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HEAT TRANSFER BETWEEN TWO PARALLEL DISKS

G.C. Hazarika
Department of Mathematics,
Dibrugarh University

ABSTRACT

The heat transfer due to flow of a viscous Newtonian fluid between two parallel circular disks of infinite extent is investigated. Two cases viz. Squeeze- film flow and coaxial disk flow are considered. For both the cases, the temperature function is assumed to consist of two functions with prescribed temperature at the plates. The problem is studied numerically for various values of the parameters Pr, the Prandtl number and E, the Eckert number. The results are presented graphically. It is observed that the temperature function increases for increase of the parameter E in both the cases. Again as Pr increases, the temperature functions in case of coaxial disk flow increase. In the case of squeeze-film flow the first temperature function decreases for increase of Pr but the second function increases at first to a certain point and then decreases from that point onwards. This point is at about one-fifth of the total distance between the plates from the lower one.

1. INTRODUCTION

The flow of a Newtonian fluid between two parallel plates has been of interest to scientists for its application to lubrication. Two cases of flow configurations viz. Squeeze-film flow and coaxial disk flow are considered here. There have already been investigations on these flows.

Asymptotic solutions for the porous squeeze-film flow have been provided by Terrill and Cornish [1], Rasmussen [2] and Wang [3,4]. Rasmussen [2] and Wang [3,4] also provided a numerical solution to this two-point boundary value problem (BVP). The disk flow problem has been well studied long back by Von Karman [5] and later by Batchelor [6] and Stewartson [7] for flow between two coaxical disks.

Phan-Thien and Bush [8] have studied the problem for small and moderate Reynolds number where Von Karman's solution was taken for granted. They assumed power series for the velocity function and solved the non-linear algebraic system by optimisation method.

In this paper, an attempt has been made to study the heat transfer (Eckert and Drake [9]) to (i) the steady continuous squeeze-film flow between two parallel circular disks where the upper disk is porous and the lower one is rigid and stationary and (ii) the fluid flow between two rigid coaxial circular disks where the upper disk is rotating with an angular velocity and the lower disk is stationary. In both the configurations the disks are assumed to be of infinite extent. The exact similarity solutions for the energy equations are presented. In case (ii) solution for velocity function obtained by Phan-Thien and Bush is used to solve the energy equation. In case (i) velocity as well as temperature functions are obtained by using shooting methods [10,11,12].

2. FORMULATION

Case 1: Continuous squeeze-film flow

2.1.A. Velocity function: In this configuration the fluid flow is considered between two parallel circular disks of infinite extent. The bottom plate is rigid and stationary and the upper one is porous. The flow is generated by the injection of fluid through the upper plate. Considering (u, v, w) as the velocity components in cylindrical polar co-ordinates (r, θ, z), the Navier-Stokes equations after simplification become

$$f^{iv} + Reff''' = 0 \qquad (1)$$

with $u = \frac{1}{2}\left(\frac{r}{d}\right)Vf'(\eta), \quad v = 0, \quad w = -Vf(\eta)$ (2)

where Re = ρVd/m, ρ, μ, V, d, η = z/d are the Reynolds number, density, coefficient of viscosity, vertical velocity of fluid at the upper plate, distance between the plates and dimensionless vertical co-ordinate respectively. The function f is to be determined. The boundary conditions on f are

$$f(0) = 0, f'(0) = 0$$
$$f(1) = 1, f'(1) = 0 \quad (3)$$

2.1.B Heat transfer: Let T_a and T_b be the temperatures of the upper plate and lower plate respectively. Thermal boundary layer developed between the plates. The energy equation with the viscous dissipation in cylindrical polar co-ordinate is

$$c_p\left(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right) + \mu\phi \quad (4)$$

where $\phi = 2\left\{\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{u}{r}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right\} + \left(\frac{\partial u}{\partial z}\right)^2$ (5)

is the dissipation function, q, c_p, k and μ are density, specific heat at constant pressure, thermal conductivity and coefficient of viscosity respectively. The boundary conditions on T are

$T = T_b$ at z = 0; $T = T_a$ at z=d (6)

To solve equation (4), consider the substitution

$$\frac{T - T_a}{T_b - T_a} = \theta_1(\eta) + \left(\frac{r}{d}\right)^2 \theta_2(\eta) \quad (7)$$

and on equating various coefficients, we get equation on θ_1 and θ_2 as

$$\theta_1'' + 4\theta_2 + \Pr \mathrm{Re} f\theta_1' + 3\Pr E f'^2 = 0 \tag{8}$$

$$\theta_2'' - \Pr \mathrm{Re}(f'\theta_2 - f\theta_2') + {}^1/_4 \Pr E f''^2 = 0 \tag{9}$$

with boundary conditions on θ_1, θ_2 as

$$\theta_1(0) = 1, \theta_2(0) = 0$$

$$\theta_1(1) = 0, \theta_2(1) = 0 \tag{10}$$

where $\Pr = c_p\mu/k$ and $E = V^2/c_p\ \Delta T_{ab}$ are Prandtl number and Eckert number respectively. $\Delta T_{ab} = T_b - T_a$ is the temperature difference between the lower and the upper plate.

Case II: coaxial disk flow

2.II.A. Velocity function: We consider here the flow between two rigid circular disks of infinite extent. The lower disk is stationary and the upper one is rotating with an angular velocity Ω. Following Von Karman's solution for velocity field as (Schlichting [13]).

$$u = r\Omega h'(\eta),\ v = r\Omega g(\eta),\ w = -2\Omega d h(\eta) \tag{11}$$

Phan_Thien and Bush [8] obtained the equations of motion as

$$g'' - 2\mathrm{Re}(h'g - hg') = 0 \tag{12}$$

$$h'' + 2\mathrm{Re}(g'g + hh''') = 0$$

with no-slip boundary conditions

$$h(0) = h'(0) = g(0) = 0$$

$$h(1) = h'(1) = 0,\ g(1) = 1 \tag{13}$$

2.II.B. Heat transfer: Consider the transfer of heat between two parallel plates where the upper one has temperature T_0 and lower one has temperature T_1.

Substituting
$$\frac{T - T_0}{T_1 - T_0} = \phi_1(\eta) + \left(\frac{r^2}{d^2}\right)\phi_2(\eta) \tag{14}$$

and u, v, w from (11) in the energy equation (4), we get

$$\phi_1'' + 2\Pr \mathrm{Re} h\phi_1' + 12\Pr E\, h' + 4\phi_2 = 0 \tag{15}$$

$$\phi_2'' - 2Pr\,Re(\phi_2 h' - h\phi_2') + 2Pr\,E(h''^2 + g'^2) = 0 \qquad (16)$$

with boundary conditions

$$\phi_1(0) = 1,\ \phi_2(0) = 0$$

$$\phi_1(1) = 0,\ \phi_2(1) = 0 \qquad (17)$$

where $E = U^2/c_p(T_0 - T_1)$ is the Eckert number and $U = \Omega d$.

In the above analysis temperature distribution for both the cases are considered to consist of two functions.

3. METHOD OF SOLUTION

To solve BVP (1-3), (8-10) and (15-17), the shooting method is applied. In this method the BVP is converted to an initial value problem (IVP) by estimating the missing initial values to a desired degree of accuracy by an iterative scheme. Hazarika [12] showed that though there is no guarantee of convergence of the iterative scheme, if the initial guesses for the missing initial values are on opposite sides of the true value, the convergence is rapid and agrees well with other methods. This is experienced in our actual computation too. In solving the system (15)-(17), Phan-Thien and Bush [8] solutions for g and h are used.

Using shooting methods, the missing initial values viz. $f''(0)$, $f'''(0)$, $\theta_i'(0)$, $\phi_i'(0)$, $(i = 1, 2)$ are estimated for various combination of the parameters and consequently the problem is solved.

4. RESULT AND DISCUSSION

In this paper shooting method is successfully applied to solve all the three Boundary Value Problems, (1-3), (8-10) and (15-17). Velocity distribution for squeeze-film flow is determined for various values of Re. The estimated values of the missing initial values are presented in Table-I for various values of Re. It may be noted here that in Phan-Thien and Bush method it is found difficult to obtain solution for high Reynolds number (>18) due to limitation of their method but solutions are easily obtainable by the shooting method for considerably greater values of Re (>70) treating the problem to be a mathematical one.

In Table-I the missing values f "(0) and f "'(0) and in Table-II various missing initial values for the temperature functions are presented.

In figures 1, 2, 5 and 6, θ_1 and ϕ_1 and in figures 3, 4, 7 and 8, θ_2 and ϕ_2 are plotted for various values of Pr and E. The following observations are made.

1. In all cases when E increases then θ_1 and ϕ_i (i=1,2) increases as observed from Figs. 2, 3, 5 and 8.
2. As Pr increases ϕ_1, ϕ_2 increase but θ_1 decreases as seen from Figs. 1, 6, and 7.
3. At first θ_2 increases to a certain point (at about one fifth of the distance from the lower plate) and then decreases from that point onwards with the increase of Pr as evident from Fig. 4.

Table - I

Re	f "(0)	f "'(0)
1	3.678517	-13.605582
5	7.966281	-21.084982
10	9.874500	-31.769304
15	11.558604	-42.852646
18	12.469564	-49.518070
20	13.042295	-53.952549
25	14.373925	-64.994324
30	15.589704	-75.971123
35	16.714468	-86.888786
40	17.765635	-97.755013
50	19.694342	-119.358887
55	20.588566	-130.106964
60	21.444021	-140.823486

Table - II

E \ Pr		.7	1.0	1.5	2.0
0.0	$\theta_1'(0)$	0	0	0	0
	$\theta_2'(0)$	-2.418750	-2.771011	-3.215393	-3.559236
	$\phi_1'(0)$	0	0	0	0
	$\phi_2'(0)$	-.830866	-.762348	-.654618	-.555950
0.5	$\theta_1'(0)$	1.082134	1.477965	2.100089	2.689449
	$\theta_2'(0)$	-1.849478	-2.065366	-2.324414	-2.515748
	$\phi_1'(0)$	.111984	.158293	.223108	.286980
	$\phi_2'(0)$	-.736094	-.622260	-.375144	-.110179
1.0	$\theta_1'(0)$	2.164268	2.988929	4.200177	5.376897
	$\theta_2'(0)$	-1.280207	-1.358921	-1.433434	-1.472239
	$\phi_1'(0)$	.211362	.288416	.406007	.515415
	$\phi_2'(0)$	-.690654	-.505492	-.57264	.220239
1.5	$\theta_1'(0)$	3.246403	4.433894	6.300265	8.068346
	$\theta_2'(0)$	-.710935	-.652476	-.542454	-.428771
	$\phi_1'(0)$	.301763	.407229	.567293	.718111
	$\phi_2'(0)$	-.632119	-.404285	.025539	.494544

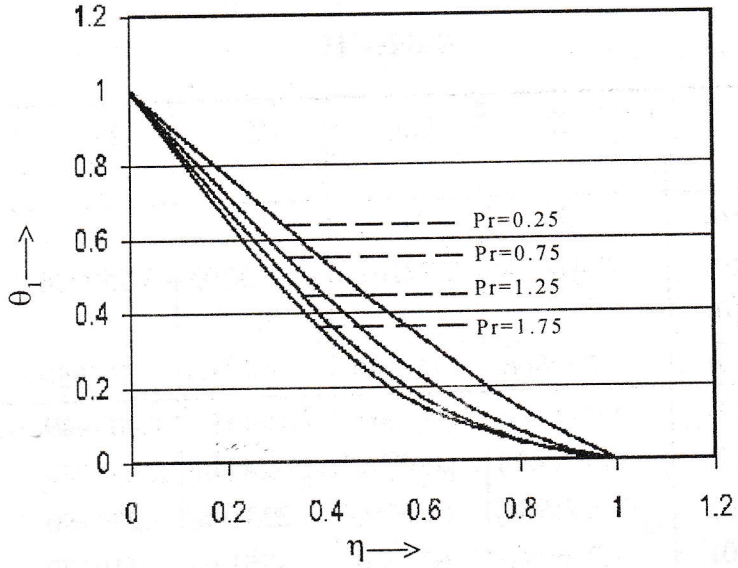

Fig.1 Variation of θ_1 for Re=10, E=.5 and Pr=0.25, .75, 1.25, 1.75 in squeezing flow.

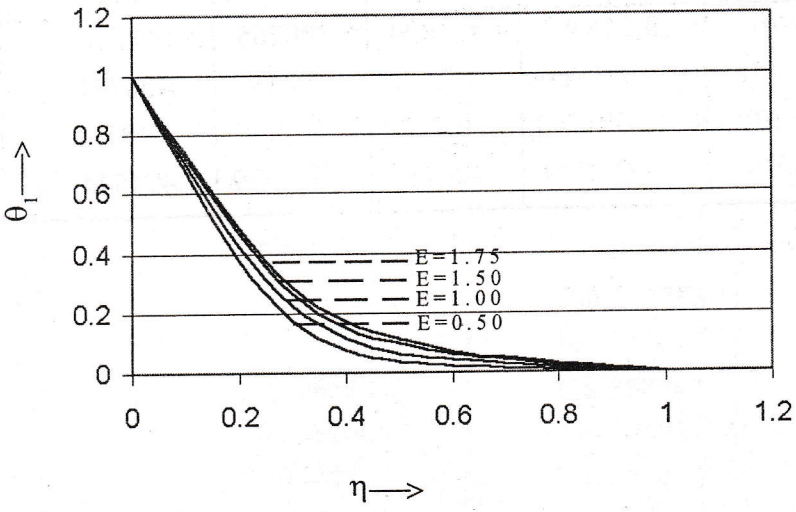

Fig. 2 Variation of θ_1 for Re=50, Pr=.7 and E=0.5, 1.0, 1.5, 1.75 in squeezing flow.

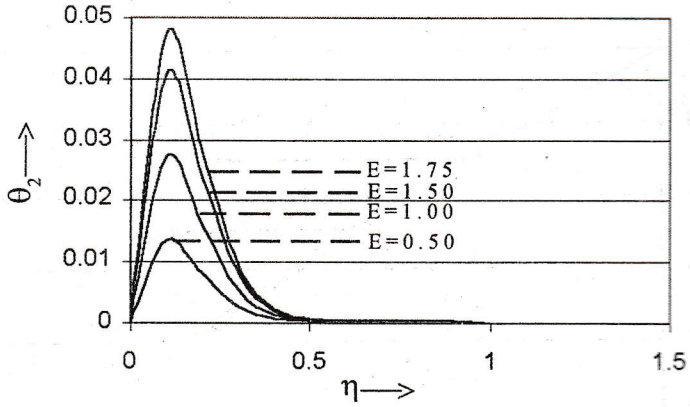

Fig. 3 Variation of θ_2 for Re=50, Pr=.7 and E=0.5, 1.0, 1.5, 1.75 in squeezing flow.

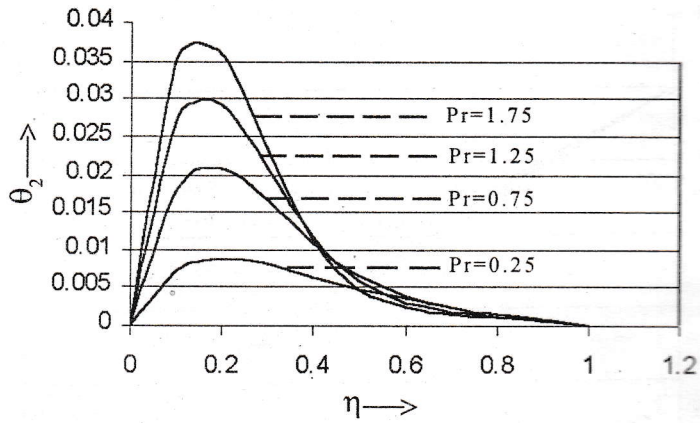

Fig.4 Variation of θ_2 for Re=10, E=.5 and Pr=0.25, .75, 1.25, 1.75 in squeezing flow.

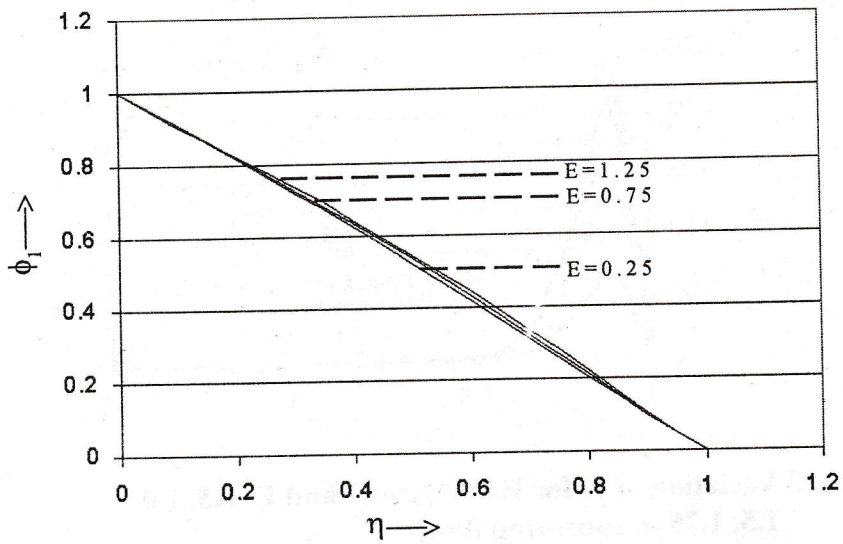

Fig. 5 Variation of ϕ_1 for Re=5, Pr=2.00 and E=0.25, 0.75, 1.25 in Co-axial disk flow.

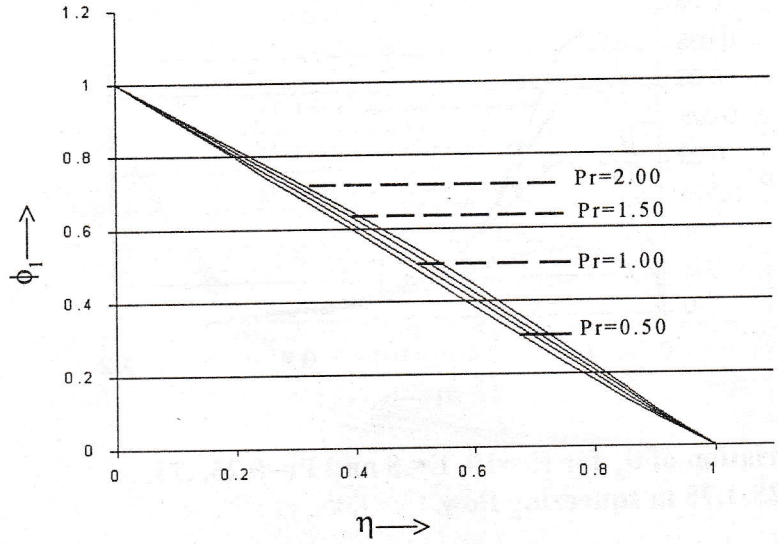

Fig.6 Variation of ϕ_1 for Re=5, E=1.25 and Pr=0.50, 1.00, 1.50, 2.00 in Co-axial disk flow.

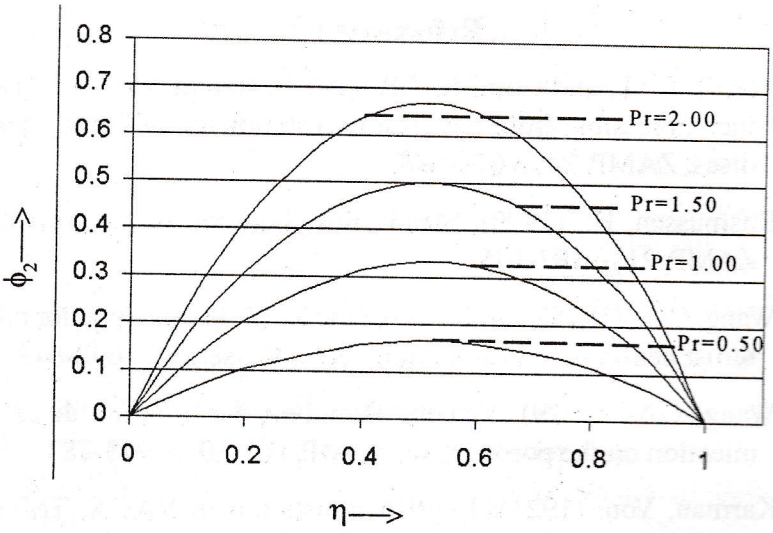

Fig.7 Variation of ϕ_2 for Re=5, E=1.25 and Pr=0.50, 1.00, 1.50, 2.00 in Co-axial disk flow.

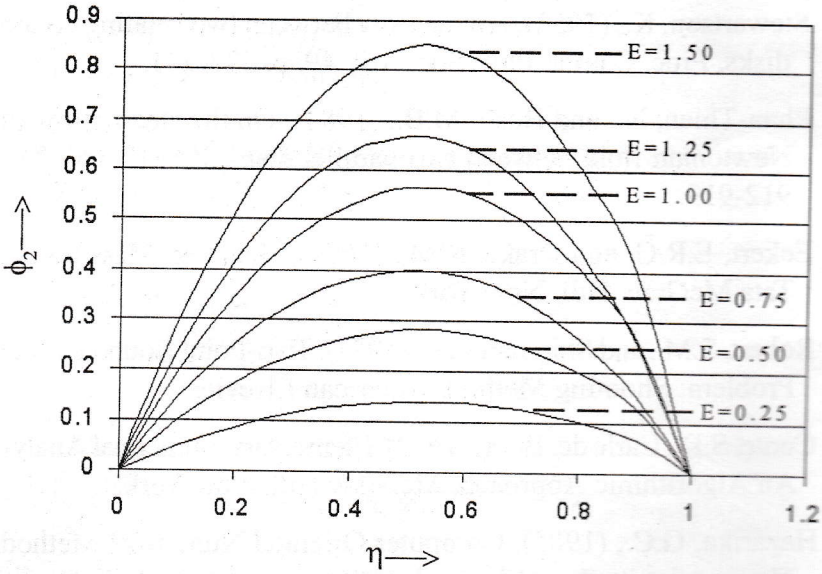

Fig. 8 Variation of ϕ_2 for Re=5, Pr=2.00 and E=0.25, 0.50, 0.75, 1.00, 1.25, 1.50 in Co-axial disk flow.

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