Mathematical Forum
Vol. XIV, 2001-2002 pp 39-52

FLOW OF A VISCOUS FLUID DUE TO A ROTATING DISK IN PRESENCE OF AN INFINITE SATURATED POROUS MEDIUM AT A SMALL DISTANCE FROM THE DISK

B. R. SHARMA

Department of Mathematics
Dibrugarh University
Dibrugarh, Assam-786004.

ABSTRACT :

The flow due to a rotating disk at a small distance from a porous medium of infinite thickness has been investigated by using Brinkman-Oberbeck-Boussinesq equations in the porous medium and Navier-Stokes equations outside the porous medium. It has been found that due to the rotation of the disk an axial flow towards the interface develops at infinity in the porous medium. This axial flow increases with the increase of the permeability of the porous medium. The shearing stresses at the interface and on the rotating disk increase with the increase of the Reynolds number and with the decrease of the permeability parameters of the porous medium. The investigation also reveals that the depth of penetration of the flow in the porous medium is inversely proportional to the square root of the permeability of the medium.

1. *Introduction*

The laminar flow of an incompressible viscous fluid due to an infinite disk rotating with constant angular velocity was solved by Karman (1921) by using the momentum-integral method. The associated heat transfer problem was solved by Millsaps and Pohlhausen (1952). Cochran (1934) integrated the Karman's (1921) disk rotation equations numerically and found the results so obtained in good agreement with those of Karman (1921). This problem attracted many researchers and consequently a lot of work was done. Bodewadt (1940) took up the inverse problem of Cochran (1934), i.e., the disk was at rest and the fluid at infinity was rotating. Batchelor (1951) generalized the solutions of Karman (1921) and Bodewadt (1940) for the flow over a single infinite rotating disk. This problem was further discussed by Stewartson (1953) who obtained approximate solutions when the Reynolds number was either large or small. Numerical solutions for this problem have been obtained by Lance and Roger (1962), Pearson (1965); and Mellor, Chapple and Stokes (1968).

The flow of an incompressible viscous fluid due to a rotating disk of infinite radius with uniform suction at the disk has been examined by Stuart (1954) and numerical solutions for small and asymptotic solutions for large values of the suction parameter were obtained. The problem was also discussed by Rogers and Lance (1960) who found that the steady flow is possible only when there is suction at the disk and the effect of the suction is to prevent the boundary layer from leaving the disk and attaching itself to the disturbing agency at infinity. Elkouh (1967, 1969), Evans (1969) and Kuiken (1971) have also studied the effect of suction or injection on the flow between disks in different cases.

Joseph (1965) has analyzed the coupled flow induced by the steady rotation of fluid saturated, naturally permeable disk and compared the results with the flow induced by the rotation of an otherwise impermeable disk over which a uniform suction has been prescribed. Further he has pointed out that the numerical solutions of Stuart (1954) may be carried over directly with the change that the suction parameter prescribed there, identified with a permeability parameter which depends on the Darcy's coefficient and can not be arbitrary. Srivastava and Sharma (1992) have discussed the flow and heat-transfer of an incompressible viscous fluid due to a rotating disk at a small distance from a porous medium of finite thickness and found that the magnitude of the velocity components decrease by increasing the permeability of the porous medium. Srivastava and Barman (1997) have discussed the flow of a second-order fluid filling the space between an impervious rotating disk and a porous medium fully saturated with a viscous fluid. They found that the depth of penetration of the flow in the porous medium is inversely proportional to the square root of the permeability of the medium. They also found that there is an axial flow in the porous medium at a large distance from the interface; and the effect of non—Newtonian terms are to increase the flow in the porous medium.

In this paper we have discussed the flow due to a rotating disk at a small distance from an infinite saturated porous medium when an incompressible viscous fluid fills the space between the disk and the porous medium. The fluid is also present in the porous region. In solving the problem we have taken Brinkman-Oberbeck-Boussinesq equations in the porous medium and the Navier-Stokes equations outside it. The solutions have been obtained by expanding the velocity components in powers of Reynolds number. First four terms of the series have been obtained and compared with exponential series. It has been found that each term in the expansion is less than the corresponding term in the exponential series indicating absolutely convergence of the series. Since we have calculated only four terms therefore the results hold good for small Reynolds number.

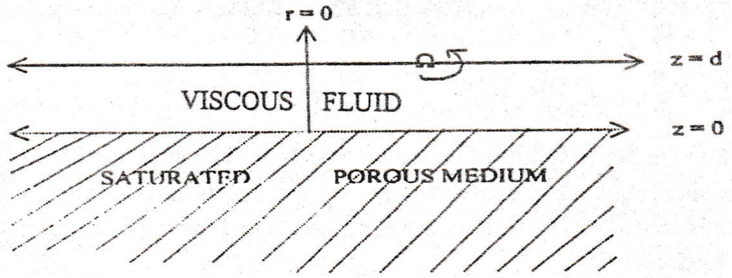

Fig. 1

2. *Formulation of the problem*

We consider the steady flow of an incompressible viscous fluid between an impervious rotating disk and a saturated porous medium at a small distance d from the disk by choosing cylindrical polar coordinates (r, θ, z) system with origin at the interface (See Fig.1). The disk at z = d is rotating with angular velocity Ω about the axis r = 0. The motion of the fluid between the porous medium and the rotating disk is axi-symmetric and is governed by the following Navier-Stokes equations and the equation of continuity :

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)=\frac{\partial T_{rr}}{\partial r}+\frac{\partial T_{rz}}{\partial z}+\frac{T_{rr}-T_{\theta\theta}}{r}, \tag{1}$$

$$\rho\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}\right)=\frac{\partial T_{r\theta}}{\partial r}+\frac{\partial T_{\theta z}}{\partial z}+\frac{2}{r}T_{r\theta}, \tag{2}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=\frac{\partial T_{rz}}{\partial r}+\frac{\partial T_{zz}}{\partial z}+\frac{1}{r}T_{rz}. \tag{3}$$

and

$$\frac{\partial u}{\partial r}+\frac{\partial w}{\partial z}+\frac{u}{r}=0, \tag{4}$$

where

$$T_{rr}=-p+2\mu\frac{\partial u}{\partial r} \qquad T_{r\theta}=\mu\left(\frac{\partial v}{\partial r}-\frac{v}{r}\right), \tag{5a}$$

$$T_{\theta\theta}=-p+2\mu\frac{u}{r} \qquad T_{\theta z}=\mu\frac{\partial v}{\partial z} \tag{5b}$$

$$T_{zz}=-p+2\mu\frac{\partial w}{\partial z}, \qquad T_{rz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) \tag{5c}$$

u, v, w are velocity components along r, θ, z directions respectively, ρ is the fluid density, μ is the viscosity and p is the hydrostatic pressure

Within the porous medium also the flow is axially symmetric and is described by Brinkman-Oberbeck-Boussinesq equations which can be written in cylindrical polar coordinates as

$$\rho\left(\bar{u}\frac{\partial \bar{u}}{\partial r}+w\frac{\partial \bar{u}}{\partial z}-\frac{\bar{v}^2}{r}\right)=\frac{\partial \bar{T}_{rr}}{\partial r}+\frac{\partial \bar{T}_{rz}}{\partial z}+\frac{\bar{T}_{rr}-\bar{T}_{\theta\theta}}{r},-\frac{\mu}{k}\bar{u}, \tag{6}$$

$$\rho\left(\bar{u}\frac{\partial\bar{v}}{\partial r}+\bar{w}\frac{\partial\bar{v}}{\partial z}+\frac{\bar{u}\bar{v}}{r}\right)=\frac{\partial\bar{T}_{r\theta}}{\partial r}+\frac{\partial\bar{T}_{\theta z}}{\partial z}+\frac{2}{r}\bar{T}_{r\theta}-\frac{\mu}{k}\bar{v}, \quad (7)$$

$$\rho\left(\bar{u}\frac{\partial\bar{w}}{\partial r}+\bar{w}\frac{\partial\bar{w}}{\partial z}\right)=\frac{\partial\bar{T}_{rz}}{\partial r}+\frac{\partial\bar{T}_{zz}}{\partial z}+\frac{1}{r}\bar{T}_{rz}-\frac{\mu}{k}\bar{w}, \quad (8)$$

where k is the permeability of the porous medium. The equation of continuity remains unchanged. Here a bar over an entity denotes its corresponding value in the porous medium. The form of stress components remains unchanged in the porous medium.

Ourside the porous medium we assume the following forms of velocity components and pressure:

$$u = r\,\Omega F'(\eta),\ v = r\,\Omega\, G(\eta),\ w = -2d\,\Omega F(\eta) \text{ and } p = -\mu\Omega P_1(\eta) \quad (9)$$

In the porous medium, we assume

$$u = r\,\Omega f'(\eta),\ v = r\,\Omega G(\eta),\ w = -2d\,\Omega f(\eta) \text{ and } \overline{P} = -\mu\Omega P_1(\eta) \quad (10)$$

where $\eta = z\,/\,d$. Here a prime denotes differentiation with respect to η. The equation of continuity is identically satisfied with the above form of velocity components.

Substituting the form of velocity components and pressure from (9) into the equations (1)— (3), we get the following ordinary non-linear differential equations for F and G:

$$\text{Re}\,(F'^2 - G^2 - 2FF'') = F''', \quad (11)$$

$$2\,\text{Re}\,(F'G - FG') = G'', \quad (12)$$

$$4\,\text{Re}\,FF' = P'_1 - 2F'', \quad (13)$$

where $\text{Re} = \rho\Omega d^2 \mid \mu$ is the Reynolds number. Substituting (10) in (6) — (8) the following ordinary non-linear differential equations are obtained:

$$\text{Re}\,(f'^2 - g^2 - 2ff'', = f''' - \sigma^2 f', \quad (14)$$

$$2\,\text{Re}\,(f'g - fg') = g'' - \sigma^2 g, \quad (15)$$

$$4\,\text{Re}\,ff' = p'_1 - 2f'' + 2\sigma^2 f, \quad (16)$$

where $\sigma = d \mid \sqrt{k}$.

The usual no-slip boundary conditions have been taken at the rotating disk, and it is assumed that there is no flow in r and θ directions at a large distance from the interface in the porous medium. These boundary conditions in terms of F, G, f and g can be written as:

$$F = F' = 0,\ G = 1 \qquad \text{at } \eta = 1 \quad (17)$$

$$f' \to 0,\ g \to 0 \qquad \text{as } \eta \to -\infty. \quad (18)$$

In a special type of porous medium, usually used in many industrial and bio-mechanical problems, the porosity is close to unity, for example a porous medium made of sparsly distributed fibrous material as in porous filter or a mushy zone in a rapidly freezing material of varying permeability (Sec Rudraiah (1983a)) the velocity is no longer uniform and one has to consider the variation of velocity (See Rudraiah (1983b)). For such cases Rudraiah (1983c). postulated that the tangential components of velocity and shearing stresses must decrease by constant factors in the porous medium to their corresponding values outside the porous medium. Hence the boundary conditions at the interface can be written in terms of F, G f and g as:

$$f = \phi f\ ,\ g = \phi G,\ f = F,\ f'' = \lambda\phi F'' \text{ and } g' = \lambda\phi G' \qquad \text{at } \eta = 0 \tag{19}$$

where λ is a constant. The condition of continuity of the normal stress at the interface gives that pressure p must be independent of r, i.e., it must be a function of only z.

3. ***Solution of the problem***

No analytic solution can be expected from the equations (11)—(16), so a regular perturbation scheme has been developed by expanding F, G, f and g in terms of Re as:

$$F = \sum_{n=0}^{\infty} \mathrm{Re}^n F_n,\ G = \sum_{n=0}^{\infty} \mathrm{Re}^n G_n,\ f = \sum_{n=0}^{\infty} \mathrm{Re}^n f_n,\ g = \sum_{n=0}^{\infty} \mathrm{Re}^n g_n, \tag{20}$$

Substituting (20) in the equations (11)—(12) and (14)—(15) and equating the coefficients of like powers of Re on both sides of the equations, we get the following system of linear ordinary differential equations:

$$F_0''' = 0, \tag{21}$$

$$G_0'' = 0 \tag{22}$$

$$F_1''' = F_0'^2 - 2F_0 F_0'' - G_0^2, \tag{23}$$

$$G_1'' = 2(F_0' G_0 - F_0 G_0'), \tag{24}$$

$$F_2''' = 2(F_0'F_1' - G_0 G_1 - F_0 F_1'' - F_0'' F_1), \tag{25}$$

$$G_2'' = 2(F_0' G_1 + F_1' G_0 - F_1 G_0' - F_0 G_1') \tag{26}$$

$$F_3''' = F_1'^2 + 2F_0' F_2' - G_1^2 - 2G_0G_2 - 2F_0F_2'' - 2F_1 F_1'' - 2F_0'' F_2, \tag{27}$$

$$G3'' = 2(F_0' G_2 + F_1'G_1 + F_2'G_0 - F_0G_2' - F_1G_1' - F_2G_0'), \tag{28}$$

... ...

... ...

$$f_0''' - \sigma^2 f_0' = 0, \tag{29}$$

$$g_0'' - \sigma^2 g_0 = 0, \tag{30}$$

$$f_1''' - \sigma^2 f_1' = f_0'^2 - 2f_0 f_0'' - g_0^2, \tag{31}$$

$$g_1'' - \sigma^2 g_1 = 2(f_0' g_0 - f_0 g_0') \tag{32}$$

$$f_2''' - \sigma^2 f_1' = 2(f_0' f_1' - g_0 g_1 - f_0 f_1'' - f_0'' f_1), \tag{33}$$

$$g_2'' - \sigma^2 g_2 = 2(f_0' g_1 + f_1' g_0 - f_1 g_0' - f_0 g_1'), \tag{34}$$

$$f_3''' - \sigma^2 f_3' = f_1'^2 + 2f_0' f_2' - g_1^2 - 2g_0 g_2 - 2f_0 f_2' - 2f_1 f_1'' - 2f_0'' f_2, \tag{35}$$

$$g_3'' - \sigma^2 g_3 = 2(f_0' g_2 + f_1' g_1 + f_2' g_0 - f_0 g_2' - f_1 g_1' - f_2 g_0'). \tag{36}$$

... ...

... ...

etc.

The boundary conditions (17), (18) and (19) can be writen in terms of F_n, G_n, f_n and g_n as:

$$F_n = F_n' = G_{n+1} = 0,\ G_0 = 1 \qquad \text{at } \eta = 1, \tag{37a}$$

$$f_n \to 0,\ g_n \to 0 \qquad \text{as } \eta \to \infty, \tag{37b}$$

$$f_n = \phi F_n',\ g_n = \phi G_n,\ f_n = F_n,\ f_n'' = \lambda\phi F_n'',\ g_n' = \lambda\phi G_n' \text{ at } \eta = 0, \tag{37c}$$

where $n = 0, 1, 2, 3, \ldots$

Solving (21)—(36) under the boundary conditions (37), we get

$$F_0 = 0,\ f_0 = 0,\ G_0 = \frac{\lambda + \phi\eta}{\lambda + \sigma},\ g_0 = \frac{\lambda\sigma}{\lambda + \sigma} e^{\sigma\eta} \tag{38}$$

$$F_1 = -\frac{1}{(\lambda + \sigma)^2}$$

$$\left\{\frac{\lambda^2}{2}\left(\frac{2}{3} - \eta + \frac{\eta^3}{3}\right) + \frac{\lambda\sigma}{3}\left(\frac{3}{4} - \eta + \frac{\eta^4}{4}\right) + \frac{\sigma^2}{12}\left(\frac{4}{5} - \eta + \frac{\eta^5}{5}\right)\right\}$$

$$-\frac{1}{(\lambda + \sigma)^3}\left(\frac{\lambda^2\sigma}{3\sigma} - \frac{\lambda^2\sigma}{2} - \frac{\sigma^3}{12}\right)\left(\frac{1}{2} - \eta + \frac{\eta^2}{2}\right) \tag{39}$$

$$f_1 = \left(\frac{1}{\lambda + \sigma}\right)^3 \left[\left(\frac{\lambda^3\phi^2}{3\sigma^3} + \frac{2\lambda^2\phi^2}{3\sigma^2} + \frac{\lambda^3\phi}{2\sigma} + \frac{\lambda^2\phi}{3} + \frac{\lambda\phi\sigma}{12}\right) e^{\sigma\eta)}\right.$$

$$-\left(\frac{\lambda^3\phi^2}{6\sigma^3} + \frac{\lambda^2\phi^2}{6\sigma^2}\right) e^{2\sigma\eta} -$$

$$\left.\left(\frac{\lambda^3\phi^2}{6\phi^3} + \frac{\lambda^2\phi^2}{2\sigma^2} + \frac{\lambda^2\sigma}{3} + \frac{3\lambda\sigma^2}{20} + \frac{\lambda^3\phi}{2\sigma} + \frac{\lambda^2\phi}{3} + \frac{\lambda\phi\sigma}{12} + \frac{\lambda^2\phi}{6\sigma} + \frac{\lambda^3}{3} + \frac{\sigma^3}{40}\right)\right] \tag{40}$$

$$G_1 = 0,\ g_1 = 0,\ F_2 = 0,\ f_2 = 0. \tag{41}$$

$$G_2 = -\frac{1}{(\lambda+\sigma)^3}\,[\lambda^3\,(\frac{5}{12} - \frac{\eta^2}{2} + \frac{\eta^4}{12}) + \frac{4\lambda^2\sigma}{3}(\frac{9}{20} - \frac{\eta^2}{2} + \frac{\eta^5}{20}) + \frac{2\lambda\sigma^2}{3}(\frac{7}{15}$$

$$- \frac{\eta^2}{2} + \frac{\eta^6}{30}) + \frac{2\sigma^3}{15}(\frac{10}{21} - \frac{\eta^2}{2} + \frac{\eta^7}{42})] - \frac{1}{(\lambda+\sigma)^4}[(\frac{2\lambda^3\phi}{3\sigma} - \lambda\,\sigma - \frac{2\lambda^2\sigma^2}{3}$$

$$- \frac{\lambda\sigma^3}{6})(\frac{1}{3} - \frac{\eta^2}{2} + \frac{\eta^3}{6}) + (\frac{\lambda^2\phi}{3} - \frac{\lambda\sigma^2}{2} - \frac{\lambda\sigma^3}{3} - \frac{\sigma^4}{12}) \times (\frac{5}{12} - \frac{\eta^2}{2} + \frac{\eta^4}{12})]$$

$$- \frac{1}{(\lambda+\sigma)^5}[\frac{\lambda^4\eta^2}{12\sigma^3} + \frac{5\lambda^3\phi^2}{12\sigma^2} + \frac{\lambda^3\sigma}{6\sigma} + \frac{\lambda^3\sigma}{3} + \frac{3\lambda^2\phi^2}{20} + \frac{\lambda^3\phi}{9} + \frac{\lambda^4}{3} + \frac{\lambda^4\phi}{2\sigma} + \frac{\lambda\sigma^3}{40} -$$

$$\frac{\lambda^3\phi\sigma}{18} - \frac{5\lambda^2\sigma}{12} - \frac{41\lambda^3\sigma^2}{60} - \frac{173\lambda^2\sigma^3}{360} - \frac{227\lambda\sigma^4}{1260} - \frac{29\sigma^5}{1008}](1-\eta), \tag{42}$$

$$g_2 = -\frac{1}{(\lambda+\sigma)^3}\frac{\lambda^3\phi^3}{24\sigma^4}e^{3\sigma\eta} + \frac{1}{(\lambda+\sigma)^4}(\frac{\lambda^4\phi^3}{6\sigma^3} + \frac{\lambda^3\phi^3}{2\sigma^2} + \frac{\lambda^3\phi^2}{6\sigma} + \frac{\lambda^3\phi\sigma}{3}$$

$$+ \frac{\lambda^3\phi^2}{3} + \frac{\lambda^4\phi}{3} + \frac{\lambda^4\phi^2}{2\sigma} + \frac{3\lambda^2\phi\sigma^2}{20} + \frac{\lambda^2\phi^2\sigma}{12} + \frac{\lambda\phi\sigma^3}{40})\eta e^{\sigma\eta} + \frac{1}{(\lambda+\sigma)^5}(\frac{\lambda^5\phi^3}{24\sigma^4}$$

$$- \frac{3\lambda^3\phi^3}{8\sigma^2} - \frac{5\lambda^5\phi}{12} - \frac{41\lambda^4\phi\sigma}{60} - \frac{173\lambda^3\phi\sigma^2}{360} - \frac{227\lambda^2\phi\sigma^3}{1260} - \frac{29\lambda\phi\sigma^4}{1008} - \frac{\lambda^3\phi^2}{6\sigma}$$

$$- \frac{\lambda^3\phi\sigma}{3} - \frac{17\lambda^3\phi^2}{36} - \frac{\lambda^4\phi}{3} - \frac{13\lambda^4\phi^2}{18\sigma} - \frac{3\lambda^2\phi\sigma^2}{20} - \frac{\lambda^2\phi^2\sigma}{12} - \frac{\lambda\phi\sigma^3}{40})\,e^{\sigma\eta}, \tag{43}$$

$$F_3 = \frac{1}{(\lambda+\sigma)^4}[\lambda^4(\frac{311}{1260} - \frac{47}{120}\eta + \frac{13}{72}\eta^3 - \frac{\eta^4}{36} - \frac{\eta^5}{120} + \frac{\eta^7}{2520}) + \lambda^3\sigma$$

$$(\frac{31}{72} - \frac{211}{315}\eta + \frac{23}{90}\eta^3 + \frac{\eta^4}{72} - \frac{\eta^5}{36} - \frac{\eta^6}{360} + \frac{\eta^8}{2520}) + \lambda^2\sigma^2(\frac{3347}{11340} -$$

$$\frac{1511}{3360}\eta + \frac{49}{360}\eta^3 + \frac{2}{45}\eta^4 - \frac{13}{720}\eta^5 - \frac{\eta^6}{108} + \frac{\eta^7}{840} + \frac{11\eta^9}{90720}) + \lambda\sigma^3(\frac{1737}{18144} -$$

$$\frac{6461\eta}{45360} + \frac{23\eta^3}{756} + \frac{7\eta^4}{270} - \frac{\eta^5}{225} - \frac{13\eta^6}{2160} + \frac{\eta^7}{1260} + \frac{11\eta^{10}}{453600}) + \sigma^4(\frac{34}{2835} - \frac{973\eta}{56700}$$

$$+\frac{\eta^3}{864}+\frac{\eta^4}{189}-\frac{\eta^6}{675}+\frac{\eta^7}{5040}+\frac{\eta^{11}}{453600})]+\frac{1}{(\lambda+\sigma)^5}[\Delta_4(\frac{17}{45}-\frac{3}{5}\eta+\frac{5}{18}\eta^3-\frac{\eta^4}{24}-\frac{\eta^5}{60}+\frac{\eta^6}{360})\ \Delta_5(\frac{439}{840}-\frac{4}{5}\eta+\frac{\eta^3}{4}+\frac{\eta^4}{12}-\frac{\eta^5}{20}-\frac{\eta^6}{120}+\frac{\eta^7}{420})+\Delta_6(\frac{739}{1680}-\frac{22}{35}\eta+\frac{\eta^3}{30}+\frac{5\eta^4}{12}-\frac{\eta^6}{15}+\frac{\eta^7}{70}-\frac{\eta^8}{1680})]+\frac{1}{(\lambda+\sigma)^6}[\Delta_7\{\lambda(\frac{5}{24}-\frac{\eta}{3}+\frac{\eta^3}{6}-\frac{\eta^4}{24})+\sigma(\frac{7}{120}-\frac{\eta}{12}+\frac{\eta^4}{24}-\frac{\eta^5}{60})\}]+A(\frac{1}{2}\eta+\frac{\eta^2}{2}) \qquad (44)$$

$$f_3=\frac{D}{\sigma}e^{\sigma\eta}+\frac{1}{(\lambda+\sigma)^6}[-\frac{\Delta_1{}^2}{6\sigma}e^{2\sigma\eta}-\frac{\lambda\phi\Delta_2}{3\sigma^3}e^{2\sigma\eta}+\frac{\Delta_1\Delta_3}{\lambda\phi}\eta e^{\sigma\eta}-\frac{\Delta_1\Delta_3}{\lambda\phi\sigma}e^{\sigma\eta}]+\frac{1}{(\lambda+\sigma)^5}[\frac{\lambda^2\phi^2}{24\sigma^4}e^{3\sigma\eta}+\frac{2\lambda\phi\Delta_3}{9\sigma^4}e^{2\eta}-\frac{2\lambda\phi\Delta_3}{6\sigma^3}\eta e^{2\sigma\eta}+\frac{\lambda\phi\Delta_3}{6\sigma^4}e^{2\eta}]-\frac{\lambda^4\phi^4}{2160\sigma^7(\lambda+\sigma)^4}e^{4\sigma\eta}+E. \qquad (45)$$

where

$$D=\frac{1}{(\lambda+\sigma)^5}(\frac{\lambda^4\phi^4}{135\sigma^5}+\frac{\lambda^5\phi^4}{540\sigma^6}-\frac{47\lambda^5\phi}{120}-\frac{211\lambda^4\phi\sigma}{315}-\frac{1511\lambda^3\phi\sigma^2}{3360}-\frac{6461\lambda^2\phi\sigma^3}{45360}-\frac{973\lambda\phi\sigma^4}{56700})+\frac{1}{(\lambda+\sigma)^6}(-\frac{3\lambda\phi\Delta_4}{5}-\frac{4\lambda\phi\Delta_5}{5}-\frac{22\lambda\phi\Delta_6}{35}-\frac{3\lambda^2\phi^2\Delta_1}{8\sigma^2}-\frac{2\lambda\phi\Delta_3}{9\sigma^2}-\frac{\lambda^3\phi^2}{8\sigma^3}-\frac{4\lambda^2\phi\Delta_3}{9\sigma^3})+\frac{1}{(\lambda+\sigma)^7}(\frac{2\sigma\Delta_1{}^2}{3}+\frac{\lambda\Delta_1{}^2}{3}+\frac{2\lambda^2\phi\Delta_2}{3\sigma^2}-\frac{\lambda^2\phi\Delta_7}{3}-\frac{\lambda\phi\sigma\Delta_7}{12}-\frac{\sigma\Delta_1\Delta_3}{\lambda\phi}+\frac{4\lambda\varphi\Delta_2}{3\sigma}),$$

$$E=\frac{1}{(\lambda+\sigma)^4}(\frac{\lambda^4\phi^4}{2160}+\frac{311\lambda^4}{1260}+\frac{31}{72}\lambda^3\sigma+\frac{3347}{11340}\lambda^2\sigma^3+\frac{1737}{18144}\lambda\sigma^3+\frac{34\sigma^4}{2835}-\frac{\lambda^3\phi^3}{270\sigma^5})+\frac{1}{(\lambda+\sigma)^5}(\frac{17\Delta_4}{45}+\frac{439}{840}\Delta_5+\frac{739}{1680}\Delta_6-$$

$$\frac{\lambda^2\phi^2\Delta_1}{24\sigma^4} - \frac{7\lambda\phi\Delta_3}{18\sigma^4} - \frac{3\lambda\phi\Delta_1}{16\sigma^2} - \frac{\Delta_3}{9\sigma^2} - \frac{\lambda^3\phi^3}{270\sigma^4} - \frac{\lambda^4\phi^3}{1080\sigma^5} + \frac{47}{240}\lambda^4\sigma + \frac{211}{630}$$

$$\lambda^3\sigma^2 + \frac{1511}{6720}\lambda^2\sigma^3 + \frac{6461}{90720}\lambda\sigma^4 + \frac{973\sigma^5}{113400} - \frac{\lambda^4\phi^4}{135\sigma^6} - \frac{\lambda^5\phi^4}{540\sigma^7} + \frac{47\lambda^5\phi}{120\sigma} +$$

$$\frac{211\lambda^4\phi}{315} + \frac{1511\lambda^3 o\sigma}{3360} + \frac{6461\lambda^2\phi\sigma^2}{45360} + \frac{973\lambda\phi\sigma^3}{56760}) + \frac{1}{(\lambda+\sigma)^6}(-\frac{5\lambda\Delta_7}{24}$$

$$+ \frac{7\sigma\Delta_7}{120} + \frac{\Delta_1^2}{6\sigma} + \frac{\lambda o\Delta_2}{3\sigma^3} + \frac{\Delta_1\Delta_3}{\lambda\phi\sigma} - \frac{\sigma\Delta_1\Delta_3}{2\lambda^2\phi^2} + \frac{\sigma\Delta_1^2}{3\lambda\phi} + \frac{2\Delta_2}{3\sigma} + \frac{3\sigma\Delta_4}{10}$$

$$+ \frac{2\sigma\Delta_5}{5} + \frac{11\sigma\Delta_6}{35} + \frac{3\lambda\phi\Delta_1}{16\sigma} + \frac{\Delta_3}{9\sigma} + \frac{\lambda^2\phi}{16\sigma^2} + \frac{2\lambda\Delta_3}{9\sigma^2} + \frac{3\lambda\phi\Delta_4}{3\sigma} + \frac{4\lambda\phi\Delta_5}{5\sigma} +$$

$$\frac{22\lambda\phi\Delta_6}{35\sigma} + \frac{3\lambda^4\phi^2\Delta_1}{8\sigma^3} + \frac{2\lambda\phi\Delta_3}{9\sigma^3} + \frac{\lambda^3\phi^2}{8\sigma^4} + \frac{4\lambda^2\phi\Delta_3}{9\sigma^4}) + \frac{1}{(\lambda+\sigma)^7}$$

$$(\frac{\sigma^2\Delta_1\Delta_3}{2\lambda^2\phi^2} + \frac{\sigma^2\Delta_7}{24} + \frac{\lambda\sigma\Delta_7}{6} \frac{2\lambda\Delta_2}{6\sigma} - \frac{\sigma\Delta_1^2}{6\phi} - \frac{\sigma^2\Delta_1^2}{3\lambda\phi} - \frac{2\Delta_2}{3} - \frac{2\Delta_1^2}{3} - \frac{\lambda\Delta_1^2}{3\sigma} -$$

$$\frac{2\lambda^2\phi\Delta_2}{3\sigma^3} + \frac{\lambda^2\phi\Delta_7}{3\sigma} + \frac{\lambda\phi\Delta_7}{12} + \frac{\Delta_1\Delta_3}{\lambda\phi} - \frac{4\lambda\phi\Delta_2}{3\sigma^2})$$

$$\Delta_1 = \frac{\lambda^3\phi^2}{3\sigma^3} + \frac{2\lambda^2\phi^2}{3\sigma^2} + \frac{\lambda^2\phi}{2\sigma} + \frac{\lambda^2\phi}{3} + \frac{\lambda\phi\sigma}{12},$$

$$\Delta_2 = \frac{\lambda^5\phi^3}{24\sigma^3} - \frac{3\lambda^3\phi^3}{8\sigma^2} - \frac{5\lambda^5\phi}{12} - \frac{41\lambda^4\phi\sigma}{60} - \frac{173\lambda^3\phi\sigma^2}{360} - \frac{227\lambda^2\phi\sigma^3}{1260}$$

$$- \frac{29\lambda\phi\sigma^4}{1008} - \frac{\lambda\phi^2}{6\sigma} - \frac{\lambda^3\phi\sigma}{3} - \frac{17\lambda^3\phi^2}{36} - \frac{\lambda^4\phi}{3} - \frac{13\lambda^4\phi^2}{18\sigma} - \frac{3\lambda^2\phi\sigma^2}{20}$$

$$- \frac{\lambda^3\phi^2\sigma}{12} - \frac{\lambda\phi\sigma^2}{40},$$

$$\Delta_3 = \frac{\lambda^4\phi^3}{6\sigma^3} + \frac{\lambda^3\phi^3}{2\sigma^2} - \frac{\lambda^3\phi^2}{6\sigma} + \frac{\lambda^3\phi\sigma}{3} + \frac{\lambda^3\phi^2}{3} + \frac{\lambda^4\phi}{3} + \frac{\lambda^4\phi^2}{2\sigma} + \frac{3\lambda^2\phi\sigma^2}{20}$$

$$+ \frac{\lambda^2\phi^2\sigma}{12} + \frac{\lambda\phi\sigma^3}{40},$$

$$\Delta_4 = \frac{\lambda^4\phi}{3\sigma} - \frac{\lambda^4\sigma}{2} - \frac{\lambda^3\sigma^2}{3} - \frac{\lambda^2}{12},$$

$$\Delta_5 = \frac{2\lambda^3\phi}{9} - \frac{\lambda^3\sigma^2}{3} - \frac{2\lambda^2\sigma^3}{9} - \frac{\lambda\sigma^4}{18},$$

$$\Delta_6 = \frac{\lambda^2\phi\sigma}{18} - \frac{\lambda^2\sigma^3}{12} - \frac{\lambda\sigma^4}{18} - \frac{\sigma^5}{72}$$

and

$$\Delta_7 = \frac{\lambda^4\phi^3}{6\sigma^3} + \frac{5\lambda^3\phi^2}{6\sigma^2} + \frac{\lambda^3\phi}{3\sigma} + \frac{2\lambda^3\sigma}{3} + \frac{3\lambda^2\sigma^2}{10} + \frac{2\lambda^3\phi}{9} + \frac{2\lambda^4}{3}$$

$$+ \frac{\lambda^4\phi}{\sigma} + \frac{\lambda\sigma^3}{20} - \frac{\lambda^3\phi\sigma}{9} - \frac{5\lambda^4\sigma}{6} - \frac{41\lambda^3\phi^2}{30} - \frac{173\lambda^2\sigma^3}{180} - \frac{227\lambda\sigma^4}{630} - \frac{29\sigma^5}{504}.$$

4. *Discussion*

The expressions (5c) and (5b) for shearing stresses T_{zr} and $T_{z\theta}$ at the interface become

$$\frac{[T_{zr}]_{z=0}}{\mu\Omega r / d} = \mathrm{Re}\,[\frac{1}{(\lambda+\sigma)^3}(\frac{\lambda\phi\sigma^3}{12} + \frac{\lambda^2\phi\sigma^2}{3} + \frac{\lambda^3\phi\sigma}{2} - \frac{\lambda^3\phi^2}{3\sigma})] + \mathrm{Re}^3\,[D\sigma -$$

$$\frac{\lambda^4\phi^4}{135\sigma^4(\lambda+\sigma)^4} + \frac{1}{(\lambda+\sigma)^5}(\frac{3\lambda^2\phi^2\Delta_1}{8\sigma^2} + \frac{2\lambda\phi\Delta}{9\sigma}\ 3) +$$

$$\frac{1}{(\lambda+\sigma)^6}(\frac{\sigma\Delta_1\Delta_3}{\lambda\phi} - \frac{2\sigma\Delta_1{}^2}{3} - \frac{4\lambda\phi\Delta_2}{3\sigma})]$$

$$\frac{[T_{z\theta}]_{z=0}}{\mu\Omega r / d} = \frac{\lambda\phi\sigma}{\lambda+\sigma} + \mathrm{Re}^2\,[\frac{\lambda^3\phi^3}{8\sigma^3(\lambda+\sigma)^3} + \frac{1}{(\lambda+\sigma)^4}(\frac{\lambda^4\phi^3}{6\sigma^3}$$

$$+ \frac{\lambda^3\phi^3}{2\sigma^2} + \frac{\lambda^3\phi^2}{6\sigma} + \frac{\lambda^3\phi\sigma^3}{3} + \frac{\lambda^3\phi^2}{3} + \frac{\lambda^4\phi}{3} + \frac{\lambda^4\phi^4}{2\sigma} + \frac{3\lambda^2\phi\sigma^2}{20} + \frac{\lambda^2\phi^2\sigma}{12} +$$

$$\frac{\lambda\phi\sigma^3}{40}) + \frac{1}{(\lambda+\sigma)^5}(\frac{\lambda^5\phi^3}{24\sigma^3} - \frac{3\lambda^3\phi^3}{8\sigma} - \frac{5\lambda^5\phi\sigma}{12} - \frac{41\lambda^4\phi\sigma^2}{60} - \frac{173\lambda^3\phi\sigma^3}{360}$$

$$\frac{227\lambda^2\phi\sigma^4}{1260} - \frac{29\lambda\phi\sigma^5}{1008} - \frac{\lambda^3\phi^2}{6} - \frac{\lambda^3\phi\sigma^2}{3} - \frac{17\lambda^3\phi^2\sigma}{36} - \frac{\lambda^4\phi\sigma}{3}$$

$$- \frac{13\lambda^4\phi^2}{18} - \frac{3\lambda^2\phi\sigma^3}{20} - \frac{\lambda^2\phi^2\phi^2}{12} - \frac{\lambda\phi\sigma^4}{40})]$$

Though the expressions (38)-(45) have been derived by taking the radius of the disk infinite, it is applicable to finite disk if the edge effects are neglgible. It is possible when the radius of the disk 'a' >> 'd'. To study the effect of permeability of the porous medium and Reynolds number on the stresses, F'' (l), f' (0), G' (l) and g' (0) are tabulated for various values of σ and Re in Table 1. It is observed from the table that the shearing stresses at the rotating disk as well as at the interface increase with the increase of the Reynolds number and with the decrease of the permeability of the porous medium.

The axial velocity component f at infinity in the porous medium for σ = 1, 2, 3 and Re = 1 takes the values 0.02673, 0.02291, 0.02200 respectively and for R_e = 0,5, it takes the values 0.16558, 0.01355, 0.01295 respectively which shows that the radial velocity at infinity increases with the increase of the Reynolds number and with the decrease of the permeability of the porous medium. The expression for g (η) given by (30) suggests that the depth of penetration $\sigma\eta = z / \sqrt{k}$ is inversely proportional to the square root of the permeability of the porous medium and is independent of the distance of the rotating disk from the interface. These conclusions are in good agreement with the results obtained by Srivastava and Barman (1997).

The graph of velocity components $f'(\eta)$, $g(\eta)$ and $-f(\eta)$ has been plotted against the distance from the interface inside the porous medium for σ = 1, 2, 3 by taking λ = 0.077 and ϕ = 0.39 (See Fig. 2). It is observed from the graph that the axial velocity has maximum value at infinity in the porous medium and decrease gradually towards the interface. The tangential velocity component in the porous medium decreases exponentially and vanishes at a large distance. All velocity components in the porous medium reduce with the decrease of the values of permeability parameter σ. The graph of $F'(\eta)$ and — $F(\eta)$ have been plotted in Fig. 3 and Fig. 4 respectively. It is observed from the graphs that due to the presence of porous medium the radial velocity component does not vanish at the interface. This problem has got direct application in Geology for the extraction of the fluid from the porous medium. Filling the gap between the porous medium and a solid plate with the fluid present in the porous medium and then rotating the disk can do this.

Table. 1 : Shearing stresses at the rotating disk and at the interface.

σ	G' (1)		g' (0)		F" (1)		f' (0)	
	Re=0.5	Re=1.0	Re=0.5	Re=1.0	Re=0.5	Re=1.0	Re=0.5	Re=1.0
1.0	0.93168	0.95786	0.00768	0.02703	-0.15364	-0.30579	0.00094	0.00181
1.5	0.95658	0.98040	0.02836	0.02771	-0.14429	-0.28877	0.00104	0.00200
2.0	0.96906	0.99184	0.02871	0.02806	-0.13951	-0.27944	0.00109	0.00210
2.5	0.97657	0.99875	0.02893	0.02828	-0.13665	-0.27326	0.00112	0.00216
3.0	0.98158	1.00338	0.02907	0.02842	-0.13473	-0.27009	0.00114	0.00220

ACKNOWLEDGEMENTS

I would like to express my gratitude to Prof. A. C. Srivastava, Emeritus Scientist (CSIR), Department of Mathematics, Lucknow University, Lucknow, for his valuable suggestions during the preparation of this paper.

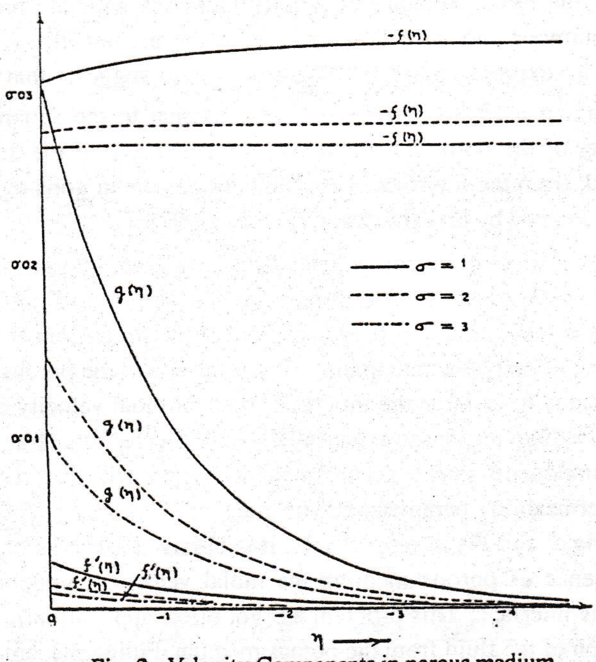

Fig. 2 : Velocity Components in porous medium

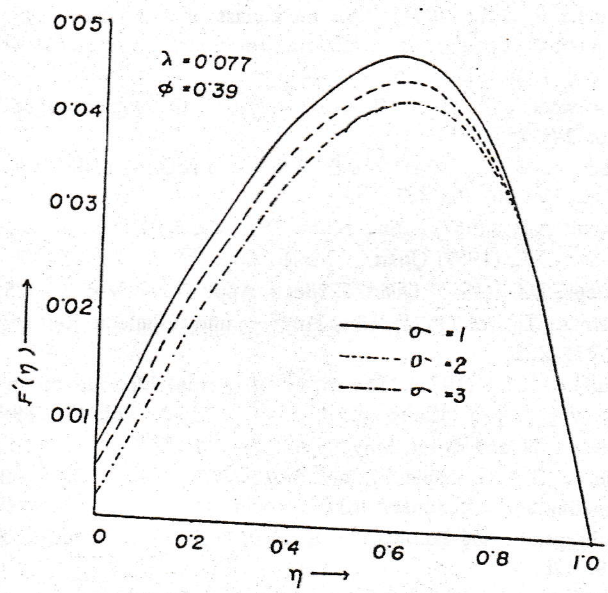

Fig. 3 : RADIAL VELOCITY COMPONENT $F'(\eta)$

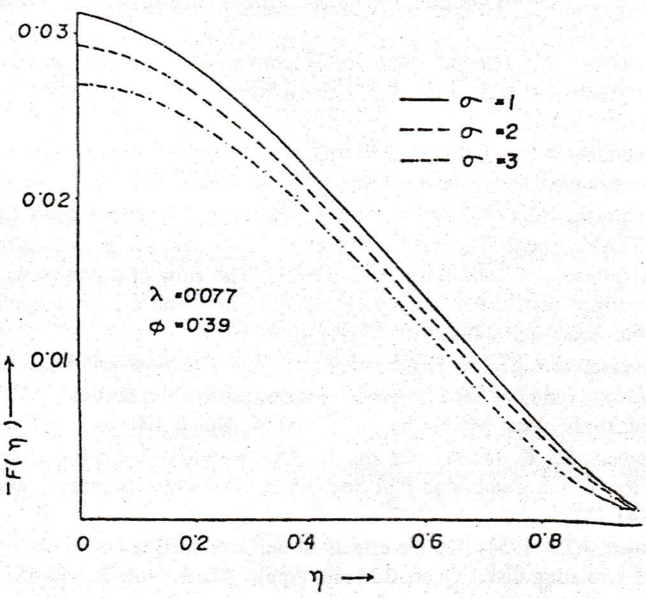

Fig. 4 : AXIAL VELOCITY COMPONENT -

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