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SHOOTING METHOD FOR THE PROBLEM OF HEAT TRANSFER TO THE THREE DIMENSIONAL STRETCHING OF A FLAT SURFACE

G. C. HAZARIKA
Department of Mathematics
Dibrugarh University

ABSTRACT :

In this study an attempt has been made to investigate the heat transfer in the three dimensional fluid motion caused by the stretching of a flat surface numerically using the shooting method. The thermal bounday layer equation is solved numerically for a set of values of the Prandtl number P, stretching parameter α and the the viscous dissipation parameter m. The effect of all the parameters are quite prominent.

Key Words : Shooting method, Heat transfer, Stretching surface.

1. INTRODUCTION :

The transfer of heat between a solid body and a fluid is important in many fields of science. In order to determine the temperature distribution, it is necessary to combine the equations of motion with those of heat conduction. The exact similarity solutions to the two-dimensional flow due to a stretching flat surface and to outside a stretching cylinder were studied by Crane (1970, 75), but to inside a stretching channel or tube was investigated by Brady & Acrivos (1981). Kuiken (1981) and Banks (1983) invsstigated the two dimensional non-uniform stretching boundary layer. Terril and Thomas (1969, 1973) obtained numerical and analytical solutions of an ordinary non-linear differential equation that arose in laminar or spiral flow through a porous pipe. Robinson (1976) studied the multiple solutions for the laminar flow in a uniformly porous channel with suction at both walls. Pulsatile flow in a channel was investigated by Secomr (1978) and the same flow in a tube with suction and injection was investigated by Skalak & Wang (1977). Wang (1984) studied the flow caused by three dimensional stretching of a flat surface. Hazarika and Goswami (1991) have studied heat transfer of a stretching surface in a rotating fluid with suction and blowing, and unsteady, fluid flow due to a stretching cylinder has been studied by Gowami and Hazarika (1992).

In this study the transfer of heat to the fluid motion caused by the three dimensional stretching of a flat surface is investigated numerically.

2. FORMULATION OF THE PROBLEM :

For our present problem the equation of continuity, the steady Navier-Stokes equations and the energy equation with viscous dissipation remain same for three dimensional cartesian co-ordinates in absence of external forces. Due to the stretching, we have,

$u = cx,\ v = dy,\ w = 0,\ T = 0$, at $z = 0$

where u, v, w are the cartesian components of velocity, c and d are stretching rate parameters and T is the temperature. The flat surface is considered as the plane $z = 0$. The velocities and temperrature at a large distance from the flat surface is zero. So

$$u = 0,\ v = 0,\ T = 0 \text{ as } z \to \infty$$

Using the transformations,

$$u = cx\phi'(\eta),\ v = cy\psi'(\eta) \qquad w = -\sqrt{c\nu}\,(\phi+\psi)$$

$$T = \frac{cx^2}{\nu}\theta_1(\eta) + \frac{cy^2}{n}\theta_2(\eta) + \theta_3(\eta) \qquad \textit{where } \eta = z\sqrt{c/\nu}$$

in the Navier-Stokes equations and the energy equation, the Navier-Stokes equations reduce to –

$$\phi''' = (\phi')^2 - (\phi+\psi)\phi'', \tag{2.1}$$

$$\phi''' = (\psi')^2 - (\phi+\psi)\psi'', \tag{2.2}$$

$$\frac{p}{r} = nw^2 - \frac{1}{2}w^2 + \text{Const.}, \tag{2.3}$$

With boundary conditions

$$\left.\begin{array}{ll} \phi'(0) = 1, & \psi'(0) = \frac{d}{c} = a \\ \phi(0) + \psi(0) = 0, & \phi'(\infty) = \psi'(\infty) = 0 \end{array}\right] \tag{2.4}$$

and the envery equation reduces to

$$\ddot{\theta}_1 + P(\phi+\psi)\,\dot{\theta}_1 - 2P\phi'\theta_1 + m\phi''^2 = 0 \tag{2.5}$$

$$\ddot{\theta}_2 + P(\phi+\psi)\dot{\theta}_2 - 2P\psi'\theta_2 + m\psi''^2 = 0, \tag{2.6}$$

$$\ddot{\theta}_3 + P(\phi+\psi)\ddot{\theta}_3 + 2(\theta_1 + \theta_2) + m(\theta+\psi)(\phi''+\psi'') + (\phi+\psi)^2(\phi'+\psi') + 4(\phi'^2 + \psi'^2 + \phi'\psi') = 0 \tag{2.7}$$

with boundary conditions

$$\theta_i(0) = \theta_i(\infty) = 0,\ i = 1, 2, 3 \tag{2.8}$$

Here ρ, ν, p are respectively density, Kinematic viscosity and pressure ; P is the Prandtl number, m is the dimensionless viscous dissipation parameter ϕ, ψ are dimensionless stream functins ; θ_1, θ_2, θ_3 are the dimensionless temperatures and η is the similarity variable.

3. SHOOTING METHOD :

In boundary value problems of second order ordinary differential equations one condition is prescribed at the either end points. Thus one condition is always missing at the initial point of integration. Shooting method (Conte and Boor, 1972, Robert and Shipmen, 1972) estimates the missing initial condition in such a way that the final estimation satisfies the condition prescribed at the other boundary too, to a desired degree of accuracy, converting the problem there by to an intitial value problem. The missing value is guessed at the beginning and refined by using an iterative technique until the desired accuracy is obtained.

To described the method, let us consider a second order ordinary differential equatin.

$$y'' = f(x,y,y') \tag{3.1}$$

with the boundary conditions

$$y\,(a) = y_a \qquad y\,(b) = y_b \tag{3.2}$$

where the initial condition y'(a) is missing. Let δ be the correct value of y'(a) and this correct value must be so determined that the resulting solution yields Y_b the prescribed value of y at x = b to some desired accuracy. We, therefore, guess the initial slope an set up and iterative procedure for converging to the correct slope. Let δ_0, δ_1 be the two guesses for the initial slope y'(a) and y(δ_i ; b), y(δ_i,b) be the values of y at x = b obtained from integration the differential equation (3.1).

A normally better approximation to δ can be obtained by linear interpolation,

$$\delta_2 = \delta_0 + (\delta_1 - \delta_0)\frac{y_b - y(\delta_0\,;\,b)}{y(\delta_1\,;\,b) - y(\delta_0\,;\,b)} \tag{3.3}$$

We now integrate the differential equation (3.1), using the initial conditions y(a) = y_a, y'(a) = δ_2, to obtain y(δ_2 ; b). Again by linear interplation based on δ_1 and δ_2, we can obtain a next approximation δ_3. The process is repeated until convergence has been obtained i.e., y(δ_k ; b) agrees with y (b) = y_b to some desired degree of accuracy, i.e., if $|y(\delta_k\,;\,b) - y_b|$ is very small for some k, then δ_k may be treated as actual missing value for y'(a). There is no guarantee of convergence of the iterative process and the rapidity of convergence will clearly depend upon how good initial guesses are. The method is easy to apply and when it does converge, it is usually more efficient than other methods. It is experienced that the convergence is more rapid if the two guess values are close to the true value or they lie opposite to the true value. Using the above procedure the boundary value problem (3.1) – (3.2) can be solved numerically.

In the system (2.5) – (2.8), there are three second order non-linear differential equations whose analytical exact solution is difficult to obtain. Hence a numerical method is essential to solve the system (2.5) – (2.8). Considering Wang's (1984) solution of the system (2.1) – (2.4) for the velocity function for some values of the parameters α, P, m, we assumed two values for the missing initial condition $\dot{\theta}_1(0)$ of equation (2.5). Once the iterative procedure is convergent with the above guesses for a particular set of values of α, P, m, the selection of the guessed values corresponding to any other set of values of α, P,m, become easier.

Also our problem is a singular one as one of the boundries is at infinity. The singularity of the problem is tackled by a method of continuation (Hazarika, 1985). At first a large value of η (say η = 2 or 3) is chosed and $\dot{\theta}_1(0)$ is estimated for a set of values of α, P,m. Next η is increased and $\dot{\theta}_1(0)$ is estimated again for the same set of values of α, P,m. If there is no change in the estimation of $\dot{\theta}_1(0)$ then any of the η may be treated as the truncated boundary at infinity. Otherwise the process is repeated until there is no change in the value of the estimation of $\dot{\theta}_1(0)$, $\dot{\theta}_2(0)$ is also estimated in the same manner. Finally $\dot{\theta}_3(0)$ is estimated using the solutions of (2.5) and (2.6).

4. RESULTS AND DISCUSSION :

The missing initial values are estimated in the manner described in the previous article and then the system (2.5) – (2.6) is solved completely for different sets of values of α, P, m.

Table (1) represents the estimated values of $\dot{\theta}_1(0)$, $\dot{\theta}_2(0)$, $\dot{\theta}_3(0)$ for the different set of values of α, P, m, . In figures 1 to 3, θ_1, θ_2, θ_3 are respectively plotted against η for P = 0.72, m = 0.5, and α = 0.25, 0.50, 0.75. It is seen that θ_1, θ_2, θ_3 increase first and then decrease gradually. Also all the three functions decrease as α increases and increases if P increses. When m increases θ_1, θ_2, θ_3 all increases. Thus the effect of all the parameters are prominent.

It may be concluded that the shooting method may be applied to solve the boundary value problem in which three initial conditions are missing such as one discussed in this paper. Inspite of the singularity of the problem this method is easily applied to solve the boundry value problem for a wide range of values of the parameters.

Table – 1

α	P	m	$\theta_1(0)$	$\theta_2(0)$	$\theta_3(0)$
0.25	0.72	1.0	0.3903771	0.02169678	2.275446
0.25	0.25	1.0	0.4515693	0.0245012	2.707466
0.25	0.25	1.5	0.6773540	0.0369519	4.061200
0.50	0.25	1.0	0.4859679	0.1125048	3.367012
0.50	0.25	1.5	0.7289517	0.1687572	5.0500516
0.75	0.25	1.0	0.5172078	0.2812564	4.191208
0.75	0.25	1.5	0.7758117	0.4218848	6.286815
1.00	0.25	1.5	0.8192674	0.8170872	7.742816

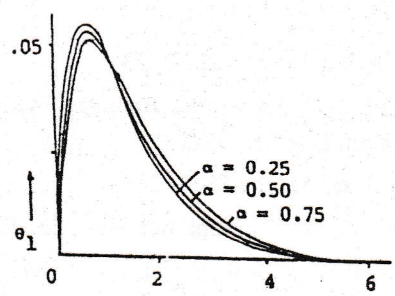

FIG. 1. θ_1 vs η

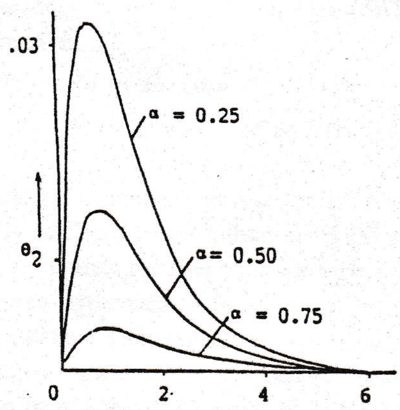

FIG. 2. θ_2 vs η

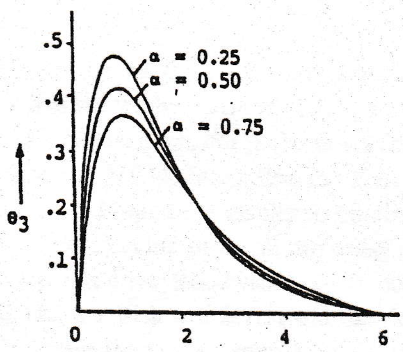

FIG. 3. θ_3 vs η

REFERENCES

1. Banks, W.H.H., (1983) J. Mech. Theor. Appl. 2, P. 375.
2. Brady, J.F. & Acrivos, A., (1981) J. Fluid Mech., 112, P. 127.
3. Crane, L.J. (1970)ZAMP, 21, P. 645.
4. Crane, L.J. (1975) ZAMP, 26, P. 619
5. Conte, S.D., C.D. Boor, (1972) Elementary Numerical Analysis, An Algorithmic Approach, (McGraw Hill Books Co., New York)
6. Goswami, P. D. and Hazarika, G.C., (1992), J. Assam. Sci. Soc. 34(4), 27-32.
7. Hazarika, G.C. (1985) Ph. D. Thesis submitted to Dibrugarh University.
8. Hazarika, G. C. and Goswami, P.D., (1991) Procedings, 36th ISTAM congress. P. 55-64.
9. Robinson, W.A. (1976) J. Eng. Math., 10, P. 23.
10. Robert, S.M. & Shipman, J.S., Two Point Boundary Value Problems ; Shooting Method, (American Elsevier, 1972)
11. Secomr, T. W., (1978) J. fluid Mech., 88, p. 273
12. Shalak, F.M. & Wang, C.Y., (1977) Appl. Sci. Res., 33, P. 269.
13. Terrill, R. M. & Thomas, P.W., (1969), Appo. scient Res., A 21, PP. 36-67.
14. Terrill, R. M. & Thomas, P.W., (1973). Phys. Fluids. 16, P. 356.
15. Wang, C.Y., (1984), Phys. fluids. Lett 27(8), P. 1917.