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A NOTE ON THE EFFECT OF PRESSURE REDISTRIBUTION ON WEAK TURBULENT SHEAR FLOW IN ABSENCE OF BUOYANT FORCES

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ABSTRACT

The present paper deals with the weak turbulent flow of an incompressible viscous fluid. We have obtained an equation governing the decay rate of turbulent energy in absence of turbulent buoyant forces when isotropy prevails. The mean flow is assumed two dimensional. It is concluded that the anisotropy affects the turbulent production and the effect of pressure redistribution is always to inhibit the production.

1. INTRODUCTION

It is generally believed that in abence of external agencies anisotropy should decrease to avoid second law of violation. The subject is discussed in classic text by Hinze [1]. The works in this direction have been made by Lumely and Tennekes [2] and others. These authors considered mainly the decaying anisotropic homogeneous turbulence. Ferrziger and Shaanan [3] discussed the effect of anisotropy and rotation on turbulence production. They have shown that turbulence production requires anisotropy and simultaneous increase in anisotropy. In this paper we have focussed our attention on pressure - redistribution in weak turbulent flow of an incompressible viscous fluid and worked out an equation governing turbulent energy in absence of turbulent buoyant forces and concluded that the anisotropy affects the turbulence production and the effect of pressure - redistribution is always to inhibit the production.

2. DISCUSSION OF THE PROBLEM

The governing equation of Reynolds stresses of an incompressible homogeneous turbulent flow in absence of buoyant forces is written by Bradshaw [11] and Jones [12] as :

$$\frac{d}{dt}\overline{u_i u_j} = -\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} - \overline{u_j u_k}\cdot\frac{\partial U}{\partial x_k} + A_{ij} - 2\nu\overline{\frac{\partial u_i}{\partial x_k}\cdot\frac{\partial u_j}{\partial x_k}}$$

$$-\frac{\partial}{\partial x_k}\left(\overline{u_i u_j u_k} + \frac{2}{3}\delta_{ij}\frac{\overline{u_k P}}{\rho}\right) \qquad \text{......... (1)}$$

Where U and u are mean velocity and fluctuating velocity. The first two term representes turbulent production and last three terms represent pressure-redistribution, dissipation viscosity, and turbulent diffusion respectively. In equation (1) A_{ij} is pressure-velocity correlation and is expressed by Lumely [4].

as :

$$A_{ij} = -\left(\overline{\frac{u_i}{\rho}\cdot\frac{\partial p}{\partial x_j}} + \overline{\frac{u_j}{\rho}\cdot\frac{\partial p}{\partial x_i}} - \frac{2}{3}\delta_{ij}\overline{\frac{u_k}{\rho}\cdot\frac{\partial p}{\partial x_k}}\right) \qquad \text{......... (2)}$$

Where, ρ is the fluid density, p represents the fluctuation in pressure about mean and $A_{ij} = 0$

In present context we are ignoring the dissipation term and turbulent diffusion term (i.e. the effect of weak turbulence) and focussing our attention on first three terms in equation (1) using mean flow to be two dimensional. We know that spatial variation form a second order tensor and it can be decomposed into symmetric and antisymmetric part as :

$$\frac{\partial U_i}{\partial x_j} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) + \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)$$

$$= \frac{1}{2}D_{ij} + \frac{1}{2}\Omega_k \in_{ijk} \qquad \text{......... (3)}$$

The symmetric part D_{ij} determines the deformation of the fluid and is called the deformation tensor. The antisymmetric part $\Omega_k \in_{ijk}$ determines rotation without deformation and for simplification we take.

$$\frac{1}{2}D_{ij} = S_{ij} \text{ and } \frac{1}{2}\Omega_k \in_{ijk} = R_{ij} \qquad \text{......... (4)}$$

The mean velocity gradient tensor can be written as :

$$\frac{\partial U_i}{\partial x_j} = S_{ij} + R_{ij} \qquad \text{......... (5)}$$

Where,

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \qquad \text{.........} (6)$$

and
$$R_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right) \qquad \text{.........} (7)$$

Here $S_{ij} = 0$ and $S_{11} = S_{22} = S$ and $R = R_{12}$, which is non zero component of rotation tensor. Thus we have, $S_{12} = 0$.

In Equation of Reynolds stresses the Reynolds stresses are so closely bound up with anisotropy that the velocity-pressure correlation term is considered to be one of the controlling factor. A simplest form of velocity pressure gradient correlation A_{ij} expressed by Jones and Musong [5] is :

$$A_{ij} = \frac{-c_1}{2\epsilon} \cdot q^2 \left(\overline{u_i u_j} - \frac{1}{3}\delta_{ij}\, q^2\right) + a_{ijkm}\frac{\partial U_k}{\partial x_m} \qquad \text{.........} (8)$$

Where, $\frac{q^2}{2} = \frac{\overline{u_i\, u_i}}{2}$, is the turbulence kinetic energy.

ϵ = turbulence energy dissipation rate (per unit mass)

c_1 = constant.

Equation (8) shows that the redistribution is composed of two parts. The first part comprises of turbulent fluctuation and second part of mean strain correlation. Considering Donaldson [7, 8], Lumely [9] and Daly and Harlow [10], it is concluded that for a narrow range of turbulent shear flows the mean strain term [i.e. second term of equation (8)] may be omitted in comparison to first term in pressure-redistribution term. This equation (8) may be written as,

$$A_{ij} = \frac{-c_1}{2\epsilon} \cdot q^2 \left(\overline{u_i u_j} - \frac{1}{3}\delta_{ij}\, \overline{q^2}\right) \qquad \text{.........} (9)$$

Now we derive the following equations by using equations (3), (6), (7) and (9) in (1) as :

$$\begin{aligned}
\frac{d}{dt}\overline{u^2} &= 2S\,\overline{u^2} - 2R\,\overline{uv} - \frac{c_1}{2\epsilon}\,\overline{q^2}\left(u^2 - \overline{q^2/3}\right) \\
\frac{d}{dt}\overline{v^2} &= 2S\,\overline{v^2} + 2R\,\overline{uv} - \frac{c_1}{2\epsilon}\,\overline{q^2}\left(\overline{v^2\ q^2/3}\right) \qquad (10) \\
\text{and} \quad \frac{d}{dt}\overline{uv} &= R\left(\overline{u^2 - v^2}\right) - \frac{c_1}{2\epsilon}\,\overline{q^2}\left(\overline{uv}\right)
\end{aligned}$$

All others of equation (1) have r.h.s. equal to zero under the assumption made here. The equation for rate of change of Kinetic Energy is simply.

$$\frac{d}{dt}\left(\overline{u^2} + \overline{v^2}\right)$$

$$= 2S\left(\overline{u^2} - \overline{v^2}\right) - \frac{c_1}{2\epsilon}\,\overline{q^2}\left[\left(\overline{u^2} + \overline{v^2}\right) - \frac{2\left(\overline{q^2}\right)}{3}\right] \quad \text{.........} (11)$$

To discuss the production of turbulence, we shall discuss following cases,

Case - I : Isotropic Case :

For isotropic case $\overline{u^2} = \overline{v^2}$. the first term on r.h.s. of (11) vanishes and does not give rise to the turbulent production. In this case (11) reduces to form.

$$\frac{d}{dt}\left(\overline{q^2}\right) = \frac{-c_1}{6\epsilon}\left(\overline{q^2}\right)^2$$

$$\Rightarrow \overline{q^2} = \left(\frac{c_1 t}{6\epsilon} + A\right)^{-1} \quad \text{.........} (12)$$

Where A is the constant of integration. Equation (12) is similar to the expression derived by Kishore and Dixit [6]. This equation shows the decay of turbulent energy with time and is slower than the decay shown by Kishore and Dixit [6] in context of homogeneous turbulent flows. Thus we conclude that in isotropic case the pressure redistribution term has decaying effect on turbulence.

Case II : Anisotropic Case :

For anisotropic case $\overline{u^2} > \overline{v^2}$, the second term taken on the r.h.s. of (11) will decrease at the rate faster than the first term. Thus the effect of pressure-redistribution is to reduce the anisotropy and inhibit the production of turbulence. This was also pointed out by Ferziger [3] but the effect of pressure-redistribution was not explicitly shown there.

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