_{j2} \Big[\sum_{k=1}^{N} \Big\{ c_{ik} + d_{ik} \int_0^{t_1} \| \Phi_i(T, T-s) \| ds$$

$$+ H_i \exp(-\alpha_i t_1) (\delta_{ik} + c_{ik}) \Big\} \| x_{kT} \|_{t_1+r}$$

$$+ \int_0^{t_1} \| \Phi_i(T, T-s) f(s, x_{is}, u_j(s)) \| ds \Big]$$

$$\leq K_{j2} \Big[\sum_{k=1}^{N} \{ \eta_{ik} + H_i \exp(-\alpha_i t_1) (\delta_{ik} + c_{ik}) \} \| x_{kT} \|_{t_1+r}$$

$$+ K_{j2} \int_0^{t_1} H_i \exp(-\alpha_i s) \exp(-\beta_i s)\, \pi(x_i(\cdot), u_j(\cdot)) ds \Big]$$

$$\leq \quad K_{j2} \sum_{k=1}^{N} \left\{ \eta_{ik} + H_i \exp(-\alpha_i t_1)\,(\delta_{ik} + c_{ik}) \right\} \| x_{kT} \|_{t_1+r}$$

$$+ K_{j2} \int_0^{t_1} H_i \exp[-(\beta_i+\alpha_i)s]\, \pi(x_i(\cdot), u_j(\cdot))ds \tag{15}$$

$$\leq \quad K_{j2} \sum_{k=1}^{N} \eta_{ik} \| x_{kT} \|_{t_1+r} + K_{i1}\, K_{j2}\, H_i \exp(-\alpha_i T)$$

$$\leq \quad a \tag{16}$$

Since $\rho(\eta_{ik}) < 1$, the first term is finite, δ_{ik} is the Kronecker delta we see that from equation (15), second term vanishes and third term is finite for large $T \geq t_1$, $\exp[-(\beta_i+\alpha_i)T] \leq \exp[-\alpha_i T]$, $\beta_i \geq 0$. For the choice of t_1 and T, $\| u_j(t) \| \leq a$, $t \in J$ implies that u_j is an admissible control. It remains to prove that the existence of a solution pair of the integral equations (13) and (14). Let B be the Banach space of all function $(x_i, u_j) : [-r, T] \times J \rightarrow E^{n_i} \times E^{m_j}$. Define an operator $F : \boldsymbol{B} \rightarrow \boldsymbol{B}$ by

$$F(x_i, u_j) \rightarrow (z_i, v_j)$$

where

$$v_j(t) = -B_j^*(t)\, \Phi_i^*(T, t)\, W^{-1} \Big[D_i(T, x_{iT}) + \Phi_i(T, T-t_1) \Big\{ x_i(T-t_1) - D_i(T-t_1, x_{iT-t_1}) \Big\} + \int_{T-t_1}^{T} \Phi_i(T, s) \Big\{ A_i(s)\, D_i(s, x_{is}) + f(s, x_{is}, u_j(s)) \Big\} ds \Big]$$

$$z_i(t) = D_i(t, x_{it}) + \Phi_i(t, t-t_1)\,[x_i(t-t_1) - D_i(t-t_1, x_{it-t_1})] + \int_{t-t_1}^{t} \Phi_i(t, s)\,[A_i(s)\, D_i(s, x_s) + B_j(s)u_j(s) + f_i(s, x_{is}, u_j(s))\,]\,ds, \quad t \geq t_1$$

$$z_i(t) = \phi(t) \qquad t \in [-r, 0]$$

From (13) it is clear that $\| v_j(t) \| \leq a$, $t \in J$. Next

$$\| z_i(t) \| \leq \sum_{k=1}^{N} c_{ik} \| x_{kt} \|_r + H_i \exp(-\alpha_i t_1) \left\{ \| x_i(t-t_1) \| + \sum_{k=1}^{N} c_{ik} \| x_{kt-t_1} \|_r \right\}$$

$$+ \int_{t-t_1}^{t} \sum_{k=1}^{N} d_{ik} \| x_{kt} \|_r \| \Phi_i(t, s) \| ds + \int_{t-t_1}^{t} \| \Phi_i(t, s) \| \| B_j(s) \| \| u_j(s) \| ds$$

$$+ \int_{t-t_1}^{t} \| \Phi_i(t, s) \| \left\| f(s, x_{is}, u_j(s)) \right\| ds$$

$$\leq \sum_{k=1}^{N} \left\{ c_{ik} + d_{ik} \int_{0}^{t_1} \| \Phi_i(t, t-s) \| ds + H_i \exp(-\alpha_i t_1) (c_{ik} + \delta_{ik}) \right\} \| x_{kT} \|_{t_1+r}$$

$$+ \int_{0}^{t_1} \|\Phi_i(t, t-s) \| \| B_j(s) \| \| u_j(s) \| ds$$

$$+ \int_{0}^{t_1} \| \Phi_i(t, t-s) \| \| f(t-s, x_{it-s}, u_j(t-s))\|ds$$

$$\leq \sum_{k=1}^{N} \eta_{ik} \| x_{kT} \|_{t_1+r} + \exp(-\alpha_i t_1) a t_1 \|B_j(s)\| + K_{i1} H_i \exp[-(\beta_i+\alpha_i)T]$$

For large t_1 and T, we have

$$\| z_i(t) \| \leq c_2 + c_3 + K_{i1} H_i = \gamma_i \tag{17}$$

where $c_2 = \sum_{k=1}^{N} \eta_{ik} \|x_{kT}\|_{t_1+r}$ and $c_3 = a\, t_1 \sup_{t \in J} \| B_j(t) \|$

Further, $\| z_i(t) \| \leq \sup \| \phi(t) \| \equiv \delta,\ t \in [-r, 0]$.

Let $h = \max \{\gamma_i, \delta\}$, $i = 1, 2, \ldots N$. Define the set

$$Q = \left\{ (x_i, u_j) : \| x_i \| \leq h, \| u_j \| \leq h \right\}.$$

Then from the inqualities (16) and (17) we see that F maps Q into itself. Further Q is closed, bounded and convex, by Riez's theorem [7], it is relatively compact under F. Hence by the Schauder fixed point theorem F has a fixed point

$(x_i, u_j) \in Q$. This fixed point (x_i, u_j) of F is a solution pair of the set of integral equations (13), (14). Thus the system (5) is null controllable on J.

4. EXAMPLE

Consider the system (5) with

$$D_i(t, x_{it}) = \frac{1}{4}\begin{bmatrix} 0 \\ x_2(t-1) \end{bmatrix} \quad A_i(t) = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \quad B_i(t) = \begin{bmatrix} 1 \\ t \end{bmatrix}$$

$$f_i(t, x_{it}, u_j) = \begin{bmatrix} \varepsilon\cos x_2(t-1) + \exp(-t)\, u_1(t) \\ \varepsilon\sin x_2(t-1) + \exp(-t)\, u_2(t) \end{bmatrix}$$

then

$$\Phi_i(t, s) = \Phi_i(t-s) = \exp A_i(t-s) = \begin{bmatrix} \exp(s-t) & 0 \\ (t-s)\exp(s-t) & \exp(s-t) \end{bmatrix}$$

it is easy to verify that it satisfies all the properties of Definition 2.1 hence $\Phi_i(t-s)$ is a transition matrix. Also

$$\Phi_i(t_1, t_1-s) = \Phi_i(s) = \begin{bmatrix} \exp(-s) & 0 \\ s\exp(-s) & \exp(-s) \end{bmatrix}$$

Now

$$\Phi_i(s)B_j(s) = \begin{bmatrix} \exp(-s) \\ 2s\exp(-s) \end{bmatrix}$$

$$\Phi_i(s)B_j(s)B_j^*(s)\Phi_i^*(s) = \begin{bmatrix} \exp(-2s) & 2s\exp(-2s) \\ 2s\exp(-2s) & 4s^2\exp(-2s) \end{bmatrix} \tag{18}$$

$$\| \Phi_i(t-s) \| \le (1+t-s)\exp[-(t-s)]$$

$$\| D_i(t, x_{it}) \| \le (1/4)\, \| x_2(t-1) \|$$

and

$$\| \Phi_i(t-s)\, A_i(t)\, D_i(t, x_{it}) \| \le (1/4)\, \| x_2(t-1) \|\, (1+t-s)\exp(s-t)$$

$$\le (1/4)\,(1+t-s)\exp(s-t)\, | x_2(t-1) |$$

Now $c(s) + \int_0^T d(s,t)dt \le (1/4) + (1/4)\exp(s)\,[-(T+s)\exp(-T) + 1]$

$$\le (1/2) < 1$$

and

$$W(t_1, T) = \int_0^T \Phi_i(t_1, t_1-s)B_j(s)\, B_j^*(s)\, \Phi_i^*(t_1, t_1-s)\, ds.$$

That is

$$W(T) = \int_0^T \Phi_i(s)B_j(s)\, B_j^*(s)\, \Phi_i^*(s)\, ds.$$

From (18) it is clear that W is nonsingular for $T \neq 0$. Hence W^{-1} exists, by Theorem 3.1, the system (5) is null controllable on J.

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