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ON FUZZY PRE-$T_{1/2}$ AND FG PRE - CONNECTED SPACES

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ABSTRACT

In this paper some generalizations of fuzzy pre-continuous functions are introduced and studied. Making use of the concept of fuzzy pre-open set several interesting concepts such as generalized fuzzy pre-extremally disconnected spaces, fuzzy generalized pre-compact spaces are studied and several examples are also given.

Key Words: Fuzzy almost pre-continuous, M-fuzzy pre-bicontinuous, fuzzy pre-$T_{1/2}$ space, generalized fuzzy pre-$T_{1/2}$ space, fuzzy generalized pre-compact space, generalized fuzzy pre-extremally disconnected and fg pre-conected spaces.

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1.Introduction

The fuzzy concept has penetrated almost all branches of Mathematics since the introduction of the concept of fuzzy set by L. A. Zadeh [11]. Fuzzy sets have applications in many fields such as information [9] and control [10]. The Theory of fuzzy topological spaces was introduced and developed by Chang[6]. The concept of fuzzy pre-open set was introduced and studied along with the concept to fuzzy pre-continuity by Bin Shahna in [5]. In [2] fuzzy separation axioms have been introduced and investigated with the help of fuzzy pre-open sets. The motivation for this paper is to introduce some generalizations of fuzzy pre-continuous functions and study their various aspects in the direction similar to that of [3].

2. Preliminaries.

By fuzzy topological space we shall mean the pair (X,T) where X is a non-empty set and T is a fuzzy topology in the sense of C.L. Chang [6].

Definition 1. Let λ be any fuzzy set in the fuzzy topological space (X,T). λ is called fuzzy pre-open [2] if $\lambda \leq$ Int cl. λ. The complement of a fuzzy pre-open set is called fuzzy pre-closed.

Definitions 2. Let λ be any fuzzy set in the fuzzy topological space (X,T). Then we define fpcl (λ) = fuzzy pre-closure of $\lambda = \wedge \{\mu/\mu$ fuzzy pre-closed and $\mu \geq \lambda\}$,

fpInt (λ) = fuzzy pre-interior of $\lambda = \vee \{\mu/\mu$ fuzzy pre-open and $\mu \geq \lambda\}$[2].

Note : For any fuzzy set λ, fpcl $(1-\lambda)$ = 1- fpIntλ and fpInt $(1-\lambda)$ = 1- fpclλ.

Definition 3. Let (X,T) and (Y, S) be any two fuzzy topological spaces.

f : (X, T) $\rightarrow$ (Y, S) is said to be fuzzy continuous [1] if $f^{-1}(\lambda)$ is fuzzy open (fuzzy closed) set of X for each fuzzy open (fuzzy closed) set λ of Y.

Definition 4. Let (X,T) and (Y, S) be any two fuzzy topological spaces. f : (X, T) $\rightarrow$ (Y, S) be a function. f is called M-fuzzy pre-continuous [2] if the inverse image of fuzzy pre-open set in Y is fuzzy pre-open in X.

Definition 5. Let (X, T) and (Y, S) be any two fuzzy topological spaces.

Let f : (X, T) $\rightarrow$ (Y, S) be a function. f is called fuzzy pre-open [2] if the image of each fuzzy pre-open set in X is fuzzy pre-open in Y.

Definition 6. Let (X, T) and (Y, S) be fuzzy topological spaces. X and Y are said to be M-fuzzy pre-homeomorphic [2] $\Leftrightarrow$ There exists f : (X, T) $\rightarrow$ (Y, S) such that f is 1-1, onto, M-fuzzy pre-continuous and fuzzy pre-open. Such an f is called fuzzy pre-homeomorphism.

Definitions 7. A fuzzy set λ of a fuzzy topological space X is called (i) a fuzzy regular open set [1] of X if Int cl $\lambda = \lambda$ and (ii) a fuzzy regular closed set [1] of X if cl Int $\lambda = \lambda$.

Definition 8. A mapping f : (X, T) $\rightarrow$ (Y, S) from a fuzzy topological space (X,T) to another fuzzy topological space (Y,S) is called a fuzzy almost continuous mapping if $f^{-1}(\lambda) \in$ (X, T) for each fuzzy regular open set λ of Y [1].

Definition 9. Let (X, T) be a fuzzy topological space and Y be an ordinary subset of X. Then $T_Y = \{ \lambda/Y/\lambda \in T \}$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology [1]. The pair (Y,T_Y) is called a fuzzy subspace of (X, T). (Y, T_Y) is called a fuzzy open / fuzzy closed / fuzzy pre-open fuzzy subspace if the characteristic function of Y i.e. χ_Y is fuzzy open / fuzzy closed / fuzzy pre-open respectively in (X,T).

Definition 10. A fuzzy topological space (X,T) is said to be fuzzy connected [7] iff the only fuzzy sets which are both fuzzy open and fuzzy closed are 0_X and 1_X.

Definition 11. Suppose (X,T) be any fuzzy topological space. X is said to be fuzzy extremally disconnected [4] if $\lambda \in T$ implies cl$\lambda \in T$.

Definition 12. Let (X,T) be any fuzzy topological space. X is called fuzzy pre-extremally disconnected if the pre-closure of a fuzzy pre-open set is fuzzy pre-open.

Definition 13. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A function f : (X,T) $\rightarrow$ (Y,S) is said to be strongly fuzzy continuous [3] if $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed for each fuzzy open set λ in Y.

Definition 14. Let (X,T) and (Y,S) be any two fuzzy topological spaces. f : (X,T) $\rightarrow$ (Y,S) is said to be strongly fuzzy pre-continuous if $f^{-1}(\lambda)$ is fuzzy pre-open and fuzzy pre-closed for each fuzzy set λ in Y.

Definition 15. Let (X,T) be a fuzzy topological space. A fuzzy set λ in X is called generalised fuzzy pre-open (in short gfpo) $\Leftrightarrow \mu \leq$ fpIntλ whenever μ is fuzzy pre-closed and $\mu \leq \lambda$.

Definition 16. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A map f : (X,T) → (Y,S) is called gfpre - continuous if the inverse image of every fuzzy pre-closed set in Y is gfpre - closed in X.

Definition 17. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A map f : (X,T) → (Y,S) is called fuzzy gc-pre-irresolute if the inverse image of every gfpre-closed set in Y is gfpre-closed in X.

Definition 18. Let (X,T) and (Y,S) be any two fuzzy topological spaces. f : (X,T) → (Y,S) is said to be perfectly fuzzy pre-continuous if the inverse image of every fuzzy pre-open set in Y is both fuzzy open and fuzzy closed in X.

Notation. We shall denote the complement of any fuzzy set λ either by λ' or by 1-λ.

Pre-closure of any non-zero fuzzy set in A⊂X is denoted as fpcl$_A\lambda$.

For concepts not defined in this paper we refer to [6].

3. Fuzzy almost pre-continuous mappings.

Definition 19. A fuzzy set λ of a fuzzy topological space X is called (i) a fuzzy regular pre-open set of X if fpInt [fpclλ] = λ and (ii) a fuzzy regular pre-closed set of X if fpcl [fpIntλ] = λ.

From this we can deduce the following simple **properties**

1. A fuzzy set λ of a fuzzy topological space X is regular pre-open $\Leftrightarrow$1-λ is fuzzy regular pre-closed.
2. Every fuzzy regular pre-open set is a fuzzy pre-open set.
3. Every fuzzy regular pre-closed set is fuzzy pre-closed.
4. (a) The pre-closure of a fuzzy pre-open set is a fuzzy regular pre-clsoed set and

(b) The pre-interior of a fuzzy pre-closed set is a fuzzy regular pre-open set.

Proof. (a) Let λ be a fuzzy pre-open set of a fuzzy space X. fpInt [fpclλ] $\leq$ fpcl λ, fpcl{fpInt [fpclλ]}$\leq$ fpcl λ (by definition)-(1) Now λ is fuzzy pre-open and $\lambda\leq$fpcl λ implies that $\lambda\leq$fpInt[fpclλ] and hence fpcl $\lambda\leq$fpcl{fpInt [fpclλ]} - (2). From (1) and (2), we get fpclλ = fpcl {fpInt [fpclλ]}.

(b) Let λ be a fuzzy pre-closed set of a fuzzy topological space X. fpcl [fpInt λ] $\geq$ fpInt λ, fpInt {fpcl[fpIntλ]} $\geq$ fpIntλ-(3) (by definition) Now λ is pre-closed and $\lambda\geq$fpIntλ. That is $\lambda\geq$fpcl[fpIntλ] and therefore fpInt $\lambda \geq$ fpInt{fpcl[fpIntλ]} - (4). From (3) and (4), we get fpIntλ = fpInt{fpclnt(fpInt - λ)}. Thus fpInt λ is a fuzzy pre-regular closed set.

Example 1. Let X = {a,b}, Define T = {0_X,1_X,λ} where λ : X $\to$[0,1] is such that λ(a) =1, λ(b)=1/2. Consider a fuzzy set μ: X$\to$ [0,1] such that μ (a)=0, μ (b)=3/4. Since clμ=1,Intclμ=1>μ. Therefore μ is fuzzy pre-open. Since Intμ=0, clInt μ=0. Hence μ is fuzzy pre-closed. Therefore fpcl μ = μ and fpInt [fpcl(μ)]=μ. Hence μ is fuzzy regular pre-open. Since fpcl fpInt μ =μ, μ is also fuzzy regular pre-closed.

Definition 20. A mapping f : (X,T) $\to$(Y,S) from a fuzzy topological space (X,T) to another fuzzy topological space (Y,S) is called a fuzzy almost pre-continuous mapping if $f^{-1}(\lambda)$ is fuzzy pre-open for each fuzzy regular pre-open set λ of Y.

Example 2. Let (X,T) be a fuzzy indiscrete topological space. Let (Y,S) be a fuzzy topological space. Let f : (X,T) $\to$ (Y,S) be any map. Then f is fuzzy almost pre-continuous.

Proposition 1. Let f : (X,T) $\to$ (Y,S) be a mapping. Then the following are equivalent.

(i) f is a fuzzy almost pre-continuous mapping.

(ii) $f^{-1}(\mu)$ is a fuzzy pre-closed set for each fuzzy regular pre-closed set μ of Y.

(iii) $f^{-1}(\lambda) \leq$ fpInt f^{-1}[fpInt(fpclλ)] for each fuzzy pre-open set λ or Y.

(iv) fpcl f^{-1}[fpcl(fpIntμ)] $\leq f^{-1}(\mu)$ for each fuzzy pre-closed set μ of Y.

Proof. (i) $\Rightarrow$ (ii) Let μ be a fuzzy regular pre-closed set of Y. Then μ' is fuzzy regular pre-open. Therefore, by (i) $f^{-1}(\mu')$ is fuzzy pre-open. This means that $(f^{-1}(\mu'))'$ is fuzzy pre-closed. That is $f^{-1}(\mu')$ is fuzzy pre-closed. Similarly one can show that (ii) $\Rightarrow$ (i).

(i) $\Rightarrow$ (iii). Let λ be a fuzzy pre-open set of Y. Then we have $f^{-1}(\lambda) \leq f^{-1}$[fpInt(fpcl$\lambda$)]. By the property 4(b), fpInt(fpclλ) is fuzzy regular pre-open set of Y, hence f^{-1}[fpInt(fpclλ)] is a fuzzy pre-open set of X. Thus $f^{-1}(\lambda) \leq$ [f^{-1}[fpInt(fpclλ)] = fpint{f^{-1}[fpInt(fpclλ)]}.

(iii) $\Rightarrow$ (i) Let λ be a fuzzy regular pre-open set of Y. Then we have $f^{-1}(\lambda) \leq$ fpIntf^{-1}[fpIntf^{-1}(fpclλ)]=fpInt$f^{-1}(\lambda)$. But fpInt $f^{-1}(\lambda) \leq f^{-1}(\lambda)$. Thus $f^{-1}(\lambda)$ = fpInt$f^{-1}(\lambda)$ shows that $f^{-1}(\lambda)$ is a fuzzy pre-open set of X.

(ii) $\Rightarrow$ (iv) Let μ be a fuzzy pre-closed set of Y. Then $\mu \geq$ fpcl(fpInt μ). Therefore, $f^{-1}(\mu) \geq f^{-1}$[fpcl(fpInt μ)]. Since fpcl(fpIntμ) is a fuzzy regular pre-closed set of X, f^{-1}[fpcl (fpInt μ)] is a fuzzy pre-closed set of X. Then $f^{-1}(\mu) \geq f^{-1}$[fpcl(fpInt μ)] = fpcl f^{-1}[fpcl(fpIntμ)].

(iv) $\Rightarrow$ (ii) Let μbe a fuzzy regular pre-closed set of Y. Hence by assumption (iv) we have fpcl f^{-1}[fpcl(fpInt μ)]$\leq f^{-1}(\mu)$. That is fpcl $f^{-1}(\mu) \leq f^{-1}(\mu)$. But $f^{-1}(\mu) \leq$ fpcl$f^{-1}(\mu)$. Therefore, $f^{-1}(\mu)$ =fpcl$f^{-1}(\mu)$. This proves that $f^{-1}(\mu)$ is fuzzy pre-closed.

Remark. Let f:(X,T)$\rightarrow$(Y,S) be any M-fuzzy pre-continuous mapping. Then f is fuzzy almost pre-continuous.

4. Fuzzy bicontinuous, fuzzy pre-bicontinuous, M-fuzzy pre-bicontinuous and fuzzy gc-pre-biirresolute maps.

Note : The following definitions are the fuzzyfied concept of the bicontinuous mapping given in [8].

Definition 21. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let f : (X,T) $\rightarrow$ (Y,S) be any map. f is called fuzzy bicontinuous if it is onto and if (λ in Y is fuzzy open)$\Leftrightarrow$($f^{-1}(\lambda)$ in X is fuzzy open). Equivalently, if

(λ in Y is fuzzy closed)$\Leftrightarrow$($f^{-1}(\lambda)$ in X is fuzzy closed).

Definition 22. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let $f : (X,T) \rightarrow (Y,S)$ be any map. f is called fuzzy pre-bicontinuous if it is onto and if (λ in Y is fuzzy open)$\Leftrightarrow$($f^{-1}(\lambda)$ in X is fuzzy pre-open. Equivalantly, if (λ in Y is fuzzy closed)$\Leftrightarrow$($f^{-1}(\lambda)$ in X is fuzzy pre-closed).

Definition 23. Let (X,T) and (Y,S) be any two fuzzy topological spaces. Let $f : (X,T) \rightarrow (Y,S)$ be any map. f is called M-fuzzy pre-bicotinuous if it is onto and if (λ in Y is fuzzy pre-open)$\Leftrightarrow$($f^{-1}(\lambda)$ in X is fuzzy pre-open). Equivalently, if (λ in Y is fuzzy closed) $\Leftrightarrow$ ($f^{-1}(\lambda)$ in X is fuzzy pre-closed).

Proposition 2. if f is M-fuzzy pre-bicontinuous and one-to-one, then f is fuzzy pre-homeomorphism.

Proposition 3. Let $f : (X,T) \rightarrow (Y,S)$ be M-fuzzy pre - bicontinuous and $g : (Y,S) \rightarrow (Z,R)$. If h = gf is M-fuzzy pre-bicontinuous, then so is g. If h is M-fuzzy pre-bicontinuous, then so is g.

Proof.

$$\begin{tikzcd}
X \arrow[r, "f"] \arrow[dr, "h"'] & Y \arrow[d, "g"] \\
& Z
\end{tikzcd}$$

Let, λ be any fuzzy pre-open set in Z. Since h is M-fuzzy pre-continuous, $h^{-1}(\lambda)$ is fuzzy pre-open. $h^{-1}(\lambda)=f^{-1}(g^{-1}(\lambda))$ is fuzzy pre-open. Since by assumption, f is M-fuzzy pre-bicontinuous, $g^{-1}(\lambda)$ is fuzzy pre-open. This proves g is M-fuzzy pre-continuous.

Next, let us assume that h is M-fuzzy pre-bicontinuous and that $g^{-1}(\lambda)$ is fuzzy pre-open. Since f is M-fuzzy pre-bicontinuous, $f^{-1}(g^{-1}(\lambda))$ is fuzzy pre-open. $f^{-1}(g^{-1}(\lambda)) = h^{-1}(\lambda)$ is fuzzy pre-open. Since h is M-fuzzy pre-continuous, λ is fuzzy pre-open. Therefore g is M-fuzzy pre-bicontinuous.

Proposition 4. If $f : (X,T) \to (Y,S)$ is M-fuzzy pre-continuous, onto and pre-open, then f is M-fuzzy pre-bicontinuous.

Definition 24. Let (X,T) and (Y,S) be any two fuzzy topological spaces. A map $f : (X,T) \to (Y,S)$ is called fuzzy gc-pre-biirresolute if it is onto and if (λ in Y is gfpre-closed) $\Leftrightarrow$ ($f^{-1}(\lambda)$ in X is gfpre-closed).

Proposition 5. Let $f : (X,T) \to (Y,S)$ be gc-pre-biirresolute and $g : (Y,S) \to (Z,R)$. If h=gf is gc-pre-irresolute then so is g. If h is gc-pre-biirresolute then so is g.

$$\begin{tikzcd} X \arrow[r, "f"] \arrow[dr, "h"'] & Y \arrow[d, "g"] \\ & Z \end{tikzcd}$$

Proof. Let λ be gfpre-open in Z. Since h is gc-pre-irresolute, $h^{-1}(\lambda)$ is gfpre-open. $h^{-1}(\lambda)=f^{-1}(g^{-1}(\lambda))$. As f is gc-pre-biirresolute, $g^{-1}(\lambda)$ is gfpre-open. Therefore g is gc-pre-irresolute.

Next, Let us assume that $g^{-1}(\lambda)$ is gfpre-open. Since f is fuzzy gc-pre-biirresolute $f^{-1}(g^{-1}(\lambda))$ is gfpre-open. But $f^{-1}(g^{-1}(\lambda)) = h^{-1}(\lambda)$. Since h is gc-pre-biirresolute λ is gfpre-open. Therefore g is fuzzy gc-pre-biirresolute.

5. Fuzzy pre - $T_{1/2}$ space and its properties.

Definition 25. A fuzzy topological space (X,T) is said to be fuzzy pre - $T_{1/2}$ space if every gfpre-closed set in (X,T) is fuzzy closed in (X,T).

Proposition 6. Let $f : (X,T) \to (Y,S)$ and $g : (Y,S) \to (Z,R)$ be mappings and Y be fuzzy pre - $T_{1/2}$. If f and g are gfpre-continuous, then g.f is gfpre-continuous. The above proposition is not vaild if Y is not fuzzy pre - $T_{1/2}$.

Example 3. Put X = {a,b,c}. Define $T_1 = \{0_X, 1_X, \lambda\}$ where $\lambda : X \to [0,1]$ is such that $\lambda(a)=\lambda(b)=1$, $\lambda(c)=0$. $T_2=\{0_X,1_X,\gamma\}$ where $\gamma:X\to[0,1]$ such that $\gamma(a)=1$, $\gamma(b)=\gamma(c)=0$. $T3=\{0_X,1_X,\rho\}$ where $\rho:X\to[0,1]$ is such that $\rho(a)=0$, $\rho(b)=1$, $\rho(c)=0$. Also define $f:(X,T_1)\to(X,T_2)$ as $f(a)=a$, $f(b)=a$, $f(c)=b$ and let $g:(X,T_2)\to(X,T_3)$ be the identity map. f is fuzzy open for $f(\lambda)=\gamma$, $f(0_X)=0_X$, $f(1_X)=1_X$. f is fuzzy continuous, for $f^{-1}(\gamma)=\lambda$, $f^{-1}(0_X)=0_X$, $f^{-1}(1_X)=1_X$. Since f is fuzzy open and fuzzy continuous, the inverse image of every fuzzy pre-open set in (X,T_2) is fuzzy pre-open in (X,T_1). Equivalently, the inverse image of every fuzzy pre-closed set in (X,T_2) is fuzzy pre closed in (X,T_1). That is the inverse image of every fuzzy pre-closed set in (X,T_2) is gfpre-closed in (X,T_1). **Therefore f is gfpre-continuous. g is gf-pre-continuous.** For, any fuzzy pre-closed set λ_1 in (X,T_3) we have Int cl $\lambda_1\leq\lambda_1$. Since g is an identity map, the above inequality can be written as Int cl $g^{-1}(\lambda_1)\leq g^{-1}(\lambda_1)$. Therefore $g^{-1}(\lambda_1)$ is fuzzy pre-closed in $(X,T_2)\Rightarrow g^{-1}(\lambda_1)$ is gfpre-closed in (X,T_2). **g.f is not gfpre-continuous.** For $(1-\rho)$ is fuzzy pre-closed in (X,T_3) and $g^{-1}(1-\rho)=1-\rho$. Therefore $f^{-1}(g^{-1}(1-\rho))=f^{-1}(1-\rho)$. But $f^{-1}(1-\rho)(a)=(1-\rho)f(a)=(1-\rho)(a)=1$, $f^{-1}(1-\rho)(b)=(1-\rho)f(b)=(1-\rho)(a)=1$, $f^{-1}(1-\rho)(c)=(1-\rho)f(c)=(1-\rho)(b)=0$. Therefore, $f^{-1}(1-\rho)=\lambda$ which is not fuzzy closed in (X,T_1). $cl_{T_1}Int_{T_1}f^{-1}(1-\rho)=1>f^{-1}(1-\rho)=\lambda$. This shows that $f^{-1}(1-\rho)$ is not fuzzy pre-closed in (X,T_1). fpcl $f^{-1}(1-\rho)\neq 1\leq\lambda$ implies that $f^{-1}(1-\rho)$ is not gfpre closed in (X,T_1). That is $(g.f)^{-1}(1-\rho)$ is not gfpre closed in (X,T_1). **(X,T_2) is not fuzzy pre - $T_{1/2}$**, since $g^{-1}(1-\rho)$ is gfpre-closed in (X,T_2) but not fuzzy closed in (X,T_2).

Proposition 7. Let (X,T) be a fuzzy pre - $T_{1/2}$ space and (Y,S) be a fuzzy topological space. Let $f:(X,T)\to(Y,S)$ be a gfpre-continuous map. Then f is fuzzy continuous.

Proposition 8. Let (X,T) be a fuzzy pre - $T_{1/2}$ space and (Y,S) be a fuzzy topological space. Let $f:(X,T)\to(Y,S)$ be a strongly fuzzy pre-continuous map. Then f is perfectly fuzzy pre-continuous.

Proposition 9. Let (X,T) be a fuzzy pre - $T_{1/2}$ space and (Y,S) be a fuzzy topological space. Let $f:(X,T)\to(Y,S)$ be a gfpre-continuous map. Then f is M-fuzzy pre-continuous.

Proposition 10. Let (X,T), (Y,S) and (Z,R) be any fuzzy topological spaces. let f : (X,T) →(Y,S), g : (Y,S) → (Z,R) be maps such that f is M-fuzzy pre-continuous and g is fuzzy pre-continuous. Then g.f is fuzzy pre-continuous.

6. Generalized fuzzy pre - $T_{1/2}$ space and its properties.

Definition 26. A fuzzy topological space (X,T) is said to be generalized -fuzzy pre - $T_{1/2}$ if every fuzzy pre-closed set in (X,T) is fuzzy closed in (X,T). It is denoted by gfpre-$T_{1/2}$.

Proposition 11. Let (X,T), (Y,S) and (Z,R) be any fuzzy topological spaces. Let f : (X,T) →(Y,S) and g : (Y,S) → (Z,R) be fuzzy pre-continuous mappings and Y be generalized fuzzy pre - $T_{1/2}$ space. Then g.f is fuzzy pre-continuous.

The above proposition is not valid if Y is not generalized fuzzy pre - $T_{1/2}$ as the following example shown.

Example 4. Let X = {a,b,c,}. Define T_1={0_X,1_X,λ) where λ : Y →[0,1] is such that $\lambda(a)=\lambda(b)=1$, $\lambda(c)=0$, T_2={0_X,1_X,μ} where μ.X→[0,1] is such that $\mu(a)=0$, $\mu(b)=0$, $\mu(c)=1$, T_3={0_X,1_X,ρ) where ρ : X→[0,1] is such that $\rho(a)=0$, $\rho(b)=1$, $\rho(c)=0$. Also define f : (X_1,T_1)→(X_1,T_2) as f(a)=a, f(b)=a, f(c)=b and let g : (X,T_2)→(X,T_3) be the identity map. **f is fuzzy pre-continuous.** For $f^{-1}(\mu)(a) = \mu f(a) = \mu(a) = 0$, $f^{-1}(\mu)(b) = \mu f(b) = \mu(a) = 0$, $f^{-1}(\mu)(c)=\mu f(c)=\mu(b)=0$. Therefore $f^{-1}(\mu)=0_X$, $f^{-1}(0_X)=0_X$, $f^{-1}(1_X)=1_X$. **g is fuzzy pre-continuous.** For $g^{-1}(1-\rho)=1-\rho$ is fuzzy pre-closed in (X,T_2). **g.f is not fuzzy pre-continuous.** For (1-ρ) is fuzzy closed in (X,T_3) and $g^{-1}(1-\rho)=1-\rho$. Therefore $f^{-1}(g^{-1}(1-\rho))=f^{-1}(1-\rho)$, $f^{-1}(1-\rho)(a)=(1-\rho)f(a)=(1-\rho)(a)=1$, $f^{-1}(1-\rho)(b)=(1-\rho)f(b)=(1-\rho)(a)=1$, $f^{-1}(1-\rho)(c)=(1-\rho)f(c)=(1-\rho)(b)=0$. Therefore $f^{-1}(1-\rho)=\lambda$, which is not fuzzy closed in (X,T_1). Again $Int_{T_1} f^{-1}(1-\rho)=\lambda$. Therefore cl_{T_1} Int_{T_1} (1-ρ)=1 Hence, cl_{T_1} Int_{T_1} $f^{-1}(1-\rho)=1$. Thus cl_{T_1} Int_{T_1} $f^{-1}(1-\rho)>f^{-1}(1-\rho)=\lambda$. This shows that $f^{-1}(1-\rho)$ is not fuzzy pre-closed in (X,T_1). **(X,T_2) is not gfpre -$T_{1/2}$** for $g^{-1}(1-\rho)$ is fuzzy pre-closed in (X,T_2) but not fuzzy closed in (X,T_2).

Proposition 12. Let (X,T) be a gf pre -$T_{1/2}$ space. Let f : (X,T)→ (Y,S) be a strongly fuzzy pre-continuous map. Then f is fuzzy continuous.

7. Interrelations between the functions

Proposition 13. Every fuzzy pre -$T_{1/2}$ space is gfpre-$T_{1/2}$.

Proposition 14. Let f : (X,T)→(Y,S) be any map from a fuzzy topological space (X,T) into a fuzzy gfpre-$T_{1/2}$ space (Y,S). If f is fuzzy pre-continuous then f is fuzzy almost pre-continuous. This proposition is not vaild if Y is not gfpre-$T_{1/2}$ See example 5.

Example 5. Let X={a,b,c}, Y={p,q,r,}. Define $T_1=\{0_X,1_X,\lambda\}$ where λ:X→[0,1] such that λ(a)=1, λ(b)=0=λ(c). Define $T_2=\{0_Y,1_Y,\mu\}$ where μ:Y→[0,1] such that μ(p)=μ(q)=1, μ(r)=0. Also define f:$(X,T_1)\to(Y,T_2)$ as f(a)=f(b)=p, f(c)=q. Then **f is fuzzy pre-continuous**. For any set ρ:Y→[0,1] such that ρ(p)=0, ρ(q)=1, and ρ(r)=0. Since $cl_{T_2}\rho=1,Int_{T_2}cl_{T_2}\rho=1$, ρ is fuzzy pre-open.Since $Int_{T_2}\rho=0,cl_{T2}Int_{T2}\rho=0$, ρ is fuzzy pre-closed. Since $fpInt_{T_2}\ fpcl_{T_2}\rho = fpInt_{T_2}\rho = \rho$, ρ is fuzzy regular pre-open.Further $f^{-1}(\rho)(a)=\rho(f(a))=\rho(p)=0,f^{-1}(\rho)(b)=\rho(f(b)=\rho(p)=0,f^{-1}(\rho)(c)=\rho(f(c)=\rho(q)=1$. Since $Int_{T_1}cl_{T_1}f^{-1}(\rho) = 0<f^{-1}(\rho)$, $f^{-1}(\rho)$ is not fuzzy pre-open. **Hence f is not fuzzy almost pre-continuous. But Y is not gfpre-$T_{1/2}$.** For ρ is fuzzy pre-open in (Y,T_2) but not fuzzy open in (Y,T_2).

Proposition 15. Let f:(X,T)→(Y,S) be any map from a gfpre-$T_{1/2}$ space (X,T) into a gfpre-$T_{1/2}$ space (Y,S). Then f is M-fuzzy pre-continuous ⇔ f is fuzzy bicontinuous. This proposition is not vaild if (X,T) is not gfpre-$T_{1/2}$. See example 6.

Example 6. Let X={a,b} and the topology T defined on X be indiscrete. Let Y={p,q} and the topology S defined on Y be discrete. Let f: (X,T)→(Y,S) be any onto map. Since T is indiscrete, inverse image of every fuzzy pre-open set in (Y,S) is fuzzy pre-open in (X,T). Since S is discrete, $f^{-1}(\mu)$ in (X,T) is fuzzy pre-open implies μ in (Y,S) is fuzzy pre-open. Therefore **f is M-fuzzy pre-bicontinuous**. But this is **not fuzzy bicontinuous and (X,T) is not gfpre-$T_{1/2}$.**

Proposition 16. Let f:(X,T)→(Y,S) be any map from a gf pre-$T_{1/2}$ space (X,T) into a fuzzy topological space (Y, S). Then f is fuzzy pre-bicontinuous$\Leftrightarrow$ f is fuzzy bicontinous.

This proposition is not valid if (X, T) is not gf pre-$T_{1/2}$, See examples 7 and 8.

Example 7. Let X = {a, b} and the topology T defined on X be indiscrete. Let Y = {p, q} and the topology S defined on Y be discrete. Let f: (X, T) $\rightarrow$ (Y, S) be any onto map. Since T is indiscrete, inverse image of every fuzzy open set in (Y, S) is fuzzy pre-open in (X, T). Since S is discrete, $f^{-1}(\mu)$ in (X,T) is fuzzy pre-open implies μ in (Y, S) is fuzzy pre-open. Therefore **f is fuzzy pre-bicontinuous.** But this is **not fuzzy bicontinuous and (X, T) is not gfpre-$T_{1/2}$.**

Example 8. Let X = {a, b, c}, Y = { p, q, r}. Define $T_1 = \{0_x, 1_x, \lambda\}$ whereλ : X$\rightarrow$[0,1] such that $\lambda(a)=1$, $\lambda(b) = 0$, $\lambda(c) = 1$. Define $T_2=\{0_y, 1_y, \mu\}$ where μ : Y$\rightarrow$[0,1] such that $\mu(p)=0$, $\mu(q)=1$, $\mu(r)=1$. Define f: (X, T_1) - (Y, T_2) as f(a) = q, f(b) = p, f(c) =r. μ is fuzzy open in (Y, T_2). $f^{-1}(\mu)(a)=\mu f(a)=\mu(q)=1$, $f^{-1}(\mu)(b)=\mu(p)=0$, $f^{-1}(\mu)(c)=\mu f(c)=\mu(r)=1$. Therefore $f^{-1}(\mu)=\lambda$ which is fuzzy open in (X,T_1). For the fuzzy open set $\lambda = f^{-1}(\mu)$in (X,T_1), μ is fuzzy open in (Y,T_2), f is also onto. Hence **f is fuzzy bicontinuous. f is not fuzzy pre-bicontinuous.** For the set λ_1:X$\rightarrow$[0,1] such that $\lambda_1(a)=1$, $\lambda_1(b)=1$, $\lambda_1(c)=0$, $cl\lambda_1=1$, $Intcl\lambda_1=1>\lambda_1 \Rightarrow \lambda_1$ is fuzzy pre-open in (X,T_1). $f(\lambda_1)(p)=1$, $f(\lambda_1)(q)=1$, $f(\lambda_1)(r)=0$. Therefore $f(\lambda_1)$ is not fuzzy open in (Y,T_2). Further **X is not gfpre-$T_{1/2}$.** For λ_1 is a fuzzy pre-open set in (X, T_1) but λ_1 is not fuzzy open in (X, T_1).

Proposition : 17. Let f: (X, T) $\rightarrow$(Y,S) be any map from a fuzzy pre-$T_{1/2}$ space (X, T) into a fuzzy pre-$T_{1/2}$ space (Y,S). Then f is fuzzy gc-pre-biirresolute $\Leftrightarrow$ f is fuzzy bicontinuous.

This proposition is not valid if (X, T) is not fuzzy pre-$T_{1/2}$. See example 9.

Example 9. Let X = (a,b) and the topology T define on X be indiscrete. Let Y = {p, q} and the topology S defined on Y be discrete. Let f : (X, T) $\rightarrow$ (Y, S) be any onto map. Since T is indiscrete, inverse image of every gfpre-open set in (Y, S) is fgpre-open. Since S is discrete, $f^{-1}(\mu)$ in (X,T) is gfpre-open implies μ in (Y,S) is gfpre-open. Therefore **f is fuzzy gc-pre-biirresolute.** But **f is not fuzzy bicontinuous** and **(X,T) is not fuzzy pre-$T_{1/2}$.**

8. Definition and some properties of fpcl*

Definition 27. Let λ be a fuzzy set in X and define fuzzy fpInt*(λ)=$\vee\{\mu/\mu\leq\lambda$ and μ is gf pre-open$\}$. fpcl*(λ)=$\wedge\{\mu/\mu\geq\lambda$ and μ is gf pre-closed$\}$.

Note. For any fuzzy set λ, fpcl*(1-λ)=1-fpInt*λ and fpInt*(1-λ)=1-fpcl*λ.

Proposition 18. f : (X, T) $\rightarrow$ (Y, S) is fuzzy gc-pre-irresolute $\Leftrightarrow$f[fpcl*λ]$\leq$fpcl*f(λ) for any set λ of X.

Proof. Suppose f is fuzzy gc-pre-irresolute. Let λ be any set in X. fpcl*f(λ) is gf-pre-closed. Since f is fuzzy gc-pre-irresolute, f^{-1}[fpcl*f(λ)] is gf pre-closed in X. Now $\lambda\leq f^{-1}f(\lambda)\leq f^{-1}$[fpcl*f($\lambda$)]. By the definition fo fpcl*, fpcl*$\lambda\leq$ f^{-1}[fpcl*f(λ)]. That is f[fpcl*λ]$\leq$fpcl*f(λ). Conversely suppose that λ is gf pre-closed set in Y. Now by hypothesis f[fpcl*$f^{-1}(\lambda)$]$\leq$fpcl*f [$f^{-1}(\lambda)$]=λ $\Rightarrow$ fpcl*$f^{-1}(\lambda)$ $\leq f^{-1}(\lambda)$. But $f^{-1}(\lambda)\leq$fpcl*$f^{-1}(\lambda)$. Therefore $f^{-1}(\lambda)$=fpcl*$f^{-1}(\lambda)$. Hence $f^{-1}(\lambda)$ is gfpre-closed. Therefore f is fuzzy gc-pre-irresolute.

Proposition 19. f : (X, T) $\rightarrow$ (Y, S) is fuzzy gc-pre-irresolute $\Leftrightarrow$For all fuzzy sets λ of Y, fpcl*$f^{-1}(\lambda)\leq f^{-1}$(fpcl*$\lambda$).

Proof. Suppose f is fuzzy gc-pre-irresolute. Now fpcl*(λ) is gfpre-closed and therefore by assumption f^{-1}(fpcl*λ) is gfpre-closed. Since $f^{-1}(\lambda)\leq f^{-1}$(fpcl*$\lambda$), it follows from the definition of fpcl* that fpcl* that fpcl* $f^{-1}(\lambda)\leq f^{-1}$(fpcl*$\lambda$). Conversely, suppose λ is gfpre-closed in Y. Then by assumption fpcl* $f^{-1}(\lambda)\leq f^{-1}$(fpcl*($\lambda$)) = $f^{-1}(\lambda)$. But $f^{-1}(\lambda)\leq$fpcl*$f^{-1}(\lambda)$. But $f^{-1}(\lambda)\leq$fpcl*$f^{-1}(\lambda)$. Therefore fpcl*$f^{-1}(\lambda)$ = f^{-1}(fpcl*λ) =$f^{-1}(\lambda)$. Thus $f^{-1}(\lambda)$ is gfpre-closed.

9. Fuzzy generalised pre-compact spaces

Definition 28. A collection $\{\lambda_i\}_{i\in\Gamma}$ of fgpre-open sets in X is called gfpre-open cover of a fuzzy set μ in X if $\mu\leq V_{i\in\Gamma}\lambda_1$.

Definition 29. A fuzzy topological space (X,T) is called fgpre-compact if every gfpre-open cover of X has a finite subcover.

Definition 30. A fuzzy set λ in X is said to be fgpre-compact relative to X if for every collection $\{\lambda_i\}_{i\in\Gamma}$ of gf pre-open sets of X such that $\lambda\leq V_{i\in\Gamma}\lambda_i$ there exists a finite subset Γ_o of Γ such that $\lambda\leq V_{i\in\Gamma o}\lambda_i$.

Definition 31. A fuzzy set λ of X is said to be fgpre-compact if λ is fgpre-compact relative to X.

Proposition 20. A gf-pre-continuous image of a fgpre-compact space is fuzzy pre-compact.

Proof. Let f : (X, T) → (Y, S) be gf pre-continuous map from a fg pre-compact space X onto a fuzzy topological space Y. Let $\{\lambda_l\}_{l\in\Gamma}$ be a collection of fuzzy pre-open sets of Y such that $1 \leq V_{l\in\Gamma}\, \lambda_l$...(1). Since f is gfpre-continuous and each λ_1 is fuzzy pre-open in Y, $f^{-1}(\lambda_1)$ is gf pre-open in X. From (1), $1_X=f^{-1}(1_Y) \leq V_{l\in\Gamma} f^{-1}(\lambda_1)$. i.e $\{f^{-1}(\lambda_l)\}_{l\in\Gamma}$ is a gf pre open cover of X. Since X is fg pre-compact, there exists a finite subset Γ_o of Γ such that $I_X \leq V_{l\in\Gamma_o} f^{-1}(\lambda_l) \Rightarrow 1_Y \leq V_{l\in\Gamma o}(\lambda_l)$. Therefore Y is fuzzy pre-compact.

Proposition 21. If a map f:(X, T) → (Y, S) is fuzzy gc-pre-irresolute and if λ is fgpre-compact relative to X then f (λ) is fgpre-compact relative to Y.

Proof. Let $\{\lambda_l\}_{l\in\Gamma}$ be a collection of gfpre-open sets of Y such that $f(\lambda) \leq V_{l\in\Gamma}\, \lambda_1$...(1). Since f is gc-pre-irresolute and each λ_1 is gf pre-open in Y, $f^{-1}(\lambda_1)$ is gfpre-open in X. From (1), $\lambda=f^{-1}[f(\lambda)] \leq V_{1\in\Gamma} f^{-1}(\lambda_1)$, i.e. $f^{-1}(\lambda_l)_{l\in\Gamma}$ is a gfpre-open cover of X. Since X is fgpre-compact, there exists a finite subset Γ_o of Γ such that $\lambda \leq V_{l\in\Gamma o} f^{-1}(\lambda_l)$, $f(\lambda) \leq f[V_{l\in\Gamma o} f^{-1}(\lambda_l)] \leq V_{l\in\Gamma o}\lambda_1$. Therefore f ($\lambda$) is fuzzy fg pre-compact.

Proposition 22. A strongly gf pre-continuous image of a fuzzy pre-compact space is fg pre-compact.

Proof. Let f : (X, T) → (Y, S) be a strongly gf pre-continuous map from a fuzzy pre-compact space X onto a fuzzy topological space Y. Let $\{\lambda_l\}$ be a collection of gfpre-open sets of Y such that $1_Y \leq V_{l\in\Gamma}\lambda_l$ – (1). Since f is strongly gfpre-continuous and each λ_l is gfpre-open in Y, $f^{-1}(\lambda_l)$ is fuzzy pre-open cover of X. From (1) $1_X=f^{-1}(1_Y) \leq V_{l\in\Gamma} f^{-1}(\lambda_l)$. $\{ f^{-1}(\lambda_l)\}_{l\in\Gamma}$ is a fuzzy pre-open in X. Since X is fuzzy pre-compact there exists a finite subset Γ_o of Γ such that $1_X \leq V_{l\in\Gamma o} f^{-1}(\lambda_l)$. This implies $1_Y=f(1_X) \leq f[V_{l\in\Gamma o} f^{-1}(\lambda_l)] \leq V_{l\in\Gamma o}\lambda_l$. Therefore Y is fgpre-compact.

10. Generalized fuzzy pre-extremally disconnected spaces

Definition 32. X is said to be generalised fuzzy pre-extremally disconnected if fpcl*(λ) is gf pre-open whenever λ is gf pre-open.

Proposition 23. For any fuzzy topological space X the following are equivalent

(a) X is generalised fuzzypre-extremally disconnected.

(b) For each gfpre-closed set λ, fpint*λ is gf pre-closed.

(c) For each gfpre-open set λ, we have fpcl*λ+fpcl*[1-fpcl*λ]=1.

(d) For each pair of a gfpre-open sets λ, μ in X with fpcl*λ+μ=1, we have fpcl*λ+ fpcl*μ = 1

Prof : a $\Rightarrow$ b Let λ be any gf pre-closed set. We claim fpInt*λ is gf pre-closed. Now 1- λ is gf pre-open and so by assumption (a) fpcl* (1-λ) is gf pre-open. That is fpInt*λ is gfpre-closed.

b$\Rightarrow$c Let λ be any gfpre-open set.Them 1-fpcl*λ = fpInt*(1-λ) ----- (1). Consider fpcl*λ + fpcl* [1-fpcl*λ=fpcl*λ+fpcl* [fpInt* (1-λ). As λ is gf pre-open 1-λ is gfpre-closed and by assumption (b) fpInt*(1-λ) is gf pre-closed and therefore pcl* fpInt y* (1-λ) = fpInt* (1-λ). Now fpcl*λ + fpcl* [1-fpcl*λ] = fpcl*λ + fpInt* (1-λ) = fpcl*λ +1-fpcl*λ = 1.

c$\Rightarrow$d Let λ and μ be any two gfpre-open sets such that fpcl*λ+μ=1... (2) Now by assumption (c) fpcl*λ + fpcl* [1 - fpcl*λ] = 1 = fpcl*λ + μ. That is μ = fpcl* [1- fpcl*λ] ...(3). But μ = 1-fpcl*λ from (2). Thus we have fpcl*μ = fpcl*[1-fpcl*λ] ... (4). From (3) and (4), it follows that μ = fpcl*μ ...(5). That is μ is gfpre - closed. From (2) and (5), it follows that fpcl*λ + fpcl*μ =1.

d$\Rightarrow$a Let λ be any gfpre - open set. Put μ =1-fpcl*λ ... (6). Clearly μ is gfpre-open and from the construction of μ, it follows that fpcl*λ + μ =1 . Hence by assumption (d) we have fpcl*λ + fpcl*μ = 1, fpcl*μ = 1- fpcl*λ ... (7). From (6) and (7), we have μ = fpcl*μ. That is μ is gfpre-closed and so fpcl*λ = 1−fpcl*μ is gf pre-open.

11. fg pre-Connected and gf pre-super connected spaces

Proposition 24 Suppose that X is gfpre-$T_{1/2}$space. Then X is fuzzy connected $\Leftrightarrow$ X is fuzzy pre-connected.

Proof. Assume that X is gfpre -$T_{1/2}$ and fuzzy connected. If possible, let X be not fuzzy pre-connected. This means that there exists a proper fuzzy set λ such that λ is both fuzzy pre-open and fuzzy pre-closed. Since X is gf pre-$T_{1/2}$, λ is fuzzy open and fuzzy closed and this means that X is not fuzzy connected. Contradiction. The converse follows easily.

Definition 33. A fuzzy topological space X is said to be fuzzy generalised pre-connected space iff the only fuzzy sets which are both gfpre-open and gfpre-closed are 0x and 1x.

Proposition 25. Every fgpre-connected space is fuzzy connected.

Prof : Let X be a fgpre-connected space and suppose that X is not fuzzy connected. Therefore there exists a proper subset $\lambda(\lambda \neq 0x, \lambda \neq 1x)$ such that λ is both fuzzy open and fuzzy closed. Since fuzzy open $\Rightarrow$ gf pre-open, it follows that X is not fgpre-connected. Contradiction. Hence X is fuzzy connected. The converse is not true. See Example 10.

Example 10. Let X = {a,b,c} and T = $\{0_X, 1_X, \lambda\}$ where $\lambda : X \rightarrow [0,1]$ is such that λ (a)=0, λ (b) = λ (c) =1. Then **(X,T) is fuzzy connected,** but it is not fgpre-connected. For $\mu : X \rightarrow [0,1]$ is such that $\mu(c) = 1$, $\mu(a) = \mu(b) = 0$, clμ =1, Intclμ = $1 > \mu \Rightarrow \mu$ is fuzzy pre-open $\Rightarrow \mu$ is gfpre-open. Intμ=0, clIntμ=0<$\mu \Rightarrow \mu$ is fuzzy-closed$\Rightarrow \mu$ gfpre-closed. Thus there exists a proper fuzzy set μ which is both gfpre-open and gfpre-closed in (X,T). Therefore **(X,T) is not fgpre-connected.**

Proposition 26. Every fgpre-connected space is fuzzy pre-connected.

Proof. Let X be a fg pre-connected space and suppose that X is not fuzzy pre-connected. Therefore there exists a proper subset $\lambda(\lambda \neq 0x, \lambda \neq 1x)$ such that λ is both fuzzy pre-open and fuzzy pre-closed. Since fuzzy pre-open $\Rightarrow$gfpre-open it follows that X is not fgpre-connected. Conradication. Hence X is fuzzy pre-connected.

Proposition 27. Suppose that X is fuzzy pre-$T_{1/2}$ space.Then X is fuzzy connected $\Leftrightarrow$ X is fuzzy pre-connected.

Proof. Assume that X is fuzzy pre-$T_{1/2}$ and fuzzy connected. If possible let X be not fuzzy pre-connected. This means there exists a proper fuzzy set λ such that λ is both fuzzy pre-open and fuzzy pre-closed. Every fuzzy pre-closed set is gfpre-closed. Since X is fuzzy pre-$T_{1/2}$, λ is fuzzy open and fuzzy closed and this means that X is not fuzzy connected. Contradiction. Therefore X is fuzzy pre-connected. The converse follows easily.

Definition 34. A gfpre-closed set is called regular gfpre-closed if λ=fpcl*[fpInt*(λ)]. The fuzzy complement of regular gf pre-closed set is called regular gfpre-open.

Definition 35. A fuzzy topological space X is called gf pre-super connected if there is no proper regular gf pre-open set in X.

Definition 36. X is said to be gf strongly pre-connected if it has no non zero fuzzy pre-closed sets λ_1 and λ_2 such that $\lambda_1+\lambda_2 \leq 1$. If X is not gf-strongly pre-connected then it will be called **gf-weakly pre-connected.**

Example 11. Let X = {a,b}. Define T = {Ox, 1x, λ} where λ:X→[0,1] is such that $\lambda(a)=0$, $\lambda(b)={}^2/_3$. Define any set λ_i:X→[0,1], (i=1,2,3,4) such that case (i) $\lambda_1(a) \neq 0$ $\lambda_1(b) = 0$, case (ii) $\lambda_2(a)=0$ $\lambda_2(b)\neq 0 \leq {}^1/_3$. case (iii) $\lambda_3(a)=0$, ${}^1/_3 < \lambda_3(b) < {}^2/_3$. case (iv) $\lambda_4(a)=0$, $\lambda_4(b)>{}^2/_3$.

Case (i) cl$\lambda_1=\lambda'$, Int cl$\lambda_1=0<\lambda_1$. Therefore λ_1 is not fuzzy pre-open. Intλ_1=0, cl Intλ_1=0, cl Int$\lambda_1=0<\lambda_1$. Therefore λ_1 is fuzzy pre-closed $\Rightarrow\lambda_1$ is gfpre-closed.

Case (ii). cl$\lambda_2=\lambda'$, Int cl$\lambda_2=0<\lambda_2$. Therefore λ_2 is not fuzzy pre-open.

Intλ_2=0, cl Int$\lambda_2=0<\lambda_2$. Therefore λ_2 is fuzzy pre-closed $\Rightarrow\lambda_2$ is gfpre-closed.

Case (iii). clλ_3=1, Int cl$\lambda_3=1>\lambda_3$. λ_3 is fuzzy pre-open. Therefore λ_3 is gfpre-open. Int λ_3=0, cl Int$\lambda_3=0<\lambda_3$. λ_3 is fuzzy pre-closed. Therefore λ_3 is gfpre-closed.

Case (iv) clλ_4=1, Int cl$\lambda_4=1>\lambda_4$. Therefore λ_4 is fuzzy pre-open.
$\Rightarrow \lambda_4$ is gf pre-open Int. $\lambda_4=\lambda$,cl Int $\lambda_4=1>\lambda_4$. Therefore λ_4 is not fuzzy pre-closed.
$\Rightarrow$ it has non-zero fuzzy pre-closed sets λ_1 and λ_2 such that $\lambda_1+\lambda_2\leq 1$.
Therefore X is gf-weakly pre-connected.

Proposition 28. If X is generalized fuzzy pre-super connected then
i) X does not have non-zero gfpre-open sets λ_1 and λ_2 such that $\lambda_1+\lambda_2=1$.
ii) X does not have non-zero fuzzy sets λ_1 and λ_2 satisfying
fpcl*$\lambda_1+\lambda_2=\lambda_1+$fpcl*$\lambda_2$=1.

Proof (i). There exist a non-zero gfpre-open sets λ_1 and λ_2 such that $\lambda_1+\lambda_2=1$. Now $\lambda_1=1-\lambda_2$ and fpcl*λ_1=fpcl*$(1-\lambda_2)=1-\lambda_2$, fpInt*[fpcl*$\lambda_1$]=fpInt*$(1-\lambda_2)$= fpInt*$\lambda_1=\lambda_1$. This is contradiction to our assumption. Therefore X does not have non-zero gf pre-open sets λ_1 and λ_2 such that $\lambda_1+\lambda_2=1$.

(ii) There exist non-zero fuzzy sets λ_1 and λ_2 such that fpcl*λ_1+λ_2=1.

λ_1+fpcl*λ_2=1.

λ_2=1-fpcl*λ_1 and λ_1+fpcl*[1-fpcl*λ_1]=1, λ_1+fpcl*[fpInt*(1-λ_1)]=1, fpcl* [fpInt* (1-λ_1)]=1-λ_1. This is a contradiction to our assumption. Therefore X does not have non-zero fuzzy sets λ_1 and λ_2 satisfying fpcl*λ_1+λ_2=λ_1+fpcl*λ_2=1.

References :

[1] Azad, K.K., On fuzzy semi-continuity fuzzy almost continuity and fuzzy weakly continuity, J. Math. Appl. 82(1981), 14-32.

[2] Balasubramanian, G. On fuzzy pre-separation axioms, Bull. Cal. Math. Soc. 90 (1998), 427-434.

[3] Balasubramanian, G. and Sundaram, P., On some generalizations of fuzzy continuous functions, Fuzzy Sets and Systems 86 (1997), 93-100.

[4] Balasubramanian, G., Fuzzy β-compact spaces and fuzzy extremally disconnected spaces, Kybernetika 33 (1997), 271-277.

[5] Binshahna, A.S., On fuzzy strong semi continuity and fuzzy pre-continuity, Fuzzy Sets and Systems 44 (1991), 303-308.

[6] Chang, C.L., Fuzzy topological spaces, J.Math.Anal.Appl.24 (1968), 182-190.

[7] Fatteh, U.V. and Bassan, D.S., Fuzzy connectedness and its stronger forms, J.Math. Anal. Appl. 111(1985), 449-464.

[8] Kuratowski, K., Topology Vol II, Academic press (1968).

[9] Smets, M., The degree of belief in a fuzzy event, Inform. Sci.25(1981), 1-19

[10] Sugeno, M., An introductory survey of fuzzy control, Inform. Sci. 36(1985), 59-83

[11] Zadeh, L.A., Fuzzy Sets, Inform. and Control 8 (1965), 338-353.

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