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BETWEEN FUZZY CLOSED SETS AND FUZZY g-CLOSED SETS

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ABSTRACT :

Veera Kumar [15] introduced the concept of g*-closed sets in General Topology. In this paper we introduce a new class of sets namely Fg* - closed sets, which is properly placed in between the class of fuzzy closed sets and the class of fuzzy g-closed sets. Applying these sets we introduce and study Fg*-continuity, Fg*-open mapping and Fg*-irresolute mappings.

KEYWORDS : Fuzzy g-closed sets ; Fg*-closed sets ; Fg*-continuous ; and Fgc-irresolute mappings.

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1. INTRODUCTION AND PRELIMINARIES.

S. S. Thakur and R. Malviya introduced the clas of fuzzy g-closed sets, a super class of closed sets in 1995. Balasubramanian and sundaram [3] also studied the same concept in 1997. In the present paper we introuce a new class of sets (using new techniques), called Fg^*-closed sets. Which is properly placed in between the class of fuzzy closed sets and the class of Fg-closed sets. We also showed that this new class is proeprly contained in the class of Fα g-closed sets, the class of Fgs-closed sets, the class of Fgsp-closed sets, the class of Fgpr-closed sets and the class of Frg-closed sets. And is independent of fuzzy semiclosedness, fuzzy preclosedness, fuzzy α-closedness, Fsp-closedness Fsg-closedness and Fgα-closedness.

Throughout this paper (X, τ), (Y, σ) and (Z,τ) (or simply X, Y and Z) represent nonempty fuzzy topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a fuzzy subset A and X, Cl (A) and nt (A) denote the closure and interior of A respectively. To reduce the size of the paper most of the proofs and omited.

DEFINITION 1.1 : A fuzzy subset A of X is called fuzzy preopen [4] (resp. fuzzy semiopen [2]. fuzzy α-open [4], Fsp - open [13], if $A \leq$ Int (Cl(A)) (resp. $A \leq$ Cl (int(A), $A \leq$ Int (Cl (Int (A)), $A \leq$ Cl (Int (Cl(A))). And their complements are respectively called fuzzy preclosed (resp. fuzzy semiclosed. fuzzy α-closed, Fsp-closed) if Cl (Int(A)) $\leq$ A (resp. Int (Cl(A) $\leq$ A. Cl (Int(Cl(A))) $\leq$ A. Int (Cl (Int(A))) $\leq$ A).

The intersection of all fuzzy semiclosed (resp. fuzzy preclosed, Fsp-closed and fuzy α-closed) sets containing A of X is called the fuzzy semiclosure (resp. fuzzy preclosure, Fsp-closure and fuzzy α-closure) of A and is denoted by sCl (A) (resp. pCl (A), spCl (A) and α Cl(A)).

DEFINITION 1.2 : A fuzzy subset A of X is called :

(a) Fg-closed [13] (Frg - closed [7]) if Cl (A) $\leq$ B whenever $A \leq B$ and B is fuzzy open (fuzzy regular open) in X.

(b) Fsg-closed [1] (Fgs-closed [12] if sCl (A) $\leq$ B whenever $A \leq B$ and B is fuzzy semiopen (fuzzy open) in X.

(c) Fgα-closed [6] (Fα g-closed [11] if α Cl (A) $\leq$ B whenever $A \leq B$ and B is fuzzy α-open (fuzzy open) in X.

(d) Fgsp-closed [10] if sp Cl (A) $\leq$ B whenever $A \leq B$ and B is fuzzy open in X.

(e) Fgpr-closed [8] if pCl (A) $\leq$ B whenever $A \leq B$ and B is fuzzy regular open in X.

DEFINITION 1.3 : A mapping $f : (X, \tau) \rightarrow (Y,\sigma)$ is called :

(a) Fuzzy semicontinuous [2] (resp. fuzzy precontinuous [4], fuzzy α-continuous [14], Fsp-continuous [13], Fg-continuous [12], Fαg-continuous [11] Fgs-continuous [1], Fgsp-continuous [10], Frg-continuous [7], Fgpr-continuous [8]) if f^{-1} (A) is fuzzy semiclosed (resp. fuzzy preclosed, fuzzy α-closed. Fsp-closed, Fg-closed, Fα g-closed, Fgs-closed, Fgsp-closed, Frg-closed, fgpr-closed) set of X. for every fuzzy closed closed set A of Y.

(b) Fgc-irresolute [13] if f^{-1} (A) is Fg-closed, set of X for every Fg-closed set A of Y.

(c) Fg-closed [9] if f (A) is Fg-closed in Y for every fuzzy closed set A of X.

DEFINITION 1.4 : [5] A fuzzy subset A is said to be quasi-coincident with a fuzzy set B in X denoted by AqB if there is a point $x \in X$ such that $A(x) + B(x) > 1$, the negation of this statement is written as $\rceil$(AqB). $A \leq B$ iff (Aq (1–B)). Two fuzzy sets A and B of (X, τ) are Q-separated iff Cl (A) $\cap$ B = 0 = Cl (B) $\cap$ A.

2. Fg* - CLOSED SETS

DEFINITION 2.1 : A fuzzy subset A of X is called Fg*-closed if $Cl(A) \leq H$ whenever $A \leq H$ and H is Fg-open in X.

REMARK 2.1 : Every fuzzy closed set is Fg* - closed and every Fg*-closed set is Fg-closed but the converse may not be true in general. For,

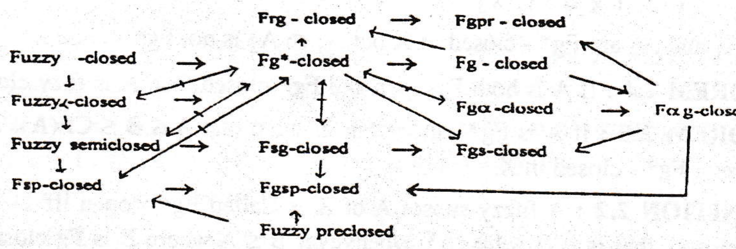
EXAMPLE 2.1 : Let X = { a, b } and A and B are defined as : A(a) = 0.3, A(b) = 0.2 ; B(a) = 0.5, B(b)_ = 0.7 ; H(a) = 0.7, H(b) = 0.6 ; E (a) = 0.3, E (b) = 0.3. And let τ_1 = { 0, B, 1. }, τ_2 = { 0, H, 1 } are fuzzy topology on X then A (resp. E) is Fg*-closed (resp. Fg-closed) but not fuzzy closed (resp. Fg*-closed) set in (X, τ_1) (resp. (X, τ_2)).

The following Remark shows that the class of Fg*-closed set is properly contained in the class of Fαg-closed sets the class of Fgs-cosed sets, the class of Fgsp-closed sets, the class of Fgpr-closed sets, and in the class of frg-closed sets.

REMARK 2.2 : Every Fg*-closed set is an Fαg-closed set and hence Fgs-closed, Fgsp-closed, Fgpr-closed set and Frg-closed set but not conversely. The set E in Example 2.1 is Fαg-closed, Fgs-closed, Fgpr-closed and Frg-closed but not Fg*-closed.

REMARK 2.3 : Fg*-closedness is independent of fuzzy α-closedness, fuzzy semiclosedness, fuzzy preclosed, Fsp-closedness, Fsg-closedness, and Fgα-closedness. For

EXAMPLE 2.2 : Let X = { a, b }. A and B are fuzzy sets in X defined as : A(a) = 0.3, A (b) = 0.7 ; B(a) = 0.7, B(b) = 0.3. Let t = { 0, A, 1 } be fuzzy topology on X. Then B is Fg*-closed but not the following sets : Fsp-closed, fuzzy α-closed, fuzzy preclosed, Fsg-closed and fuzzy semiclosed. The set E defined in Example 2.1 is fuzzy α-closed and hence fuzzy semiclosed, fuzzy preclosed, Fgα-closed, Fgα-closed and Fsp-closed in (X, τ_2). But it is not Fg*-closed in (X, τ_2). Thus we have the following diagram of implication.



Where A → B (resp. A → B) represent A implies B but not conversely (resp. A and B are independent.)

THEOREM 2.1 : Let X be a fuzzy topological space and A is a fuzzy subset of X. Then A is Fg*-closed iff $\rceil(AqH) \rightarrow \rceil(Cl(A)qH)$ for every Fg-closed set H of X.

PROOF : Let H be Fg-closed subset of X, and $\rceil(AqH)$ then by Definition 1.4, $A \leq 1 - H$ and $1 - H$ is Fg-open in X. So $Cl(A) \leq 1 - H$. Hence, $\rceil(Cl(A)qH)$.

SUFFICIENCY : Let B be a Fg-open set of X, such that $A \leq B$. Then, $\rceil(Aq(1-B))$ and $(1-B)$ is Fg-closed in X. By hypothesis $\rceil(Cl(A))q(1-B))$. Therefore, $Cl(A) \leq B$. Hence A is Fg*-closed.

THEOREM 2.2 : Let A be Fg* - closed set in x and x_α be a fuzzy point of X such that $x_\alpha qCl(A)$ then $cl(x_\alpha)qA$.

REMARK 2.3 : Theorem 3.14 of Veera Kumar [15] (A is g*-closed in topological space X iff Cl(A) – A does not contain any non-empty g-closed sets.) is no longer valid in fuzzy topological spaces. For, the fuzzy set A in the fuzzy topological space (X, τ_1) in Example 2.1 is Fg* - closed but the fuzzy set $1 - B$ is a non zero Fg*-closed subset of $cl(A) — A = Cl(A) \cap (1-A)$.

THEOREM 2.3 : If A and B are fg*-closed in a fuzzy topological space X then $A \cup B$ is Fg*-closed.

REMARK 2.4 : The intersection of any two Fg*-closed sets in X may not be Fg*-closed For,

EXAMPLE 2.3 : Let $X = \{x_1, x_2, x_3\}$. Define $f_1, f_2, f_3 : X \rightarrow [0, 1]$ as follows : $f_1 = 0$, $f_2 = 1$,

$$f_3(x) = \begin{cases} 0 & \text{If } x = x_2, x_3 ; \\ 1 & x = x_1 \end{cases}$$

clearly $\tau = \{f_1, f_2, f_3\}$ is a fuzzy topology on X. Define A_1 and A_2 as : $A_1(x)$

$$= \begin{cases} 0 \text{ if } x = x_3 ; \\ 1, \text{ if } x = x_1 \ \ x_2 \end{cases}$$

$$A_2(x) = \begin{cases} 0 \text{ if } x = x_2 ; \\ 1 \text{ if } x = x_1, x_3 \end{cases}$$

Then A_1 and A_2 are Fg* - closed in X but $A_1 \cap A_2$ is not Fg* - closed.

THEOREM 2.4 : If A is both Fg-open and Fg*-closed then A is fuzy closed

THEOREM 2.5 : If A is Fg* - closed in X, such that $A \leq B \leq Cl(A)$. Then B is also a Fg* - closed in X.

DEFINITION 2.2 : A fuzzy subset A of X is Called Fg* - open iff 1—A is Fg* - closed, that is If $B \leq Int(A)$, whenever $B \leq A$ where B is Fg-closed in X. The family of all Fg* - closed (resp. Fg* - open) set of a fuzzy topological space X will be denoted by (Fg*C(X) (resp. Fg* O(X)).

REMARK 2.5 : Every fuzzy open set is Fg* - open and every Fg*-open set is Fg-open but the converse may not be true in general.

THEOREM 2.6 : Let A and B be Q-separated Fg* - open subsets of a fuzzy toplogical space X, then $A \cup B$ is Fg*-open

PROOF : Let H be a Fg-open subset of $A \cup B$. Then $H \cap Cl(A) \leq (A \cup B) \cap Cl(A) = (A \cap Cl(A)) \cup (B \cap Cl(A)) \leq Int(A)$. Similarly $H \cap Cl(B) \leq Int(B)$. Now $H = H \cap (A \cup B) \leq (H \cap Cl(A)) \cup (H \cap Cl(B)) \leq Int(A) \cup Int(B) \leq (A \cup B)$. Hence $A \cup B$ is Fg* - open.

THEOREM 2.7 : Let A and B are two Fg*- closed sets in fuzzy topological space X and suppose that $1 - A$ and $1 - B$ are Q-separted, then $A \cap B$ is Fg*-closed.

THEOREM 2.8 : Let A be Fg*-open subset of X and $Int(A) \leq B \leq A$ then B is fg*-open.

THEOREM 2.9 : Let (Y, τ_y) be a fuzzy subspace of a fuzzy topological spce (X, τ) and A be fuzzy set in Y. If A is Fg* - closed in X, then A is Fg* - closed in Y.

DEFINITION 2.3 : Let A be a fuzzy sub set in a fuzzy toplogical space X and x_β is a fuzzy point of X. A is called g* - neighbourhood (resp. g*-Q-neighbourhood) of x_β if thee exists a Fg*-open set B in X such that $x_\beta \in B \leq A$ (resp. $x_\beta \, q \, B \leq A$).

THEOREM 2.10 : A fuzzy set A is Fg*-open in X iff for each fuzzy point x_β of A, A is a g*-neighbourhood of x_β.

Fg* - CONTINUOUS MAPPING

DEFINITION 3.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is called Fg*-continuous if $f^{-1}(A)$ is Fg*-closed in X for every fuzzy closed set A of Y.

REMARK 3.1 : Every fuzzy continuous mapping is Fg*-continuous and every Fg*-continuous mapping is Fg-continuous, hence Fαg-continuous, Frg-continuous, Fgsp-continuous, Fgs-continuous and Fgpr-cntinuous. But the converse may not be true in general. For

EXAMPLE 3.1 : Let X = { a, b } and Y = { x, y }. Fuzzy subset A and B are defined as : A { a) = 0.5, A (b) = 0.7 ; B (x) = 0.7, B (y) = 0.8 Let τ = { 0, A, 1} and σ = { 0, B, 1 } be fuzzy topology in X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f(a) = x and f(b) = y is fg* - cntinuous but not fuzzy continuous.

EXAMPLE 3.2 : Let X = { a, b } and Y = { x, y }. fuzzy sets A and B are defined as A (a) = 0.7, A (b) = 0.6 ; B (x) = 0.7, B (y) = 0.7. Let τ = { 0, A, 1 } and σ = { 0, B, 1 } be fuzzy topology in X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined as f (a) = x and f(b) = y is Fg - continuous and hence Fαg-continuous, Fgs - continuous, Fgsp - continuous, Frg - continuous and Fgpr - continuous but not Fg* - continuous.

REMARK 3.2 : The composition of two Fg* - continuous mappings may not be Fg* - continuous For,

EXAMPLE 3.3 : Let X = { a, b }, Y = {x, y} and Z = { p, q }. Fuzzy sets A, B ane H are drfined as : A (a) = 0.5, A (b) = 0.7 ; B (x) = 0.3, B (y) = 0.2 ; H (p) = 0.6, H (q) = 0.4. Let $\tau = \{0, A, 1\}$ $\sigma = \{0, B, 1\}$ and $\Gamma = \{0, H, 1\}$ be fuzzy topologies on X, Y and Z respectively. Let the mapping $f : (X, \tau) \to (Y, \sigma)$ be defined by f (a) = x f(b) = y. and the mapping $g : (Y, \sigma) \to (Z, \Gamma)$ be defined by g (x) = p, g(y) = q. Then f and g are Fg* - continuous but g of : $(X, \tau) \to (Z, \Gamma)$ is not Fg* - continuous.

REMARK 3.3 : Fg*-continuity is independent from fuzzy semicontinuity, Fsp-continuity, fuzzy precontinuity and fuzzy α-continuity. For , the mapping f defined in Example 3.3, is Fg*-continuous but neither Fsp-continuous and fuzzy semicontinuous nor Fuzzy α-continuous and fuzzy precontinuous. The mapping f in Example 3.2, is fuzzy α-continuous hence fuzzy semicontinuous, fuzzy precontinuous and also Fsp-continuous but not Fg*-continuous.

THEOREM 3.1 : A mapping $f : (X, \tau) \to (Y, \sigma)$ is Fg*-continuous iff the inverse image of every fuzzy open set of Y is Fg*-open in X.

THEOREM 3.2 : Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from a fuzzy topological space X to a fuzzy topological space Y. Then the following statements are equivalent :

(a) f is Fg*-continuous

(b) For every fuzzy closed set A in Y, $f^{-1}(A) \in Fg^*C(X)$

(c) for every fuzzy point x_β in X and every fuzzy open set A such that $f(x_\beta) \in A$ there is a fuzzy set $B \in Fg^*O(X)$ such that $x_\beta \in B$ and $f(B) \leq A$.

(d) for every fuzzy point x_β of X and every neighbourhood A of $f(x_\beta)$, $f^{-1}(A)$ is g*-neigbbourhood of x_β.

(e) For every fuzzy point x_β of X and every neighbourhood A of $f(x_\beta)$, there is a g*-neighbourhood H of x_β such that $f(H) \leq A$.

(f) For every fuzzy point x_β and every fuzzy open set A of X such that $f(x_\beta)qA$, there is a fuzzy set $B \in Fg^*O(X)$ such that $x_\beta\, qB$ and $f(B) \leq A$.

(g) for every fuzzy point x_β of X and every Q-neighbourhood A of $f(x_\beta)$, $f^{-1}(A)$ is a g*-Q-neighbourhood of x_β.

(h) for every fuzzy point x_β of X and every Q-neighbourhood A of $f(x_\beta)$ there is a g*-Q-neighbourhood H of $x\beta$ such that $f(H) \leq A$

PROOF : (a)<→ (b) ; (a)<→ (c) ; (a) ↔ (f) : obvious.

(c) → (d) : Let x_β be a fuzzy point of X and A be a neighbourhood of $f(x_\beta)$. Then there is a fuzzy open set B such that $f(x_\beta) \leq B \leq A$. Now $f^{-1}(B) \in Fg^*O(X)$ and $x\beta \in f^{-1}(B) \leq f^{-1}(A)$. Thus $f^{-1}(A)$ is a g*-neighbourhood of $x\beta$ in X.

(d) ↔ (e) : Let x_β be a fuzzy point of X and A be a neighbourhood of $f(x_\beta)$. Then $H = f^{-1}(A)$ is a g*-neighbourhood of x_β and $f(H) = f(f^{-1}(A)) \leq A$.

(e) ↔ (c) : Let x_β be a fuzzy point of X and A be a fuzzy open set such that $f(x_\beta) \in A$. Then A is a neighbourhood of $f(x_\beta)$. So there is g*-neighbourhood H of x_β in X such that $x_\beta \in H$ and $f(H) \leq A$. Hence there is a fuzzy set $B \leq Fg^*O(X)$ such that $x_\beta \in B \leq H$ and so $f(B) \leq f(H) \leq A$.

(f) → (g) → (h) → (f) obvious.

THEOREM 3.3 : If $f : (X, \tau) \to (Y, \sigma)$ be Fg* - continuous and $g : (Y, \sigma) \to (Z, \Gamma)$ is fuzzy continuous, then $gof : (X, \tau) \to (Z, \Gamma)$ is Fg*-continuous.

4. Fg* - CLOSED MAPPINGS

DEFINITION 4.1 : A mapping $f : (X, \tau) \to (Y,\sigma)$ is called Fg*-closed (resp Fg*-open) if for every fuzzy closed (resp. fuzzy open) Set A of X the image f (A) is Fg*-closed (resp. Fg*-open) in Y.

REMARK 4.1 : Every fuzzy closed (resp. fuzzy open) mapping is Fg*-closed (resp. Fg*-open) and every Fg*-closed (resp. Fg*-open) mapping is Fg-closed (resp. Fg-open) but the converse may not be true in general. May leave to the reader.

THEOREM 4.1 : (i) A mapping $f : (X, \tau) \to (Y, \sigma)$ is Fg*-open iff for any fuzzy subset A of Y and any fuzzy closed set H in X containing $f^{-1}(A)$, there exists a Fg*-closed subset B of Y containing A such that $f^{-1}(B) \leq H$.

(ii) A mapping $f : (X, \tau) \to (Y, \sigma)$ is Fg*-closed iff for any fuzzy subset A of Y and any fuzzy open set E in X containing $f^{-1}(A)$, there exists a Fg*-open set B of Y such that $A \leq B$ and $f^{-1}(B) \leq E$.

PROOF : (i) Necessity : Suppose $f : (X, \tau) \to (Y, \sigma)$ is Fg*-open mapping. Let A be any fuzzy subset of Y and H is an arbitrary closed set of X containg $f^{-1}(A)$. We put $B = Y - f(X-H)$. Then by because of f is Fg*-open mapping, B is Fg*-closed in Y. Since $f^{-1}(A) \leq H$ it follows from straight forward calculation that $A \leq B$. Now $f^{-1}(B) = X - f^{-1}(f(X - H)) \leq H$ completes the proof of this Part.

SUFFICIENCY : Suppose E is fuzzy open set of X. Then $f^{-1}(Y-f(E)) \leq X - E$ and $X - E$ is fuzzy closed. By hypothesis, there exists a Fg*-closed subset B of Y containing $Y - f(E)$ such that $f^{-1}(B) \leq X - E$. Therefore $E \leq X - f^{-1}(B)$. Hence $Y-B \leq f(E) \leq f(X-f^{-1}(B)) \leq Y - B$, which implies $f(E) = Y - B$. since $Y - B$ is Fg* - open, f(E) is Fg-open and thus f is Fg*-open mapping.

(ii) Obvious.

THEOREM 4.2 : If A is Fg*-closed set in (X, τ) and if f : (X, τ) → (Y,σ) is fuzzy gc-irresolute and Fg*-closed then f (A) is Fg*-closed in Y.

PROOF : If f(A) ≤ H where H is Fg - open in Y, then A ≤ f^{-1} (H) and hence, Cl(A) ≤ f^{-1} (H) since A is Fg*-closed. Thus f (Cl)(A)) ≤ H and f (Cl)(A)) is a Fg* - closed set. Then Cl (f (Cl(A))) ≤ H, it follows that Cl (f(A)) ≤ Cl (f(Cl (A))) ≤ H. Thus Cl (f(A)) ≤ H and f(A) is Fg*-closed.

THEOREM 4.3 : If A is Fg* - open in X and if f : (X, τ) → (Y,τ) is bijective fuzzy gc-irresolute and Fg* - closed. Then f(A) is Fg* - open in (Y, τ)

THEOREM 4.4 : If f : (X, τ) → (Y, σ) is bijective Fg* - continuous and fuzzy pre generalized open (i.e. f image of Fg-open set of X is Fg-open in Y) and if B is Fg* - closed subset of Y, then f^{-1} (B) is Fg* - closed in X.

COROLLARY 4.1 : If f : (X τ) → (Y, σ) is bijective Fg* - continuous and fuzzy pre generalized open. Then the inverse image f^{-1} (A) of each Fg* - open subset A of Y is Fg* - open in X.

THEOREM 4.5 : If f : (X, τ) → (Y, σ) is Fg* - closed and fuzzy pre g-closed and if H is Fg* - open (or Fg* - closed). Subset of Y, then f^{-1} (H) is Fg* - open (or Fg* - closed) in X.

PROOF : Let H be a Fg* - open set in Y. Let E ≤ f^{-1} (H) where E is fg-closed in X. Therefore f(E) ≤ H holds. Since f(E) is Fg-closed and H is Fg* - open In Y, f(E) ≤ Int (H)). Hence E ≤ f^{-1} (Int (H)). Since f is Fg* - continuous and Int (G) is fuzzy open in Y, E ≤ Int (f^{-1} (Int (H))) ≤ Int (f^{-1} (H)). Therefore f^{-1} (H) is Fg* - open in X. Other part is obvious by taking complement.

THEOREM 4.6 : (i) If f : (X, τ) → (Y, σ) is Fuzzy open (resp. fuzzy closed), and g : (Y, σ) → (Z, Γ) is Fg* - open (resp. Fg* - closed), then g of : (X, τ) → (Z, Γ) is Fg* - open (resp. Fg* - closed)

(ii) Let f : (X, τ) → (Y, σ) be Fg* - closed and g : (Y, σ) → (Z, Γ) be Fg* - closed and Fuzzy gc-irresolute then g of : (X, τ) → (Z, Γ) is Fg* - closed.

(iii) If f : (X, τ) → (Y, σ) is Fg* - open and g : (Y,Γ) → (Z, Γ) is bijective Fg* - closed and Fuzzy gc-irresolute then g of : (X, τ) → (Z,Γ) is Fg* - open.

THEOREM 4.7 : Let f : (X, τ) → (Y, σ) and g : (Y,σ) → (Z,Γ) be two mappings and let g of : (X, τ) → (Z, Γ) is Fg* - closed (resp Fg*-open). Then,

(i) If f is surjective and continuous then g is Fg* - closed (resp. Fg*-open)

(ii) If g is bijective, Fg* - continuous and fuzzy pre generalized open, then f is Fg* - closed (resp. Fg* - open).

COROLLARY 4.2 : If f : (X, τ) → (Y, σ) is Fg* - closed and fuzzy pre g - closed and g : (Y, σ) → (Z, Γ) is Fg* - continuous, then g of : (X, τ) → (Z, Γ) is Fg* - continuous.

THEOREM 4.8 : For any bijective, Fg* - continuous and fuzzy pregeneralized open mapping f : (X , τ) → (Y, σ) and any Fg* - continuous mapping g : (Y, σ) → (Z, Γ), the composition gof : (X, τ) → (Z, Γ) is Fg* - continuous.

5. Fg* - IRRESOLUTE MAPPINGS

DEFINITION 5.1 : A mapping f : (X, τ) → (Y, σ) is called Fg* - irresolute if f^{-1} (A) is Fg* - closed in X for every Fg* - closed set of Y.

THEOREM 5.1 : Every Fg* - irresolute mapping is Fg* - continuous.

The following examples supports that the converse of the above theorem is not true in general

EXAMPLE 5.1 : The mapping f defined in Example 4.4 of [3] is Fg*-continuous but not Fg* - irresolute.

THEOREM 5.2 : Let f : (X, τ) → (Y, σ) and g : (Y, σ) → (Z, Γ) be any two mapings. Then,

(a) g of : (X, τ) → (Z, Γ) is Fg* - irresolute if both f and g are Fg* - irresolute

(b) g of : (X, τ) → (Z, Γ) is Fg* - continuous if g is Fg* continuous and f is Fg* - irresolute.

THEOREM 5.3 : Let f : (X, τ) → (Y, σ) be a Fg* - continuous If (X, τ) is $T^*_{1/2}$ (a space in which every Fg* - closed set is fuzzy closed), then f is fuzzy continuous.

THEOREM 5.4 : Let f : (X, τ) → (Y, σ) be a fuzzy gc - irresolute and fuzzy closed mapping. Then f(A) is a Fg* - closed set of Y for every Fg* - closed set A of X.

PROOF : Let A be a Fg* - closed set of X. Let H be a Fg-open set of Y such that f (A) ≤ H.

Since f is fuzzy gc - irresolute f^{-1} (H) is a Fg-open set of X. Since A ≤ f^{-1} (H) and A is a Fg* - closed set of X, Cl (A) ≤ f^{-1} (H). Then f (Cl) (A)) ≤ f (f^{-1} (H)) ≤ H. Since f is fuzzy closed, f (Cl (A)) = Cl (f (Cl (A))). This implies Cl (f (A)) ≤ cl (f (Cl (A))) ≤ f (Cl (A)) H. Therefore f(A) is a Fg* - closed set of Y.

THEOREM 5.5 : Let f : (X, τ) → (Y, σ) be onto, Fg* - irresolute and fuzzy closed. If (X, τ) is fuzzy $T^*_{1/2}$, then (Y, σ) is also a fuzzy $T^*_{1/2}$ space.

THEOREM 5.6 : Let (X, τ) and (Y, σ) be fuzzy topological spaces and f : (X, τ) → (Y, σ). Then the following conditions are equivalent :

(a) f is Fg* - irresolute

(b) For every fuzzy point x_β of X and for every A ∈ Fg* O (Y) containing f(x_β) there exists a jifuzzy set B ∈ Fg* O (X) such that x_β ∈ B ≤ f^{-1} (A)

(c) For every fuzzy point x_β of X and for every A ∈ Fg* O (Y) containing f(x_β) there exists a fuzzy set B ∈ Fg* O (X) such that x_β ∈ B and f (B) ≤ A.

(d) For every fuzzy point x_β of X and for every A ∈ Fg* O (Y) Satisfying f(x_β) qA, There exists a fuzzy set B ∈ Fg* O (X), such that x_β qB ≤ f^{-1} (A).

(e) For every fuzzy point x_β of X and for every A ∈ Fg* O (Y) satisfying f (x_β) q A, there exists a fuzzy set B ∈ Fg* O (Y) such that x_β qB and f (B) ≤ A.

(f) For every B ∈ Fg* C (Y), f^{-1} (A) ∈ Fg* C (X).

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