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HEAT TRANSFER OF AN OSCILLATORY FLOW OF A CONDUCTING FLUID BETWEEN TWO NON-CONDUCTING POROUS FLAT PLATES

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ABSTRACT

An analysis is made on oscillatory laminar flow and heat transfer of a viscous, incompressible and electrically conducting fluid between two parallel non-conducting and non-magnetic infinite porous flat plates subject to a constant suction (or injection) in the presence of uniform transverse magnetic field. Expression for temperature is obtained when the magnetic Prandtl number is unity. The effects of the Hartmann number and suction parameter, are analysed on the temperature of the conducting fluid.

1. INTRODUCTION

Shukla [1] considered the flow of a conducting fluid between two parallel non-conducting infinite porous flat plates when the upper plate is at rest and lower one is moving with a velocity exponentially varying with time. Soundalgekar [2] studied the heat transfer in crossed MHD laminar flow between conducting walls. Dube [3] considered the unsteady flow of a viscous incompressible and electrically conducting fluid between two porous flat plates.

Similar type of problem with or without magnetic field have been studied by Dube [4], Kakutani [5], Messiha [6], Verma and Gaur [7], Ariel [8] and Das and Ahmed [9].

In this paper, we have studied the heat transfer in the unsteady flow of a viscous, incompressible and electrically conducting fluid between two non-conducting and non-magnetic infinite flat plates with suction (or injection) in the presence of a uniform transverse magnetic field when the upper plate is at rest and the lower one is oscillating with frequency 'ω' in its own plane. Small uniform suction has been imposed along the surface of walls. Expression for temperature of the fluid is obtained when magnetic Prandtl number is unity. Numerical computations are performed to study the effects of suction and magnetic parameter on the temperature of the conducting fluid.

2. FORMULATION OF THE PROBLEM :

We consider a conducting, viscous, incompressible fluid confined in the space between two non-conducting and non-magnetic infinite porous parallel flat plates one of which (lower one) is oscillating with velocity Uo μ_0^{-1} Rl exp $\left(iw\bar{t}\right)$ and frequency ω in its own plane.

Let $(\bar{x}, \bar{y})$ denote the rectangular Cartsian co-ordinates such that $\bar{y} = 0$ is the lower plate with the origin lying on it, $\bar{y} = L$ the upper plate which is held fixed, and $\bar{x}$ is the co-ordinate parallel to the direction of motion of the conducting fluid. Let U_x denote the velocity component parallel to the plates and U_y the perpendicular to the plates. Introducting the non-dimensional variables defined by (Sec, Dube [3])

$$U_x = \frac{U_0}{\mu_0} U,\ H_x = \frac{H_0}{\lambda_0} H,\ \bar{y} = Ly,\ \bar{P} = \rho U_0^2 P,$$

$$\bar{t} = \frac{L}{U_0} t,\ \omega = \frac{U_0}{L}\Omega,\ \theta = \frac{\bar{T} - \bar{T_1}}{\bar{T_2} - \bar{T_1}},$$

the equations governing the unsteady flow of a viscous incompressible and electrically conducting fluid between two non-conducting porous parallel plates, in the presence of a uniform transverse magnetic field of strength H_0, are given by

$$\frac{\partial^2 U}{\partial y^2} + \left(M \frac{\partial H}{\partial y} - V_c R \frac{\partial U}{\partial y}\right) - R \frac{\partial U}{\partial t} = 0 \qquad (2.2)$$

$$\frac{\partial^2 H}{\partial y^2} + \left(M \frac{\partial U}{\partial y} - V_c R_m \frac{\partial H}{\partial y}\right) - R_m \frac{\partial H}{\partial t} = 0 \qquad (2.3)$$

$$\frac{\partial^2 \theta}{\partial y^2} + V_c R P_r \frac{\partial \theta}{\partial y} - R P_r \frac{\partial \theta}{\partial t} - E P_r \left(\frac{\partial U}{\partial y}\right)^2 = 0 \qquad (2.4)$$

subject to the boundary conditions

$U = Rl \exp(iwt),\ H = 0,\ \theta = 0$ at $y = 0$

$U = 0,\ H = 0,\ \theta = Rl \exp(iwt)$ at $y = 1$

where y, t are dimension less independent variables,

$R = U_0 L\, \rho/\mu$ is the local Renolds number, $R_m = U_0 L/\varepsilon$ the magnetic Reynolds number, $S = \mu e H_0^2/\rho\, U_0^2$ the magnetic pressure number, $M = L\, H_0\, \mu_e \sqrt{6/\mu}$ the Hartmann number,

$V_c = V_0/U_0$ the suction parameter, $P_r = \mu C_p/k$ the Prandtl number, $E = U_0^2/C_p\left(\overline{T_2} - \overline{T_1}\right)$ the Eckert number, $\eta = 1/\sigma\mu_e$ the magnetic diffusivity, $\lambda_0 = \sqrt{SR}\ \mu_0 = \sqrt{R_m}$, Ω are dimensionless number, μ the co-efficient of viscosity, ρ the density, μ_e the magnetic permeability, σ the electrical conductivity, V_0 the constant suction velocity, H_x the induced magnetic field in x-direction, C_p the specific heat at constant pressure, $\overline{T_1}$ and $\overline{T_2}$ are the temperature of the lower and upper place respectively.

We assume

$$U = f(y)\, Rl \exp(i\Omega t) \qquad (2.6)$$

$$H = h(y)\, Rl \exp(i\Omega t) \qquad (2.7)$$

$$\theta = g(y)\, Rl \exp(i\Omega t) \qquad (2.8)$$

and $R = R_m$ as in Dube [3], equations (2.2) — (2.4) become then

$$\frac{d^2f}{dy^2} + \left(M\frac{dh}{dy} - \alpha_0\frac{df}{dy}\right) - i\,\beta_0^2 f = 0 \qquad (2.9)$$

$$\frac{d^2h}{dy^2} + \left(M\frac{df}{dy} + l_0\frac{dh}{dy}\right) - i\beta_0^2 h = 0 \qquad (2.10)$$

$$\frac{d^2h}{dy^2} - l_0 P_r\frac{dg}{dy} - i\, P_r\, \beta_0^2 g = \frac{E\, P_r}{R_m}\left(\frac{df}{dy}\right)^2 Rl \exp(i\Omega t)$$

where $\lambda_0 = V_c R = V_c R_m$ and $\beta_0^2 = \Omega R = \Omega R_m$ (2.11)

The appropriate boundary conditions are

$f = 1$	at $y = 0$,	$f = 0$	at $y = 1$	(2.12)
$h = 0$	at $y = 0$,	$h = 0$	at $y = 1$	(2.13)
$g = 0$	at $y = 0$,	$g = 1$	at $y = 1$	(2.14)

3. Solution of the Problem :

Equations (2.9) – (2.10) have been solved subject to the boundary conditions (2.12) and (2.13) by Dube [3]. Using the solutions of (2.9) and (2.10) obtained by Dube [3] we obtain the solution of (2.11) subject to the boundary conditions (2.14) as below

$$
\begin{aligned}
g(y) = {} & Ae^{\alpha_1 y} + Be^{\alpha_2 y} + \\
& \frac{1}{4}\left\{\frac{\lambda_2 e^{2(\lambda_1+\lambda_2 y)}}{(2\lambda_2-\alpha_1)(2\lambda_2-\alpha_2)} + \frac{\lambda_1^2 e^{2(\lambda_1 y+\lambda_2)}}{(2\lambda_1-\alpha_1)(2\lambda_1-\alpha_2)} - \right. \\
& \left. \frac{2\lambda_1\lambda_2 e^{(\lambda_1+\lambda_2)(1+y)}}{(\lambda_1+\lambda_2-\alpha_1)(\lambda_1+\lambda_2-\alpha_2)}\right\}\frac{1}{\left(e^{\lambda_1}-e^{\lambda_2}\right)^2} + \\
& + \left\{\frac{\mu_2^2 e^{2(\mu_1+\mu_2 y)}}{(2\lambda_1-\alpha_1)(2\lambda_1-\alpha_2)}\right. \\
& \left. + \frac{\mu_1^2 e^{2(\mu_1 y+\mu_2)}}{(2\mu_1-\alpha_1)(2\mu_1-\alpha_2)} - \frac{2\mu_1\mu_2 e^{(1+y)(\mu_1+\mu_2)}}{(\mu_1+\mu_2-\alpha_1)(\mu_1+\mu_2-\alpha_2)}\right\} \\
& \frac{1}{\left(e^{\mu_1}-e^{\mu_2}\right)^2} + \left\{\frac{2\lambda_2\mu_2 e^{(\lambda_1+\mu_1)+(\lambda_2+\mu_2)y}}{(\lambda_2+\mu_2-\alpha_1)(\lambda_2+\mu_2-\alpha_2)}\right. \\
& - \frac{2\lambda_2\mu_1 e^{(\lambda_2+\mu_1)y+\mu_2+\lambda_1}}{(\lambda_2+\mu_1-\alpha_1)(\lambda_2+\mu_1-\alpha_2)} \\
& - \frac{2\lambda_1\mu_2 e^{(\mu_2+\lambda_1)y+(\lambda_2+\mu_1)}}{(\lambda_1+\mu_2-\alpha_1)(\lambda_1+\mu_2-\alpha_2)} \\
& \left. + \frac{2\lambda_1\mu_1 e^{(\lambda_1+\mu_1)y+(\mu_2+\lambda_2)}}{(\lambda_1+\mu_1-\alpha_1)(\mu_1+\lambda_1-\alpha_2)}\right\} \\
& \left.\frac{1}{\left(e^{\lambda_1}-e^{\lambda_2}\right)\left(e^{\mu_1}-e^{\mu_2}\right)}\right]\frac{E\,P_r}{R_m}\,\mathrm{Cos}\,(i\Omega t)
\end{aligned}
\tag{3.1}
$$

where

$$A = \frac{1}{e^{\alpha_1} - e^{\alpha_2}} - \frac{1}{4\left(e^{\alpha_1} - e^{\alpha_2}\right)}$$

$$\left[\left\{\frac{\lambda_2^2 e^{2\lambda_1}\left(e^{2\lambda_2} - e^{\alpha_2}\right)}{\left(2\lambda_2 - \alpha_1\right)\left(2\lambda_2 - \alpha_2\right)} + \frac{\lambda_1^2 e^{2\lambda_2}\left(e^{2\lambda_1} - e^{\alpha_2}\right)}{\left(2\lambda_2 - \alpha_1\right)\left(2\lambda_1 - \alpha_2\right)}\right.\right.$$

$$\left. - \frac{2\lambda_1\lambda_2 e^{\lambda_1+\lambda_2}\left(e^{\lambda_1+\lambda_2} - e^{\lambda_2}\right)}{B(\lambda_1+\lambda_2-\alpha_1)\left(\lambda_1+\lambda_2-\alpha_2\right)}\right\} \times \frac{1}{\left(e^{\lambda_1} - e^{\lambda_2}\right)^2}$$

$$+ \left\{\frac{\mu_2^2 e^{2\mu_1}\left(e^{2\mu_2} - e^{\alpha_2}\right)}{\left(2\mu_2 - \alpha_1\right)\left(2\mu_2 - \alpha_2\right)}\right.$$

$$\left. + \frac{\mu_1^2 e^{2\mu_2}\left(e^{2\mu_1} - e^{\alpha_1}\right)}{\left(2\mu_1 - \alpha_1\right)\left(2\mu_1 - \alpha_2\right)} - \frac{2\mu_1\mu_2 e^{\mu_1+\mu_2}\left(e^{\mu_1+\mu_2} - e^{\alpha_2}\right)}{\left(\mu_1+\mu_2-\alpha_1\right)\left(\mu_1+\mu_2-\alpha_2\right)}\right\}$$

$$\times \frac{1}{\left(e^{\mu_1} - e^{\mu_2}\right)^2}$$

$$+ \left\{\frac{2\lambda_2\mu_2 e^{\mu_1+\lambda_1}\left(e^{\lambda_2+\lambda_2} - e^{\lambda_2}\right)}{\left(\lambda_2+\mu_2 - \alpha_1\right)\left(\lambda_2+\mu_2-\alpha_2\right)} - \frac{2\lambda_2\mu_1 e^{\mu_2+\lambda_1}\left(e^{\lambda_2+\mu_1} - e^{\alpha_2}\right)}{\left(\lambda_2+\mu_1 - \alpha_1\right)\left(\lambda_2+\mu_1-\alpha_2\right)}\right.$$

$$- \frac{2\lambda_1 \mu_2 e^{\left(\lambda_2+\mu_1\right)}\left(e^{\lambda_1+\mu_2} - e^{\lambda_2}\right)}{\left(\lambda_1+\mu_2-\alpha_1\right)\left(\lambda_1+\mu_2-\alpha_2\right)}$$

$$\left. + \frac{2\lambda_1\mu_1 e^{\lambda_2+\mu_2}\left(e^{\mu_1+\lambda_1} - e^{\alpha_2}\right)}{\left(\mu_1+\lambda_1-\alpha_1\right)\left(\mu_1+\lambda_1-\alpha_2\right)}\right\}$$

$$\left.\times \frac{1}{\left(e^{\lambda_1} - e^{\lambda_2}\right)\left(e^{\mu_1} - e^{\mu_2}\right)}\right] \frac{E\ P_r}{R_m} \mathrm{Cos}\ (i\Omega t)$$

$$B = \frac{1}{e^{\alpha_1} - e^{\alpha_2}} + \frac{1}{4\left(e^{\alpha_1} - e^{\alpha_2}\right)}$$

$$\left[\left\{\frac{\lambda_2^2 e^{2\lambda_1}\left(e^{2\lambda_2} - e^{a_1}\right)}{\left(2\lambda_2 - \alpha_1\right)\left(2\lambda_2 - \alpha_2\right)} + \frac{\lambda_2^2 e^{2\lambda_1}\left(e^{2\lambda_2} - e^{\alpha_1}\right)}{\left(2\lambda_1 - \alpha_1\right)\left(2\lambda_1 - \alpha_2\right)}\right.\right.$$

$$- \frac{2 l_1 \lambda_2 e^{\lambda_1+\lambda_2}\left(e^{\lambda_1+\lambda_2} - e^{\alpha_1}\right)}{(\lambda_1 + \lambda_2 - \alpha_1)(\lambda_1 + \lambda_2 - \alpha_2)}\Bigg\} \frac{1}{\left(e^{\lambda_1} - e^{\lambda_2}\right)^2}$$

$$+ \Bigg\{ \frac{\mu_2^2\, e^{2\lambda_2}\left(e^{2\mu_2} - e^{\alpha_1}\right)}{(2\lambda_1 - \alpha_1)(2\lambda_1 - \alpha_2)}$$

$$+ \frac{\mu_1^2\, C^{2\mu_2}\left(C^{2\mu_1} - C^{\alpha_1}\right)}{(2\mu_1 - \alpha_1)(2\mu_1 - \alpha_2)} - \frac{2\mu_1\mu_2 C^{\mu_1+\mu_2}\left(C^{\mu_1+\mu_2} - C^{\alpha_1}\right)}{(\mu_1+\mu_2-\alpha_1)(\mu_1+\mu_2-\alpha_2)}\Bigg\}$$

$$\times \frac{1}{\left(e^{\lambda_1} - e^{\lambda_2}\right)^2} + \Bigg\{ \frac{2\lambda_2\mu_2 C^{\lambda_1+\mu_1}\left(C^{\lambda_1+\mu_2} - C^{\alpha_2}\right)}{(\lambda_2+\mu_2-\alpha_1)(\lambda_2+\mu_2-\alpha_2)}$$

$$- \frac{2\lambda_2\mu_1 C^{\mu_2+\lambda_1}\left(C^{\mu_1+\lambda_2} - C^{\alpha_1}\right)}{(\lambda_2+\mu_1-\alpha_1)(\lambda_2+\mu_1-\alpha_2)}$$

$$- \frac{2\lambda_1\mu_2 C^{\lambda_1+\mu_1}\left(C^{\lambda_2+\mu_2} - C^{\alpha_1}\right)}{(\lambda_1 + \mu_2 - \alpha_1)(\lambda_1 + \mu_2 - \alpha_2)}$$

$$+ \frac{2\mu_1\lambda_1\, C^{\lambda_2 + \mu_2}\left(C^{\mu_1 + \mu_1} - C^{\alpha_1}\right)}{(\lambda_1+\mu_1 - \alpha_1)(\lambda_1+\mu_1 - \alpha_2)}\Bigg\}$$

$$\times \frac{1}{\left(C^{\lambda_1} - C^{\lambda_2}\right)\left(C^{\mu_1} - C^{\mu_2}\right)}\Bigg] \frac{E\, P_r}{R_m} \operatorname{Cos}(i\Omega t),$$

$$\mu_1, \mu_2 = \frac{1}{2}\left\{(\alpha_o + M) \pm \sqrt{(\lambda_o + M)^2 + i4\beta_0^2}\right\}$$

$$\lambda_1, \lambda_2 = \frac{1}{2}\left\{(\alpha_o - M) \pm \sqrt{(\lambda_o - M)^2 + i4\beta_0^2}\right\}$$

$$\alpha_1, \alpha_2 = \frac{1}{2}\left\{\alpha_o - P_r\right\} \pm \sqrt{(\alpha_o P_r)^2 + i\, 4\, P_r \beta_0^2}$$

Using (2.8) and (3.1) we can easily compute the temperature, of the conducting fluid confined in the space between two non-conducting and non-magnetic plates.

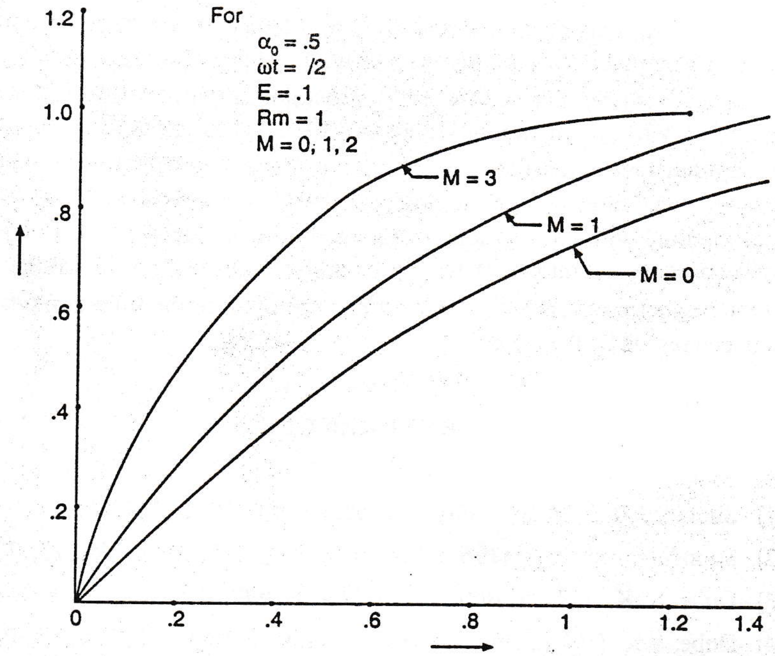

Fig 2. : Temperature distribution θ of the fluid for different Hartamann number M.

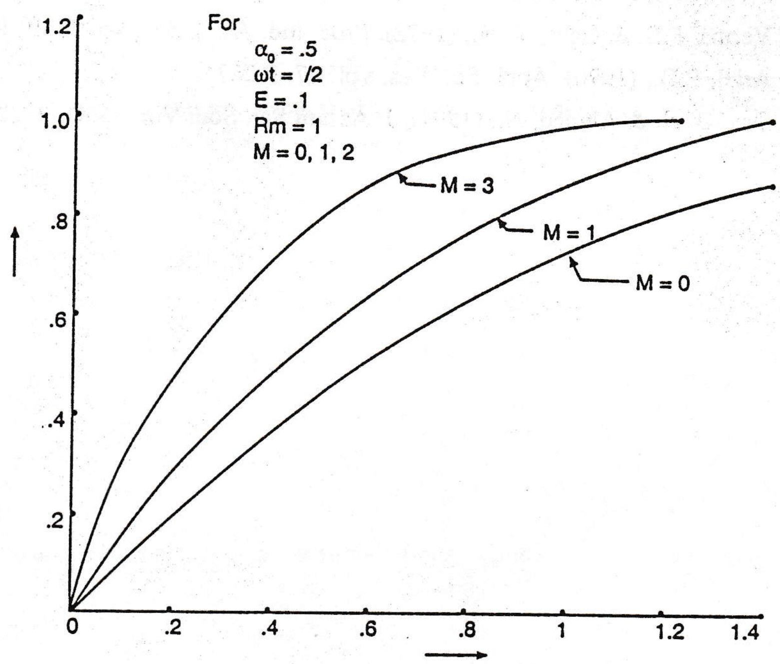

Fig 2. : Temperature distribution θ of the fluid for different Hartamann number M.

4. Results and Discussion :

The variations of temperature θ of the fluid with y for various values of suction parameter α_0, are shown in Fig. 1 when $\Omega t = \pi/2$ and $M = 1$. It is observed from the Fig. 1 thst the dimensionless temperature θ of the fluid increases with y corresponding to the decrease of suction parameter α_0 throughout the region between the plates. Hence the effect of suction is to decrease the temperature of the fluid. fig. 2 shows the variations of temperature θ of the fluid with y for various values of Hartmann number M when $\Omega t = \pi/2$ for a constant suction parameter α_0. It can be seen from Fig. 2 that the effects of the magnetic field are to increase the temperature of the fluid throughout the bounded region by the plates.

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