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CHARACTERIZATIONS OF SPACES AND MAPS VIA FUZZY β-OPEN SETS

MIGUEL CALDAS CUEVA
Departamento de Matemática Aplicada
Universidade Federal Fluminense
Rua Mário Santos Braga s/n°
CEP : 24020-140, Niteroi-RJ.
Brasil
e-mail : gmamccs@vm.uff.br

and

RATNESH KUMAR SARAF
Department of Mathematics
Government Kamla Nehru
Girls college
Damoh (M.P.) – 470661.
India.

ABSTRACT

The aim of this paper is to obtain several characterizations of fuzzy topological spaces and maps between them by utilizing the notion of β-open sets in fuzzy setting.

Key words and phrases : Fuzzy topological spaces, fuzzy semi-open sets, fuzzy β-open sets, fuzzy θ-closure, fuzzy almost regular space, fuzzy almost continuous.

1. INTRODUCTION

In general topology β-open sets were introduced by Abd El-Monsef et al. [1] and investigated by Andrijevic [2] under the name semi-preopen sets, and since then different topological properties have been defined and investigated by several authors. In 1986, A.S. Mashhour et al. [7] extend the notion of β-openness to fuzzy topological spaces. This class contains the class of fuzzy semi-open sets and the class of fuzzy preopen sets, introduced by Azad [3] and Singal and Prakash in [12] respectively. The purpose of this note is to obtain several characterizations of spaces and maps by utilizing fuzzy β-open sets. The results are parallel to ones which have been found in general topology. A systematic discussion of these properties in general topology is given in [2], [10], [13] and [15].

2. PRELIMINARIES

Now we introduce some basic notions and results that are used in the sequel.

In this work by (X, τ) (or simply X) we will denote a fuzzy topological space due to Chang [4]. A fuzzy point in X with support $x \varepsilon X$ and value p $(0 \leq p \leq 1)$ is denoted by x_p. Two fuzzy sets λ and μ are said to be q-coincident if there exists $x \varepsilon X$ such that $\lambda(x) + \mu(x) > 1$ and by $\overline{q}$ we denote "is not q-coincident". Unions and intersections of fuzzy sets are denoted by V and Λ respectively and defined by $V\lambda_i = Sup\ \{\lambda_i(x) : i\varepsilon I$ and $x \varepsilon X\}$, $\Lambda\lambda_i = \mathrm{Inf}\{\lambda_i(x) : i\varepsilon I$ and $x\varepsilon X\}$. The interior, closure and the complement of a fuzzy λ in X are denoted by Int (λ), $\mathrm{Cl}(\lambda)$ and λ^c respectively and defined by Int $(\lambda) = \mathrm{V}\ \{\mathrm{v} : \mathrm{v} \leq \lambda, \mathrm{v} \in \tau\}$, $\mathrm{Cl}(\lambda) = \Lambda\ \{v : v \leq \lambda, \mathrm{v}^c \varepsilon \tau\}$ and $\lambda^c = 1 - \lambda$.

Definition 1 Let (X,τ) be a fuzzy topological space. Then a fuzzy set λ in X is called a,

(i) fuzzy semi-open set if there exists a fuzzy open set μ such that $\mu \leq \lambda \leq Cl(\mu)$ equivalently, if $\lambda \leq ClInt\ (\lambda)$ [3].

(ii) fuzzy semi-closed set if λ^c is a fuzzy semi-open set, equivalently if $InCl(\lambda) \leq \lambda$ [3].

(iii) fuzzy semi-clopen set if λ is both fuzzy semi-open and fuzzy semi-closed.

(iv) fuzzy preopen set if $\lambda \leq IntCl(\lambda)$ [12].

(v) fuzzy preclosed set if λ^c is a fuzzy preopen set, equivalently if $ClInt\ (\lambda) \leq \lambda$ [12].

(vi) fuzzy regular open set if $\lambda = IntCl(\lambda)$ [3].

(vii) fuzzy regular closed set if λ^c is a fuzzy regular open set, equivalently if $ClInt(\lambda) = \lambda$ [3].

(viii) fuzzy α-set if $\lambda \leq IntClInt(\lambda)$ [11].

Definition 2 *Let λ be a fuzzy set of a fuzzy topological space. Then,*

(i) $sCl(\lambda) = \Lambda\{\beta : \beta \geq \lambda, \beta$ is a fuzzy semi-closed of $X\}$ is called a fuzzy semi-closure of λ [14].

(ii) $sInt(\lambda) = \vee\{\beta : \beta \leq \lambda,\ \beta$ *is a fuzzy semi-open of X} is called a fuzzy semi-interior of* λ *[14].*

(iii) $pCl(\lambda) = \wedge\{\beta : \beta \geq \lambda,\ \beta$ *is a fuzzy preclosed of X} is called a fuzzy preclosure of* λ *[12].*

(iv) $pInt(\lambda) = \vee\{\beta : \beta \leq \lambda,\ \beta$ *is a fuzzy preopen of X} is called a fuzzy preinterior of* λ *[12].*

3. Characterizations via fuzzy β-open sets

We use the notion of fuzzy β-open in order to characterize already known classes of fuzzy topological spaces.

Definition 3 *[7] A fuzzy set* λ *in a fuzzy topological space X is called :*

(i) fuzzy β*-open if* $\lambda \geq ClIntCl(\lambda)$.

(ii) fuzzy β*-closed if* λ^c *is a fuzzy* β*-open set, equivalently if* $Int\,ClInt(\lambda) \leq \lambda$.

All fuzzy semi-open (resp. fuzzy semi-closed) sets and all fuzzy preopen (resp. fuzzy preclosed) sets are fuzzy β-open (resp. fuzzy β-closed). But not conversely [7].

Theorem 3.1 *For a fuzzy subset* λ *of a fuzzy topological space X, the following conditions are equivalent :*

(1) λ *is an* β*-open set in X.*

(2) $Cl(\lambda) = ClIntCl(\lambda)$.

(3) $Cl(\lambda)$ *is fuzzy regular closed.*

(4) $Cl(\lambda)$ *is fuzzy semi-open.*

(5) $sCl(\lambda)$ *is fuzzy semi-open*

(6) $pCl(\lambda) \leq Cl\,Int(pCl(\lambda))$.

Proof. *(1)* $\rightarrow$ *(2) : By (1)* $Cl(\lambda) \leq Cl\ IntCl(\lambda)$. *Since* $Cl\ Int(\lambda) \leq Cl(\lambda)$, *then* $Cl\ Int\,(Cl)(\lambda) \leq Cl(Cl)(\lambda)) = Cl(\lambda)$. *Therefore* $ClInt(\lambda) = Cl(\lambda)$.

(2) $\rightarrow$ (1) : By (2) $\lambda \leq Cl(\lambda) = ClInt\ Cl\ (\lambda)$. *Hence,* λ is β-open.

(2) $\rightarrow$ (3) : By definition of a fuzzy regular closed set.

(3) $\rightarrow$ (4) : Every fuzzy regular closed set is fuzzy semi-open.

(4) $\rightarrow$ (3) : Since $Cl(\lambda)$ is fuzzy semi-open and fuzzy closed, then it is fuzzy regular closed.

(4) $\rightarrow$ (5) : By (4) $Cl(\lambda)$ is fuzzy semi-open and since $sCl(\lambda)$ is a fuzzy semi-closed set,

$IntCl(\lambda) \leq IntCl(sCl(\lambda)) \leq sCl(\lambda) \leq Cl(\lambda) = ClIntCl(\lambda)$. Hence $sCl(\lambda)$ is fuzzy semi-closed.

(5) $\rightarrow$ (1) : By (5) $\lambda \leq sCl(\lambda) \leq ClInt(sCl(\lambda)) \leq ClIntCl(\lambda)$. Thus λ is an β-open set.

(2) $\to$ (6) : By (2) $pCl(\lambda) \le Cl(\lambda) = ClIntCl(\lambda) = ClIntCl(pCl(\lambda))$.

(6) $\to$ (1) : By (6) $\lambda \le pCl(\lambda) \le ClInt(pCl(\lambda)) = ClIntCl(\lambda)$. Hence λ is a β-open set.

Theorem 3.2 *For a fuzzy topological space X the following are equivalent :*

(1) X is fuzzy extremally disconnected (i.e., [5], if the closure of each fuzzy open subset of X is fuzzy open).

(2) $ClInt(\lambda) \le IntCl(\lambda)$ for every fuzzy subset λ of X.

(3) Every fuzzy semi-open subset is fuzzy preopen.

(4) The closure of every fuzzy β-open subset of X is fuzzy open.

(5) Every fuzzy β-open subset of X is fuzzy preopen.

Proof. (1) $\to$ (2) : The set $ClInt(\lambda)$ is fuzzy open by (1). Thus $ClInt(\lambda) = Int\ ClInt(\lambda) \le IntCl(\lambda)$.

(2) $\to$ (3) : If λ is fuzzy semi-open by (2) $\lambda \le ClInt(\lambda) \le IntCl(\lambda)$, i.e., λ is fuzzy preopen.

(3) $\to$ (4) : Let λ be fuzzy β-open. Then $Cl(\lambda)$ is fuzzy semi-open (Theorem 3.1), hence by (3) fuzzy preopen. Thus $Cl(\lambda) \le Incl(\lambda)$, and $Cl(\lambda)$ is fuzzy open.

(4) $\to$ (5) : Let λ be fuzzy β-open. Then by (4) $Cl(\lambda)$ is fuzzy open and hence $\lambda \le Cl(\lambda) = IntCl(\lambda)$ that is λ is fuzzy preopen.

(5) $\to$ (1) : Let λ be fuzzy open. Then $Cl(\lambda)$ is fuzzy semi-open and hence fuzzy β-open. By (5) $Cl(\lambda)$ is fuzzy preopen. Thus $Cl(\lambda) \le IntCl(\lambda)$ or equivalently $Cl(\lambda) = IntCl(\lambda)$.

This shows that λ is fuzzy open.

Corollary 3.3 *Let X be a fuzzy extremally disconnected topological space. Then $sCl(\lambda) = Cl(\lambda)$ for every λ fuzzy β-open.*

Proof. Since $sCl(\lambda)$ is a fuzzy semi-closed set, we have $IntCl(\lambda) \le IntCl(sCl(\lambda)) \le sCl(\lambda)$. Thus $\lambda V IntCl(\lambda) \le sCl(\lambda) \le Cl(\lambda)$. Since X is fuzzy extremally disconnected, by Theorem 3.2, $Cl(\lambda)$ is fuzzy open in X for every λ fuzzy β-open. Therefore we have $sCl(\lambda) = Cl(\lambda)$ for every λ fuzzy β-open.

Proposition 3.4 *Let λ be a fuzzy set of a fuzzy topological space X. Then the following implication is hold : (1) $\to$ (2) where ;*

(1) Every fuzzy β-open subsets λ is fuzzy semi-open.

(2) If λ is fuzzy β-open, then $Cl(\lambda) = \lambda V ClInt(\lambda) = pCl(\lambda)$.

Proof. (1) $\to$ (2) : Let λ be a fuzzy β-open subset. By (1) λ is fuzzy semi-open. Then $Cl(\lambda) = \lambda V ClInt(\lambda)$. Since $pCl(\lambda)$ is a fuzzy preclosed set ([12], Theorem 3.8(2)), we have $ClInt(\lambda) \le ClInt(pCl(\lambda)) \le pCl(\lambda)$. Thus $Cl(\lambda) = \lambda V ClInt(\lambda) \le pCl(\lambda)$. Hence $Cl(\lambda) = \lambda V ClInt(\lambda) = pCl(\lambda)$.

Theorem 3.5 *For a fuzzy topological space X the following are equivalent :*

(1) Every subset is fuzzy β-open

(2) Every fuzzy closed subset is fuzzy β-open.

Proof. (1) $\to$ (2) : Is trivial.

(2) $\to$ (1) : Let λ be a fuzzy subset of X. By (2) cl(λ) is fuzzy β-open and hence $Cl(\lambda) \leq ClIntCl(\lambda) \leq Cl(Cl)(\lambda)) = Cl(\lambda)$. Thus $Cl(\lambda) = ClIntCl(\lambda)$ and hence λ is fuzzy β-open (Theorem 3.1).

Definition 4 *(i) A fuzzy subset λ of a fuzzy topological space X is a fuzzy S-set in X if and only if from $\lambda \leq \vee_{i\in I} \lambda_i$ where each λ_i is fuzzy regular closed, it follows that $\lambda \leq \vee_{i\in J} \lambda_i$ for some finite $J \leq I$.*

(ii) A finite dense subsystem of a cover of X is a finite subcollection whose closures cover X.

Theorem 3.6 *For a fuzzy topological space X the following are equivalent :*

(1) λ is an S-set in X.

(2) Every fuzzy β-open cover of λ has a finite dense subsystem.

Proof. (1) $\to$ (2) : Let $\lambda \leq \vee_{i\in I} \lambda_i$, where each λ_i is fuzzy β-open in X. Since λ_i is fuzzy β-open, then Cl(λ_i) is fuzzy regular closed. Hence from the inclusion $\lambda \leq \vee_{i\in I} Cl(\lambda_i)$, we have $\lambda \leq \vee_{i\in I} Cl(\lambda_i)$, where J is finite.

(2) $\to$ (1) : Every fuzzy regular closed sets is fuzzy β-open.

Recall that, A fuzzy point x_p is said to be a fuzzy θ-cluster point of a fuzzy set λ [8] if for every open q-nbd U of x_p, Cl(U) is q-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and will be denoted by $Cl_\theta(\lambda)$. A fuzzy set λ will be called θ-closed if and only if $\lambda = Cl_\theta(\lambda)$ and the complement of a fuzzy θ-closed set is fuzzy θ-open.

It is easy to see that $Cl(\lambda) \leq Cl_\theta(\lambda)$, for any fuzzy set λ in a fuzzy topological space. But the reverse implication is false (see[7]).

Theorem 3.7 *For a fuzzy topological space X the following are equivalent :*

(1) *X is a fuzzy almost regular space (i.e., for every fuzzy regular closed subset λ of X, we have $Cl(\lambda) = Cl_\theta(\lambda)$).*

(2) *If λ is fuzzy β-open then $Cl(\lambda) = Cl_\theta(\lambda)$.*

Proof. (1) $\to$ (2) : Since λ is fuzzy β-open, then Cl(λ) is fuzzy regular closed (Theorem 3.1). by (1) $Cl(\lambda) = Cl_\theta(Cl(\lambda))$. Since $Cl(\lambda) \leq Cl_\theta(\lambda) \leq Cl_\theta(Cl(\lambda)) = Cl(\lambda)$, Then $Cl(\lambda) = Cl_\theta(\lambda)$.

(2) $\to$ (1) : Every fuzzy regular closed set is fuzzy β-open.

The following result was proved in [8].

Proposition 3.8 For a fuzzy open set λ in a fuzzy topological space X, $Cl(\lambda) = Cl_\theta(\lambda)$.

Proof. Since $Cl(\lambda) \leq Cl_\theta(\lambda)$, it is sufficient to show that $Cl(\lambda) \geq Cl_\theta(\lambda)$. Let $x_p \in Cl_\theta(\lambda)$ such that $x_p \notin Cl(\lambda)$. Then there exists a q-nb V of x_p such that $V \bar{q} \lambda$. Then $V \leq 1 - \lambda$. Hence $Int(V) \leq Int(1-\lambda) \leq 1 - \lambda$. Therefore $ClInt(V) \leq Cl(1-\lambda) = 1 - \lambda$; i.e., $ClInt(V) \bar{q} \lambda$. Thus $x_p \notin Cl_\theta(\lambda)$, a contradiction.

Proposition 3.9 For a fuzzy semi-open set λ in a fuzzy extremally disconnected topological space X, $sCl(\lambda) = Cl_\theta(\lambda)$.

Proof. Since $sCl(\lambda) \leq Cl(\lambda) \leq Cl_\theta(\lambda)$, for any fuzzy set γ in X, then it is sufficient to prove that $Cl_\theta(\lambda) \leq Cl(\lambda)$ for a fuzzy semi-open set λ in X. Let $x_p \in sCl(\lambda)$, then there exists a fuzzy semi-open set β such that $x_p q \beta$ and $\beta \bar{q} \lambda$, which implies that $\beta \leq \lambda^c$. Hence $Cl(\beta) \leq Cl(\lambda^c)$. since X is extremally disconnected, then $Cl(\beta)$ is fuzzy open. Therefore $Cl(\beta) \leq IntCl(\lambda^c) \leq sCl(\lambda^c) = \lambda^c$. Hence $Cl(\beta) \bar{q} \lambda$ and so $x_p \notin Cl_\theta(\lambda)$.

Theorem 3.10 *For a map $f : X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y, the following are equivalent :*

(1) f is almost fuzzy weakly continuous (i.e., for every fuzzy open β of Y, $pCl(f^{-1}(\beta)) \leq f^{-1}(Cl(\beta))$).

(2) *If β is a fuzzy β-open subset of Y, then $pCl(f^{-1}(IntCl(\beta))) \leq f^{-1}(Cl(\beta))$.*

Proof. (1) $\to$ (2) : If β is a fuzzy β-open subset of Y, then $Cl(\beta) = ClIntCl(\beta)$ (Theorem 3.1). By (1) and since $IntCl(\beta)$ is fuzzy open.

$pCl(f^{-1}(IntCl(\beta))) \leq f^{-1}(ClIntCl(\beta)) = f^{-1}(Cl(\beta))$.

(2) $\to$ (1) : Let β be a fuzzy open subset of Y. Then $\beta \leq Incl(\beta)$. By (2) $pCl(f^{-1}(\beta)) \leq pCl(f^{-1}(IntCl(\beta))) \leq f^{-1}(Cl(\beta))$ and hence f is almost fuzzy weakly continuous.

Recall, that a map $f : X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y is said to be almost fuzzy closed in the sense of Nanda [9], if the image of every fuzzy regular closed set of X is fuzzy closed in Y.

Theorem 3.11 *For a map $f : X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y, the following are equivalent :*

(1) f is almost fuzzy closed.

(2) If λ is a fuzzy β-open subject of X, then $Cl(f(\lambda)) \leq f(Cl(\lambda))$.

Proof. (1) $\to$ (2) : If λ is fuzzy β-open, then $Cl(\lambda)$ is fuzzy regular closed (Theorem 3.1). By (1) $f(Cl(\lambda)$ is fuzzy closed. Thus $Cl(f(\lambda)) \leq Cl(f(Cl(\lambda))) = f(Cl(\lambda))$.

(2) $\to$ (1) : Every fuzzy regular closed set is fuzzy β-open. By (2) $Cl(f(\lambda)) \leq f(Cl(\lambda)) = f(\lambda)$ and hence $f(\lambda)$ is fuzzy closed.

Theorem 3.12 *For a map $f : X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y, the following are equivalent :*

(1) f is fuzzy almost continuous [3].

(2) $Cl(f^{-1}(\lambda)) \leq f^{-1}(Cl(\lambda))$ for every λ β-open.

(3) $Cl(f^{-1}(\lambda)) \leq f^{-1}(Cl(\lambda))$ for every λ semi-open.

Proof. (1) $\to$ (2) : Let λ be a β-open of Y. By Theorem 3.1, $Cl(\lambda)$ is fuzzy regular closed in Y. By (1), $f^{-1}(Cl(\lambda))$ is fuzzy closed in X and we obtain $Cl(f^{-1}(\lambda)) \leq Cl(f^{-1}(Cl(\lambda))) \leq f^{-1}(Cl(\lambda))$.

(2) $\to$ (3) : Every fuzzy semi-open set is fuzzy β-open.

(3) $\to$ (1) : Let β be any fuzzy regular closed set of Y. Then $\beta = ClInt(\beta)$ and hence β is fuzzy semi-open in Y. Therefore, we have $Cl(f^{-1}(\beta)) \leq f^{-1}(Cl(\beta)) = f^{-1}(\beta)$. Hence $f^{-1}(\beta)$ is fuzzy closed and f is fuzzy almost continuous.

Proposition 3.13 (1) Any union of fuzzy β-open sets is a fuzzy β-open set.

(2) Any intersection of fuzzy β-closed sets is a fuzzy β-closed.

Proof. (1) Let $\{\lambda_i : i\varepsilon I\}$ be any family of fuzzy β-open sets. For each $i\varepsilon I$, $\lambda_i \leq ClIntCl(\lambda_i)$. Hence we have $\bigvee_{i\varepsilon I} \lambda_i \leq \bigvee_{i\varepsilon I} ClIntCl(\lambda_i) \leq ClIntCl\left(\bigvee_{i\varepsilon I} \lambda_i\right)$.

(2) Let $\{\lambda_i : i\varepsilon I\}$ be any family of fuzzy β-closed sets. Thus $\{\lambda_i^c : i\varepsilon I\}$ is a family of fuzzy β-open sets. According to (1), $\bigvee_{i\varepsilon I} \lambda_i^c$ is a fuzzy β-open set. From $\left(\bigvee_{i\varepsilon I} \lambda_i^c\right)^c = \bigwedge_{i\varepsilon I} \lambda_i$ we obtain the conclusion.

A map $f : X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y is said to be pre-fuzzy β-closed if the imasge of every fuzzy β-closed set of X is fuzzy β-closed in Y.

Theorem 3.14 *For a map surjective $f : X \to Y$ from a fuzzy topological space X to another fuzzy topological space Y, the following are equivalent :*

(1) f is pre-fuzzy β-closed.

(2) For each fuzzy set λ of Y and each fuzzy β-open set μ in X containing $f^{-1}(\lambda)$, there exists a fuzzy β-open set γ of Y such that $\lambda \leq \gamma$ and

$f^{-1}(\gamma) \leq \mu$.

Proof. (1) → (2) : Suppose λ is a fuzzy set of Y and μ is any fuzzy β-open set in X containing $f^{-1}(\lambda)$. Put $\gamma = 1 - f(1-\mu)$. Since f is pre-fuzzy β-closed, then γ is a fuzzy β-open set of Y and since $f^{-1}(\lambda) \leq \mu$ we have $f(1-\mu) \leq 1 - \lambda$. Moreover $f^{-1}(\gamma) = 1 - f^{-1}(f(1-\mu)) \leq 1 - (1-\mu) = \mu$.

(2) → (1) : Suppose λ is fuzzy β-closed in X, Let x_p be a fuzzy point in $1 - f(\lambda)$. Then $f^{-1}(x_p) \leq 1 - f^{-1}(f(\lambda)) \leq 1 - \lambda$ and $1 - \lambda$ is fuzzy β-open in X. By (2) there exist a fuzzy β-open set μ_{x_p} containing x_p such that $f^{-1}\left(\mu_{x_p}\right) \leq 1 - \lambda$. This implies that $x_p \in \mu_{x_p} \leq 1 - f(\lambda)$. We obtain that $1 - f(\lambda) = \vee\{\mu_{xp} : x_p \in 1 - f(\lambda)\}$ and consequently by Proposition 3.13, $1 - f(\lambda)$ is fuzzy β-open.

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