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SOME STRONGER FORMS OF ψ-CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT :

In this paper we introduce a new class of sets namely ψ^*-closed sets which is the stronger form of ψ-closed sets. We study some relation between ψ^*-closed sets and g-closed set, g*-closed set, ψ-closed set, sg-closed set, gs-closed set, α-closed set, gα-closed set, αg-closed set and preclosed set.

1. Introduction

N Levine introduced semi open sets [5] and generalized closed (briefly g-closed) sets [6] in topological spaces. S.P. Arya and T. Nour [1] defined the notion of generalized semi closed sets. Bhattacharya and Lahiri [2] introduced and studied semi generalized closed sets. Veerakumar [12] introduced a new class of sets namely ψ-closed sets. In this paper we introduce a new class of sets called ψ^*-closed sets and study some of their properties.

As an application of ψ^*-closed sets we have introduced three new spaces namely $T\psi$-space, $T\psi^*$-space and $T\psi^{**}$-space. Devi etal [4] and Levine[6] introduced T_b, T_d-spaces and $T_{1/2}$, semi $T_{1/2}$-spaces respectively. Some relations between newly introduced spaces and the existing spaces like T_b, semi $T_{1/2}$, T_{gs}^* and T_{sg}^*-spaces are investigated.

Throughout this paper X, Y and Z denote topological spaces on which no separation axioms are assumed unless otherwise explicitly stated.

2. Preliminaries

Here we recall the following known definitons.

Definition 2.1

A subset A of a topological space (X, τ) is called :

a) Semi open [5] if A $\subseteq$ Cl (int(A)) and semi closed if int (Cl(A))$\subseteq$A.

b) Generalized closed (briefly g-closed) [6] if Cl (A)$\subseteq$ U whenever A $\subseteq$ U and U is open in X.

C) Semi generalised closed (briefly sg-closed) [2] if scl (A)$\subseteq$ U whenever A $\subseteq$ U and U is semi open in X.

d) Generalized semi closed (briefly gs-closed) [1] if scl (A)$\subseteq$U whenever A$\subseteq$U and U is open in X.

e) α-Open [9] if A$\subseteq$ int (cl (int (A))) and α-closed if cl (int (cl (A)))$\subseteq$ A.

f) ψ - Closed [11] if scl (A)$\subseteq$U whenver A$\subseteq$U and U is sg-open in X.

Definition 2.2 :

A function f: (X,τ) $\rightarrow$(Y,σ) is said to be

(1) Semi continuous [5] if f^{-1} (V) is semi-open in (X,τ) for every open set V of (Y,σ).

(2) Sg-continuous [10] if f^{-1} (V) is sg-closed in (X,τ) for every closed set V of (Y,σ).

(3) Irresolute [3] if f^{-1} (V) is semi-open in (X,τ) for every semi-open set V of (Y,σ).

(4) Sg-irresolute [10] if f^{-1} (V) is sg-closed in (X,τ) for every sg-closed set V of (Y,σ).

Defination 2.3:

A Topological space (X,τ) is said to be

(a) a $T_{1/2}$ space [6] if every g closed set in it is closed.

(b) a semi $T_{1/2}$ space [2] if every sg-closed set in it is semi closed.

(c) a T_b space [4] if every gs-closed set in it is closed.

(d) a T_{d^*} space [4] if every gs-closed set in it is g-closed.

(e) a T_{gs^*} -space [10] if every gs-closed set of (X,τ) is - ψ closed.

(f) a T_{sg^*} - space [10] of every sg-closed set of (X,τ) is - ψ closed.

3.Basic properties of - ψ^* closed sets

We intrroduce the following definitions.

Definition 3.01:

A subset A of (X,τ) is called a ψ^* - closed set if scl (A) $\subseteq$ Int (U) whenever A$\subseteq$U and U is sg-open.

Definition 3.02:

A subset A of (X,τ) is called a ψ^{**}- closed set if scl (A) $\subseteq$ Int (cl (U)), whenever A$\subseteq$U, U is sg-open.

Theroem 3.03

Every ψ^* - closed set is ψ - closed.

Proof :

Let A $\subset$ U and U is sg-open. Since A is ψ^* - closed, Scl (A) $\subseteq$ Int (U). This implies Scl (A)$\subseteq$ U. Therefore, A is ψ - closed.

Remarks : 3.04

The converse of the above theorem need not be true by the following example.

Example : 3.05

Let X = {a,b,c,} and τ = {X,ϕ,{a}, {c}, {a,c}}. In this topological space the set {b} is ψ - closed but not ψ^* - closed.

Theorem : 3.06

Every ψ^* - closed set is ψ^{**} - closed.

Proof :

Let A $\subset$ U and U is sg open. Since A is ψ^* - closed, Scl (A) $\subseteq$ Int (U). This implies Scl (A) $\subseteq$ Int (Cl (U)). Therefore A is ψ^{**} - closed.

Remark : 3.07

The converse of the above theorem need not be true. It is seen by the following example.

Example : 3.08

Let X={a,b,c,} and τ ={X,ϕ,{b},{a,c,}}. Let A = {a}. Then A is ψ^{**}- closed but it is not ψ^*- closed.

Theorem 3.09

Every ψ^*- closed set is sg-closed and also gs-closed.

Proof :

Let A be any ψ^*- closed set in (X,τ). By theorem 3.03, A is ψ- closed. By theorem 3.03 (ii) [12] A is sg-closed, and thus semi-preclosed and also gs-closed. Therefore, every ψ^*- closed set is sg-closed, semi-preclosed and gs-closed.

Remarks : 3.10

The converse of the above theorem need not be true. It is seen by the following example.

Example : 3.11

Let X = {a,b,c,} and τ = {X,ϕ,{b} {b,c}}. In this topological space the set {a} is sg-closed but not ψ^*- closed.

Result : 3.12

ψ^*- closedness and g-closedness are independent notions. It can be seen by the following example.

Example : 3.13.

In example 3.11 let A = {a} and B = {c}. Then A is g-closed however A is not ψ^*- closed and B is ψ^*- closed however B is not g-closed. Hence ψ^*- closedness and g-closedness are independent.

Remark : 3.14

The union of two ψ^*- closed sets need not be ψ^*- closed. It is seen by the following example.

Example : 3. 15

Let X={a,b,c,} and τ={X,ϕ,{a},{b}, {a,b}}. Let A={a} and B={b}. Then A and B are ψ^*- closed sets but their union A U B = {a,b} is not ψ^*- closed.

Theorem : 3:16

If a subset A is ψ^* - closed in (X,τ) then scl(A)/A does not contain a nonempty sg-clsoed set.

Proof :

Suppose that A is ψ^* - closed and let F be a non-empty sg-closed set such that $F \subset$ Scl (A)/A. Then F $\subseteq$ scl (A) and A $\subset$ X/F. Since X/F is sg-open and A is ψ^* - closed, Scl (A) $\subset$ Int (X/F) $\subset$ X/F. Thus we have F$\subset$ X/scl (A). This is a contradiction. Hence F = ϕ.

Remark : 3.17

The converse of the above theorem need not be true. It is seen by the following example.

Example : 3.18

Let X = {a,b,c,d} and τ = {X,ϕ,{a,b}, {a,b,d,}}. For a subset {c}, scl ({c})/{c} does not contain non-empty sg-closed set. However {c} is not ψ^* - closed in (x,τ).

Definition : 3.19 [11]

A subsets A of (X,τ) is called g^*- closed if cl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is g-open in (x,τ).

Remark : 3.20

g^*- closedness and ψ^* - closedness are independent notions. It can be seen by the following example.

Example : 3.21

Let X = {a,b,c} and τ = {X,ϕ,{c},{b,c}}. Let A = {a} and B ={b}. Then A is g^*-closed however A is not ψ^* - closed and B is ψ^* - closed however B is not g^*-closed.

Remark : 3.22

Every gs-closed set need not be ψ^* - closed. It is seen by the following example.

Example : 3:23

Let X = {a,b,c,} and τ = {X,ϕ,{a}, {a,b}}. In this topolgoical space the set {a,c} is gs-closed but not ψ^* - closed.

Remark : 3.24

ψ^* - closedness and α-closedness are independent notions. It can be seen by the following example.

Example : 3.25

Let X = {a,b,c,} and τ = {X,ϕ,{b},{c},{b,c}}. Let A = {b} and B = {a,c}. Then A is ψ^* - closed but not α - closed and B is α - closed but not ψ^* - closed.

Remark : 3.26

The following two examples show that ψ^* - closedness is independent from gα-closedness, αg-closedness and pre closedness.

Example : 3.27

In example 3.15 the set {a} is ψ^* - closed but it is neither a gα-closed set nor an αg-closed set. Also {a} is not a pre closed set.

Example : 3.28

Let X = {a,b,c} and τ = {X,ϕ,{a},{b,c}}. Let B = {b}. Then B is a gα-closed set and hence it is an αg-closed set. Moreover B is also a pre closed set of (x,τ). But B is not a ψ^* - closed set.

Remarks : 3.29

ψ^* - closedness and semi-closedness are indepenent notions. It can be seen by the following two examples.

Example : 3.30

Let X = {a,b,c,} and τ = (X,ϕ, {a}, {a,c}}. In this topological space the set {b} is semi-closed but not ψ^* - closed.

Example : 3:31

Let X = {a,b,c} and τ = {X,ϕ,{a,b}}. In this topological space the set {a,c} is ψ^* - closed but not semi-closed.

Remark 3:32

Closedness and ψ^* - closedness are independent notions. It can be seen by the following example.

Example : 3.33

In example 3.30 let A = {b} and B = {c}. Then A is closed but not ψ^* - closed and B is ψ^* - closed but not closed.

Remark : 3.34

ψ - Closedness and ψ^{**} - closedness are independent notions. It can be seen by the following example.

Example 3.35

In example 3.15 let A = {c} and B = {a,b}. Then A is ψ - closed but not ψ^{**} - closed and B is ψ^{**} - closed but not ψ - closed.

Proposition : 3.36

Let (X,τ) be an extremely disconnected topological space. Then the union of two ψ^{*} - Closed sets is ψ^{*} - closed.

Proof :

Let $A \cup B \subset U$, where U is sg-open. Then $A \subset U$ and $B \subset U$. Then scl (A) $\subset$ Int (U), scl (B) $\subset$ Int (U). Therefore, scl $(A\cup B)$=scl (A) $\cup$ scl(B)$\subseteq$ Int (U)$\cup$ Int (U)$\subseteq$ Int(U). This implies scl$(A\cup B)\subseteq$ Int (U). Hence the union of two ψ^{*} - closed sets is ψ^{*} - closed.

Proposition : 3.37

If A is ψ^{*} - closed in (X,τ) and $A\subset B \subset$ scl (A), then B is ψ^{*} - closed in (X,τ).

Proof :

Let $B \subset U$ and suppose that U is sg-open. Then scl (A)$\subset$Int (U) and Scl (B) $\subset$ scl (A). Therefore, we have scl (B) $\subset$ Int (U). Hence B is a ψ^{*} - closed set.

Definition 3.38

Let $B\subset Y\subset X$. A subset B of Y said to be ψ^{*} - closed relative to Y if B is ψ^{*} - Closed in the subspace $(Y,\tau/Y)$.

Theorem 3.39

Let $B\subset Y\subset X$.(i) If B is ψ^{*} - closed relative to Y and Y is open and ψ^{*} - closed in (X,τ) then B is ψ^{*} - closed in (X,τ).

(ii) If B is ψ^{*} - closed in (X,τ) and Y is open in (X,τ) then B is ψ^{*} - closed relative to Y.

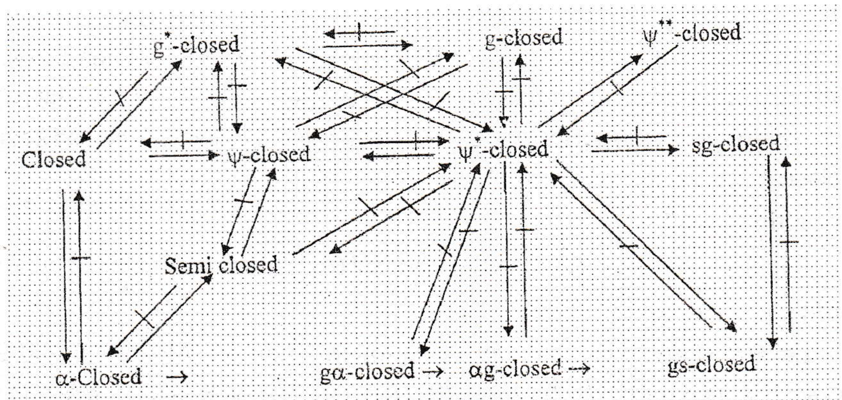
Proof :

(i) Let U be an sg-open set of (X,τ) such that $B \subset U$. Since B is ψ^{*} - closed relative to Y, $(\tau/Y)^{s}$ cl (B) $\subseteq$ (τ/Y) Int $(U \cap Y)$. We know if Y is open in (X,τ) then $(\tau/Y)^{s}=\tau^{s}/Y$. Therefore we have τ^{s}-cl $(B)\cap Y$= (τ^{s}/Y) cl (B) = $(\tau/Y)^{s}$ cl (B) $\subset$ Int $(U \cap Y)\cap Y$. Then Int $(U \cap Y) \cup \{X-(\tau^{s}\text{- cl}(B))\}$ is sg-open in (X,τ) and it contains Y. Since Y is ψ^{*} - closed in (X,τ), τ^{s}- cl (B) $\subset$ τ^{s}- cl (Y) $\subset$ Int[Int$(U\cap Y)\cup$ $\{X-(\tau^{s}\text{-cl}(B))\}$] $\subset$ Int(U)$\cup\{X-(\tau^{s}\text{- cl}(B))\}$. Therefore, scl (B) $\subset$ Int (U). Hence B is ψ^{*} - closed in (X,τ).

(ii) Let $B \subset U$ and suppose that $U \in (\tau/Y)^s$. We know if Y is open in (X,τ) then $(\tau/Y)^s = (\tau^s/Y)$. Therefore, $U \in \tau^s/Y$. Hence there exist a sg-open set V of (X,τ) such that $U = V \cap Y$. Then τ^s- cl (B) $\subset Y$, $B \subset V$ and τ^s- cl (B) $\subset$ Int (V). Therefore, we have $(\tau/Y)^s$ - cl(B) = τ^s- cl (B) $\cap Y \subset$ Int (V) $\cap Y = (\tau/Y)$ Int (U). Hence B is ψ^* - closed relative to Y.

Remark : 3.40

From the above theorems and results we have the following diagram.



where A → B (resp. A ——+——→ B) represents that A implies B. (resp. A does not always imply B)

4. Ψ*- CONTINUOUS, Ψ* - IRRESOLUTE FUNCTIONS AND GROUPS

We introduce the following definitions.

Definition 4.01 :

(i) A map $f : (X,\tau) \to (Y,\sigma)$ is said to be Ψ^* - continuous if for every closed set B of (Y,σ), $f^{-1}(B)$ is Ψ^* - closed in (X,τ).

(ii) Ψ^* - Open if the image f(U) is Ψ^* - open in (Y,σ) for every open set U of (X,τ).

(iii) Ψ^* - Closed if the image f(F) is Ψ^* closed in (Y,σ) for every closed set F of (X,τ).

(iv) Ψ^*- Irresolute if the inverse image f^{-1} (B) is Ψ^*-closed in (X,τ) for every

Ψ^*-closed set B of Y.

(v) Ψ^*-Homeomorphism if f is a bijective, Ψ^*-continuous function and f^{-1} is Ψ^*-continuous.

(vi) Ψ^*C-homeomorphism if f is a bijective Ψ^*-irresolute function and f^{-1} is Ψ^*-irresolute.

Theorem 4.02:

If f is Ψ^*-continuous, then f is Ψ-continuous

Proof :

Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a Ψ^*-continuous map. Let V be a closed set of (Y,σ). Since f is Ψ^*-continuous, f^{-1} (V) is a Ψ^*-closed set in (X,τ). But every Ψ^*-closed set is Ψ-closed (by theorem 3.03). So f^{-1} (V) is a Ψ-closed set of (X,τ). Hence f is a Ψ-continuous map.

Remark 4.03 :

The converse of the above theorem need not be true as can be seen from the following example.

Example 4.04:

Let X ={a,b,c}, τ = {X,φ, {b}, {b,c}}, Y = {p,q,r} σ={Y,φ, {q,r}}. Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a map defiend by f(a) = p,f(b) = q,f(c) = r, Then f is Ψ-continuous but not Ψ* continuous.

Remark 4.05 :

Pasting Lemmas for some "generalized maps" were investigated in [7], [8]. Here we prove an analogous result in the case of Ψ^*-closed sets in exteremely disconnected space (X,τ).

Lemma 4.06 :

Suppose that subsets A and B of (X,τ) are both open and Ψ^*-closed in extremely disconnected space (X,τ). Let f: $(A,\tau/A)$ (Y,σ) and h: $(B,\tau/B)\rightarrow(Y,\sigma)$ be compatible mappings. If f and g are Ψ^*-continuous then its combination $f\nabla$ h: $(X,\tau)\rightarrow(Y,\sigma)$ is also Ψ^*-continuous.

Proof :

Let F be a closed subset of (Y,σ). By definition $(f\nabla h)^{-1}$ (F) = f^{-1} (F)$\cup$ h^{-1} (F). By assumptions f^{-1} (F) is Ψ^*-closed in $(A,\tau/A)$ and h^{-1} (F) is Ψ^*-closed in $(B,\tau/B)$. By theorem 3.39 (i) and by assumption that f^{-1} (F) and h^{-1} (F) are Ψ^*-closed in (X,τ) and using proposition 3.36, we have that its union $f^{-1}(F) \cup h^{-1}(F)$ is Ψ^*-closed in (X,τ) for extremely disconnected spaces. Hence $f \nabla h$ is Ψ^*-continuous.

Theorem 4.07 :

Let $f:(X,\tau)\to(Y,\sigma)$ be a bijection. Then the following conditions are equivalent.

(a) f is Ψ^*-open and Ψ^*-continuous.

(b) f is Ψ^*-homeomorphism.

(c) f is Ψ^*-closed and Ψ^*-continuous.

Proof: (a)$\Rightarrow$(b)

Given that f is a bijection. Let f be a Ψ^*-open and Ψ^*-continuous map and G an open set in (X,τ). Since f is Ψ^*-open f (G) is Ψ^*-open in (Y,σ). That is $(f^{-1})^{-1}(G) = f(G)$ is Ψ^*-open in (Y,σ). Thus f^{-1} is Ψ^*-continuous. Then by definition f is Ψ^*-homeomorphism.

(b)$\Rightarrow$(a)

Let f be a Ψ^*-homeomorphism and $f^{-1} = g$ then g^{-1} f. Since f is bijective, g is also objective. If G is an open set in (X,τ), then $g^{-1}(G)$ is Ψ^*-open in (Y,σ). (Since g is Ψ^*-continuous). That is f (G) is Ψ^*-open in (Y,σ). Therefore f is Ψ^*-open and Ψ^*-continuous.

Therefore (b)$\Rightarrow$(a)

Hence (a)$\Leftrightarrow$(b).

(b)$\Rightarrow$(c)

Assume that f is a Ψ^*-homeomorphism. Let B be a closed set in (X,τ). Then X-B is open. Since $f^{-1} = g$ is Ψ^*-continuous, g^{-1} (X-B) is Ψ^*-open. That is $g^{-1}(X\text{-}B) = Y\text{-}g^{-1}(B)$ is Ψ^*-open. Thus g^{-1} (B) is Ψ^*-closed. That is f (B) is Ψ^*-closed Hence f is a Ψ^*-closed map.

(c)$\Rightarrow$(b)

If f is Ψ^*-closed and Ψ^*-continuous then we have to prove f^{-1} is also Ψ^*-continuous. Let G be an open set in (X,τ), then X-G is closed. Since f is Ψ^*-closed, f (X-G) is Ψ^*-closed in (Y,σ). That is $g^{-1}(X\text{-}G) = Y\text{-}g^{-1}(G)$ is Ψ^*-closed. This implies that g^{-1} (G) is Ψ^*-open. Thus the inverse image under g of every open set is Ψ^*-open. That is $g = f^{-1}$ is Ψ^*- continuous. Thus f is Ψ^*-homeomorphism.

Thus (c)$\Rightarrow$(b). Hence (b)$\Leftrightarrow$(c).

Lemma 4.08 :

Let f: $(X,\tau) \to (Y,\sigma)$ be a function.

(i) If f: $X \to Y$ is bijection, pre-semi-open and sg-continuous then f is sg-irresolute.

(ii) If f: $X \to Y$ is bijection and f is a homeomorphism then f^{-1} and f are Ψ^*- irresolute.

Proof :

(i) Let A be sg-closed set in (Y,σ). To prove f^{-1} (A) is sg-closed in (X,τ). Let $f^{-1}(A) \subset O$, O is semi-open in X. This implies $A \subset f(O)$ ....(1). But f (O) is semi - open in Y. Therefore f (O) is semi - open and A is sg-closed.

(1) Implied by definition of sg-closed sets, scl (A) $\subset$ f (O).

This implies f^{-1} (scl (A)) $\subset$ O

Since F is sg-continuous, f^{-1} (scl (A)) is sg-closed in X.

Therefore scl (f^{-1} (scl (A)) $\subset$ O .... (2).

But scl (f^{-1} (A))$\subset$ scl f^{-1}(scl(A)) .... (3)

(2) & (3) $\Rightarrow$ scl (f^{-1} (A)) $\subset$ O.

Therefore f^{-1} (A) is sg-closed in (X,τ). Therefore f is sg-irresolute.

(ii) To prove f^{-1} is Ψ^* irresolute. Let A be a Ψ^* closed set of (X,τ). To show $(f^{-1})^{-1}$ (A) = f(A) is Ψ^* closed in (y,σ). Let U be a sg-open set such that f(A) $\subset$ U. Then A = f^{-1} (f(A))$\subset$ f^{-1} (U) and by (i) f^{-1} (U) is sg-open. Since A is Ψ^* Closed, scl (A) $\subset$ Int (f^{-1}(U)). Since f is homeomorphism and bijection, f (scl (A)) = scl (f(A)) we have f (scl (A)) $\subseteq$ f (Int(f^{-1}(U)). Therefore Scl (f(A) $\subseteq$ Int (U). Therefore f(A) is Ψ^* closed. Thus we have showed that f^{-1} is Ψ^* irresolute. Since f^{-1} is also homemorphism, by the above proof $(f^{-1})^{-1}$ = f is Ψ^* irresolute.

Theorem 4.09 :

(i) Let f : $(X,\tau) \to (Y,\sigma)$ and g : $(Y,\sigma) \to (z,\eta)$ be two functions between topological spaces. If f and g are Ψ^* C-homeomorphism, then their composition g $\circ$ f : $(X,\tau) \to (z,\eta)$ is a Ψ^* C-homeomorphism.

(ii) If f : $(X,\tau) \to (Y,\sigma)$ is a homeomorphism then f is a Ψ^* C-homeomorphism.

Proof :

(i) let V be a open set in (Z,η). Then it is Ψ^*-open in (Z, η).

Consider $(g \circ f)^{-1}$ (V) = f^{-1} (g^{-1} (V)) = f^{-1} (U), where U = g^{-1} (V). As g isΨ^*

C-homeomorphism, g is Ψ^*-irresolute. Therefore g^{-1} (V) is Ψ^*-open in (Y,σ). Now f is Ψ^*C-homeomorphism, f is Ψ^*- irresolute, then f^{-1} (U) is Ψ^*-open in (X,τ). That is $(g \circ f)^{-1}$ (V) is Ψ^*-open in (X,τ). Hence $g \circ f$ is Ψ^* irresolute. Let A be an open set in (X,τ) then it is also Ψ^* open in (X,τ). Consider $(g \circ f)$ (A) = g (B) where B = f (A). As f is Ψ^* C-homeomorphism, f^{-1} is Ψ^* irresolute. Therefore f(A) is Ψ^* open in (Y,σ). Now g is Ψ^* C-homeomorphism, g^{-1} is Ψ^*-irresolute, then we have g (B) is Ψ^* open. That is $(g \circ f)$ (A) is Ψ^* open. Thus $(g \circ f)^{-1}$ is Ψ^* irresolute. Hence gof is Ψ^* C-homeomorphism.

(ii) By lemma 4.08 (ii), (ii) obtained.

Definition 4.10 :

For a topological space (X,τ), we define the following functions.

(i) Ψ^* Ch (X,τ) = {f/f : $(X,\tau) \to (X,\tau)$ is a Ψ^* C-homeomorphism.

(ii) Ψ^* h (X,τ) = {f/f : $(X,\tau) \to (X,\tau)$ is a Ψ^* C-homeomorphism.

We recall h (X,τ) = {f/f : $(X,\tau) \to (X,\tau)$ is a homeomorphism}.

Theorem 4.11 :

(a) The set Ψ^* Ch (X,τ) forms the group under composition of maps.

(b) The set h (X,τ) is a subgroup of Ψ^* Ch (X,τ).

Proof :

(a) (i) Operation is closed by the theorem 4.09 (i)

(ii) Since composition of mapping satisfies the associativity, associativity holds.

(iii) Since the identity is Ψ^* C-homeomorphism, it is an identity element of Ψ^* Ch (X,τ).

(iv) As the element in Ψ^* Ch (X,τ) are bijection f^{-1} exists in Ψ^* Ch (X,τ).

Hence Ψ^* Ch (X,τ) forms the group under the composition of mappings.

(b) It is obtained by using theorem 4.09 (ii).

Theorem 4.12 : If there exists a Ψ^* C-homeomorphism between (X,τ) and (Y,σ) then there exists a group isomorphism : Ψ^* Ch $(X,\tau) \cong \Psi^*$ Ch (Y,σ).

Proof :

Let f : $(X,\tau) \to (Y,\sigma)$ be a Ψ^* C-homeomorphism. For an element a $\in \Psi^*$ Ch (X,τ). Let f*(A) = $f \circ a \circ f^{-1}$. Then by theorem 4.09 (i), f*(a)$\in \Psi^*$ Ch (Y,σ). Thus f* is a requried group isomorphism.

Remarks 4.13 :

The converse of the above theorem need not be true as can be seen from the following example.

Example 4.14 :

Let (X,τ) be a topological space where $X = \{a,b,c\}$ and $\tau = \{X,\varphi,\{a\},\{a,b\}\}$. Ψ^* closed set of $(X,\tau) = \{X,\varphi,\{b\},\{b,c\}\}$. Let $h_a : (X,\tau) \to (X,\tau)$ be a function such that $h_a(a)=a$, $h_a(b)=c$ and $h_a(c)=b$. Let (Y,σ) be a topological space where $Y=\{a,b,c\}$ and $\sigma = \{Y,\varphi,\{a,b\}\}$ Ψ^* closed set of $(Y,\sigma) = \{Y,\varphi,\{c\},\{a,c\},\{b,c\}\}$. Let $f : (X,\tau) \to (Y,\sigma)$ be a bijection defined by $f(a) = c$, $f(b)=a$ and $f(c) = b$. Then f is not a Ψ^* C-homeomorphism. Indeed, for a Ψ^* closed set $\{c\}$, $f^{-1}(\{c\}) =\{a\}$ is not Ψ^*-closed. It is proved that Ψ^* Ch $(X,\tau) = \{1,h_a\}$ and Ψ^* Ch $(Y,\sigma) = \{1,h_c\}$ where $h_c : (Y,\sigma) \to (Y,\sigma)$ is bijection defined by $h_c(a) = b$, $h_c(b) = a$, $h_c(c) = c$. Since $f_*(1) = 1$ and $f_*(h_a) = f \circ h_a \circ f^{-1} = h_c$. Therefore f_* is an isomorphism.

5. Applications of Ψ^* closed sets

As applications of Ψ^* closed sets three new spaces namely T_Ψ space T_Ψ^* space and T_Ψ^{**} space are introduced.

Definition : 5.01

A subset A of (X,τ) is called a Ψ^* closed set if scl (A) $\subseteq$ Int (U) whenever $A \subseteq U$ and U is sg-open.

Definition : 5.02

A space (X,τ) is called a T_Ψ space if every Ψ closed set is Ψ^* closed.

Definition : 5.03

A space (X,τ) is called T_Ψ^* space if every sg-closed set is Ψ^* closed.

Definition : 5.04

A space (X,τ) is called a T_Ψ^{**} space every gs-closed set is Ψ^* closed.

Definition : 5.05

Every T_Ψ^* space is a T_{sg}^* space.

Proof :

Let (X,τ) be a T_Ψ^* space and A be a sg-closed set of (X,τ). Since (X,τ) is a T_Ψ^* space, A is Ψ^* closed. But every Ψ^* closed set is Ψ closed.[By theorem 3.02] Therefore A is Ψ closed. Thus (X,τ) is a T_{sg}^* space.

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