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MHD FLOW AND HEAT TRANSFER BETWEEN TWO VERTICAL PARALLEL PLATES IN PRESENCE OF UNIFORM INCLINED MAGNETIC FIELD

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Abstract:

The laminar convection flow of a viscous electrically conducting incompressible fluid between two heated vertical parallel plates is considered in presence of a uniform inclined magnetic field. The inclined magnetic field is a strong field and the induced field produced is along the flow direction. Analytical solutions for velocity, induced field and the temperature distributions are obtained; skin frictional factors are computed for different angle of inclinations and for different magnetic field strengths. The computed results of velocity distributions, induced field strengths and skin frictional factors are plotted against distances from the fixed vertical plates for different magnetic field strength and different angle of inclinations. The observed results predicts change in fluid velocity as well as induced magnetic field strength occurs in different order as changes in angle of inclination occur in different imposed field strength environment.

1. INTRODUCTION: The Study on the heat transfer in a rotating channel with Hall current under the action of a transverse uniform magnetic field was discussed by Borkakati and Srivastava (1976). Recently Das and Sanyal (1995) have studied the laminar convection flow of a conducting incompressible fluid between two vertical porous plates in presence of a uniform transverse magnetic field under different permeablities of the medium.

In this problem we have discussed the Magnetohydrodynamic flow and Heat transfer between two vertical parallel plates in presence of a uniform inclined magnetic field applied in the direction making an angle θ with the vertical axis. The field is considered to be strong enough and the induced field is along the flow direction. The analytical solution for velocity, induced field and the temperature distribution are sought for, from the system of governing equations. The skin friction and their computed values for different magnetic field strengths and different angle of inclinations of the magnetic field with the horizontal direction are plotted against the distances form the plates. Mathematica v 3.0 is used in the computational process. Here M_0 is the field parameter when external field is vertical to the flow direction and $\lambda = \cos\theta$, theta being angle of inclination with the horizontal.

The fluid viscosity is considered to be constant. In this analysis we have investigated the influence of applied magnetic field (strengths and inclinations) and the induced magnetic field on fluid variables and on skin friction and rate of heat transfer as well.

2. FORMULATION OF THE PROBLEM: We consider an unsteady laminar convection flow of a viscous incompressible electrically conducting fluid between two vertical parallel plates. We take X-axis along vertically upward direction through the central line of the channel and Y-axis is taken perpendicular to the plates. The plates of the channel are at y = ±h. A uniform magnetic field $\vec{B}_0$ is applied in the direction making an angle θ to the horizontal line. It induces another magnetic field $\vec{B}$ along the lines of motion.

The fluid velocity and magnetic field distributions are $\vec{v} = [u(y),0,0]$ an[d] $\vec{B} = [B(y) + B_0\sqrt{1-\lambda^2}, B_0\lambda, 0]$ respectively. Here $\lambda = \cos\theta$, $\vec{B}_0$ and $\vec{B}(y)$ ar[e] applied and induced magnetic field respectively.

In order to derive the governing equations of the problem the followin[g] assumptions are made:

(i) The fluid is finitely conduction and the viscous dissipation and the Joule he[at] are neglected.

(ii) Hall effect and Polarization effect are neglected.

(iii) Initially (i.e. at time $t = 0$) the plates and the fluid are at zero temperature (i.e. $T = 0$) and there is no flow within the channel.

(iv) At time $t > 0$, the temperatures of the plates $y = \pm h$ change according to $T = T_0(1 - e^{-nt})$ where T_0 is a constant temperature and $n > 0$ is a real number denoting the decay factor.

(v) The plates are considered to be infinite and all the physical quantities are functions of y and t only.

3. GOVERNING EQUATIONS: under the above assumptions the governing equations are

$$\nabla \cdot \vec{v} = 0. \tag{1}$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}.\nabla\right)\vec{v} = -\left(\frac{1}{\rho}\right)\nabla p + \frac{\mu}{\rho}\nabla^2 \bar{v} + \left(\vec{J} \times \vec{B}\right) + \vec{Z} \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} - \nabla \times \left(\vec{v} \times \vec{B}\right) - \left(\frac{1}{\sigma \mu_e}\right)\nabla^2 \vec{B} = 0 \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}\right) = \frac{d}{dy}\left(k\frac{dT}{dy}\right) \tag{4}$$

where the third term in the right hand side of equation (2) is the magnetic body force and $\vec{J}$ is the current density due to the magnetic field defined by

$$\vec{J} = \sigma\left(\vec{v} \times \vec{B}\right) \tag{5}$$

$\vec{Z}$ is the force due to buoyancy $Z = \beta g\left(T_0 - T\right)$ (6)

Using the velocity and magnetic field distribution as stated above, the equation (1) to (4) are as follows:

$$\frac{\partial u}{\partial t} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma}{\rho}uB_0^2\lambda^2 + \beta g(T_0 - T) \quad (7)$$

from equation (3) we have

$$\frac{\partial B}{\partial t} - B_0\lambda\frac{\partial u}{\partial y} - \left(\frac{1}{\sigma\mu_e}\right)\frac{\partial^2 B}{\partial y^2} = 0 \quad (8)$$

from equation (4) we have

$$\frac{\partial T}{\partial t} = \alpha_1\frac{\partial^2 T}{\partial y^2} \quad (9)$$

Here, $\alpha_1 = k/(\rho Cp)$.

The boundary conditions are

t = 0, u = 0, B = 0, T = 0, y = ±h.

t > 0, (10)

Considering the non-dimensional terms

$t^* = vt/h^2$, $b = B/B_0$, $y^* = y/h$, $u^* = u/u_0$, $\overline{T} = (T_0 - T)/T_0$, (11)

and then removing the asterisks, the non-dimensional forms of the Eq.(7-9) are as follows:

from (7)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \left(\frac{R_a}{P_r R_e}\right)\overline{T} - M^2\lambda^2 u \quad (12)$$

from (8)

$$\frac{\partial b}{\partial t} - R_e\lambda\frac{\partial u}{\partial y} - \left(\frac{1}{R_m P_r}\right)\frac{\partial^2 b}{\partial y^2} = 0 \quad (13)$$

from (9)

$$\frac{\partial \overline{T}}{\partial t}=\frac{1}{P_r}\frac{\partial^2 \overline{T}}{\partial y^2} \tag{14}$$

where,

M is the Hartmann number, $M=\sqrt{(B_0^2h^2\sigma)/(\rho v)}$,

P_r is the Prandtl number, $P_r = v/\alpha_1$,

R_e is the Reynold number, $R_e = u_0h/v$,

R_a is the Rayleigh number, $R_a = (\beta g h^3 T_0)/(v\alpha_1)$,

$R_m = \alpha_1\mu_e\sigma$,

α_1 is the thermal diffusivity, $\alpha_1 = k/\rho c_p$,

k is the thermal conductivity,

λ is the magnetic diffusivity, $\lambda = 1/\sigma\mu_e$,

v is the viscosity, $v = \mu/\rho$,

σ is the electrical conductivity,

ρ is the fluid density.

μ_e is the permeability of the medium and

μ is the co-efficient of viscosity.

The boundary conditions (10) reduces to

$t = 0,\ u = 0,\ b = 0,\ \overline{T} = 1 \ at\ y = \pm h$

$t > 0,\ u = 1,\ b = 1,\ \overline{T} = e^{-nt} \ at\ y = \pm h$ (15)

4. SOLUTIONS

To solve the equation (12) to equation (14) subject to the boundary condition (15), we apply the transformation of variables

$u = f(y)\, e^{-nt},\ b = g(y)e^{-nt}\ and\ = \phi(y)e^{-nt}$ (16)

Substitution (16) in Eq. (12-14), we have

From (12)

$$\frac{d^2 f}{dy^2} + \left(n - M^2\lambda^2\right)f + k_1\phi = 0 \quad (17)$$

where $k_1 = R_a/(P_r R_e)$.

From Eq. (13)

$$\frac{d^2 g}{dy^2} + \left(nR_m P_r\right)g + \left(R_m P_r R_e \lambda\right)\frac{df}{dy} = 0 \quad (18)$$

From Eq. (14)

$$\frac{d^2\phi}{dy^2} + \left(nP_r\right)\phi = 0 \quad (19)$$

The corresponding boundary conditions are:

for $t = 0: f = 0,\ g = 0,\ f = 1\ at\ y = \pm 1$ (20)

for $t > 0: f = e^{nt},\ g = e^{nt}, f = 1\ at\ y = \pm 1.$ (21)

From Eq. (19) and using the boundary condition (20) we have

$$\phi_0(y) = \frac{cos(a_0 y)}{cos\ a_0} \quad (22)$$

From Eq. (17) we have

$$\frac{d^2 f}{dy^2} + nf + k_1\frac{cos(a_0 y)}{cos\ a_0} - M^2\lambda^2 f = 0$$

$$\Rightarrow \frac{d^2 f}{dy^2} + k_2 f = k_3 cos\left(a_0 y\right)$$

where $k_2 = n - M^2\lambda^2$, $k_3 = -k_1/\cos a_0$.

Using the boundary condition (20) the solution is found as

$$f(y) = k_5 \cos(\sqrt{k_2}y) + k_4 \cos(a_0 y) \quad (23)$$

where $k_4 = k_3/(k_2 - a_0^2)$, $k_5 = -k_4 \cos a_0 / \cos\sqrt{k_2}$.

From Equation (23) we have

$$\frac{df}{dy} = -\sqrt{k_2 k_5}\, sin\left(\sqrt{k_2 y}\right) - a_0 k_4 sin(a_0 y) \quad (24)$$

Using equation (24) in Equation (18) we have,

$$\frac{d^2 y}{dy^2} + k_6 g = \sqrt{k_2 k_5 k_7}\ sin\left(\sqrt{k_2 y}\right) + a_0 k_4 k_7\ sin\left(a_0 y\right) \quad (25)$$

where $k_6 = nR_m P_r$, $k_7 = R_m P_r R_e \lambda$.

Using boundary condition (20) we have solution of equation (25)

$$g(y) = k_{10} sin(\sqrt{k_6}y) + k_8 sin(\sqrt{k_2}y) + k_9 sin(a_0 y) \quad (26)$$

where $k_8 = (\sqrt{k_2 k_5 k_7})/(k_6 - k_2)$, $k_9 = (a_0 k_4 k_7)/(k_6 - a_0^2)$,

$k_{10} = -(k_8 \sin\sqrt{k_2} + k_9 \sin a_0)/\sin\sqrt{k_6}$.

SKIN FRICTION: The skin friction at the plates $y = \pm 1$, is defined as

$$\tau = -\left[\mu \frac{du}{dy}\right]_{y=\pm 1} \quad (27)$$

Substituting the non-dimensional quantities (11) and then removing the asterisks we have Eq. (27)

$$\tau = -((\mu u_0)/h)\left[\frac{du}{dy}\right]_{y=\pm 1} \quad (28)$$

using relation (16)

$$\tau = -(\mu u_0/h)\left[\frac{df}{dy}e^{-nt}\right]_{y=\pm 1} \quad (29)$$

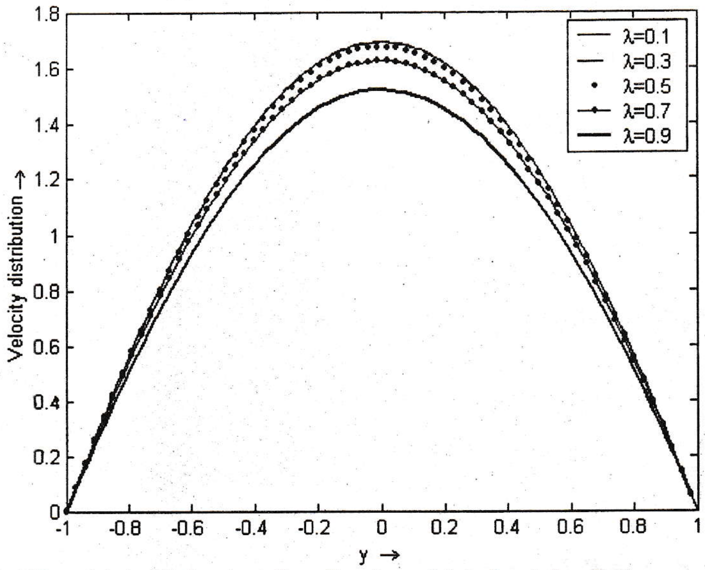

Fig.-1(a): Velocity distribution f (y) for M_0 =0.5

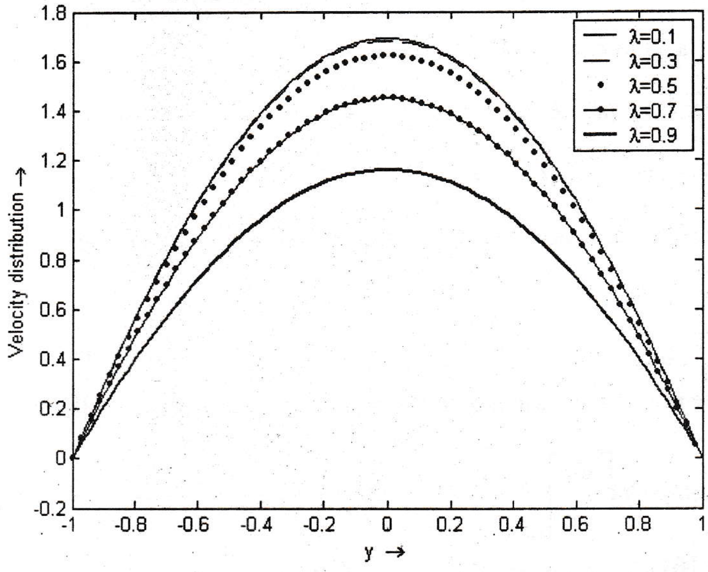

Fig.-1(b): Velocity distribution f (y) for M_0 =1.0

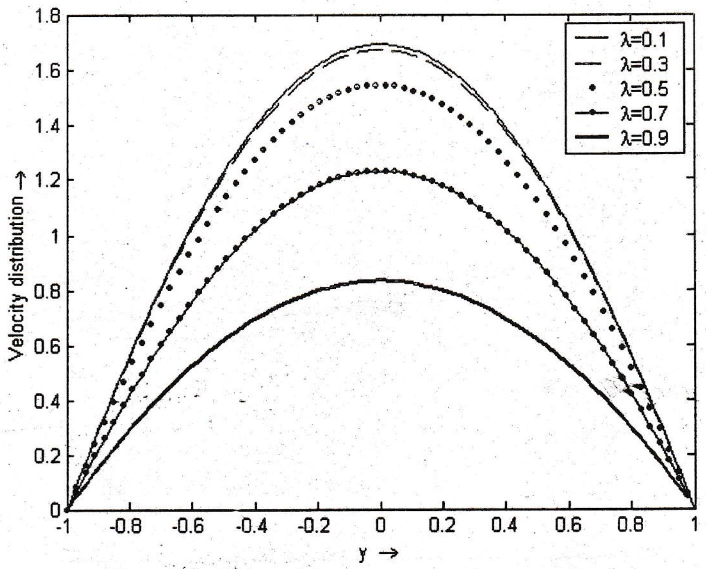

Fig.-1(c): Velocity distribution f (y) for M_0 =1.5

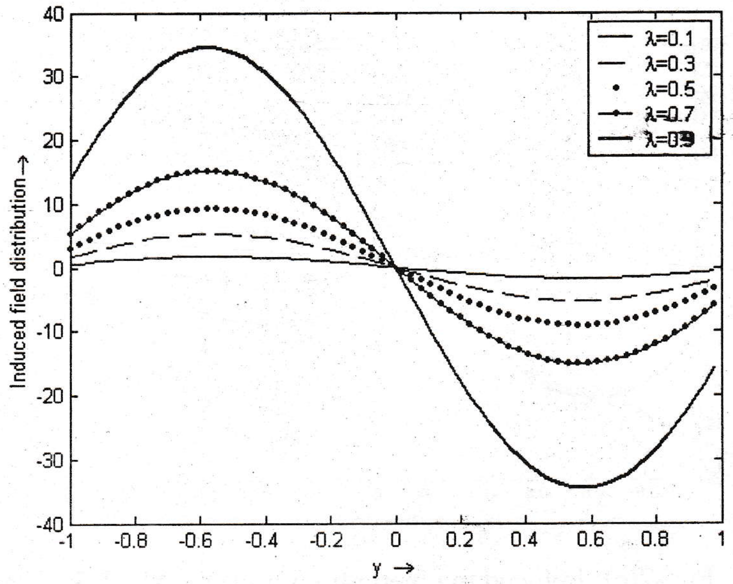

Fig.-2(a): Induced magnetic field g (y) for M_0=0.5

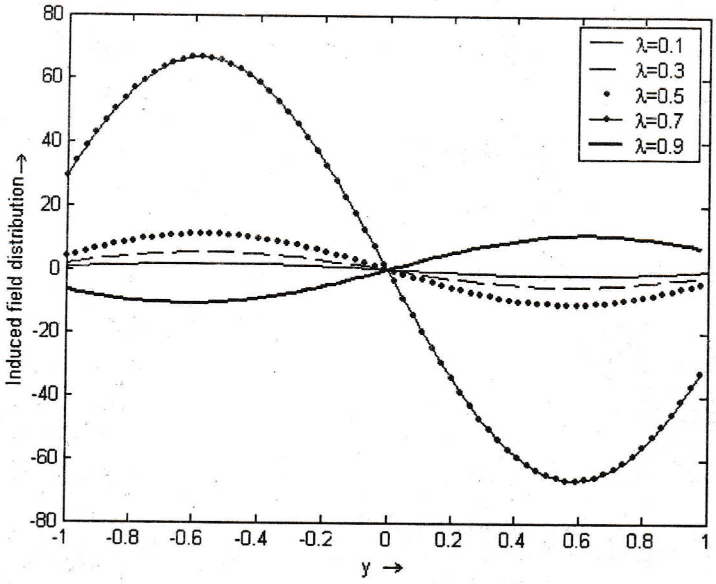

Fig.-2(b): Induced magnetic field g (y) for $M_0=1.0$

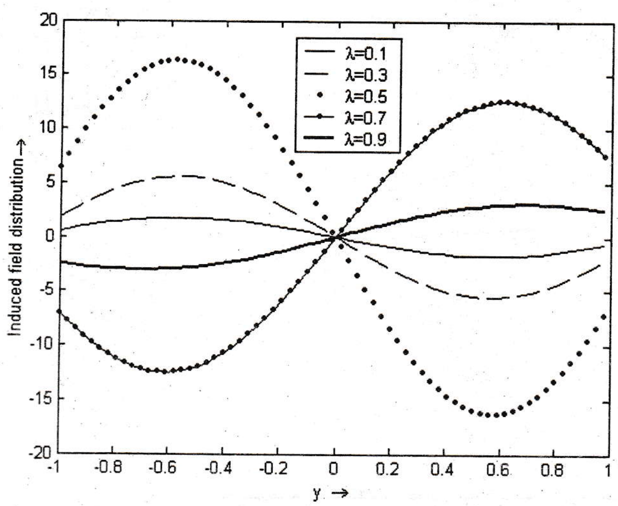

Fig.-2(c): Induced magnetic field g (y) for $M_0=1.5$

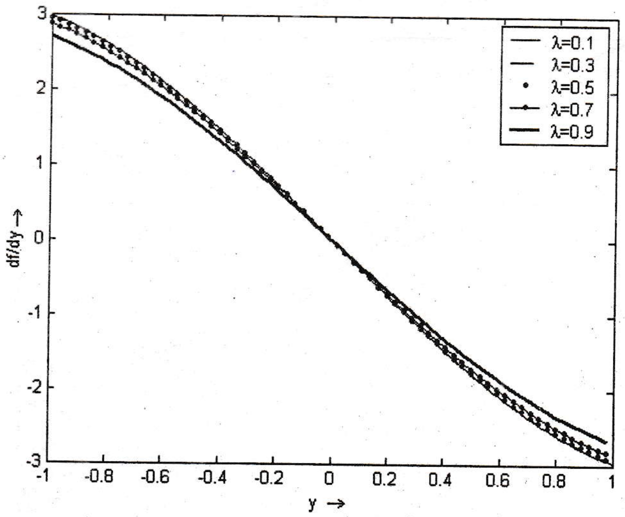

Fig.-3(a): Variation of friction factor $\left(\frac{df}{dy}\right)$ for M_0=0.5

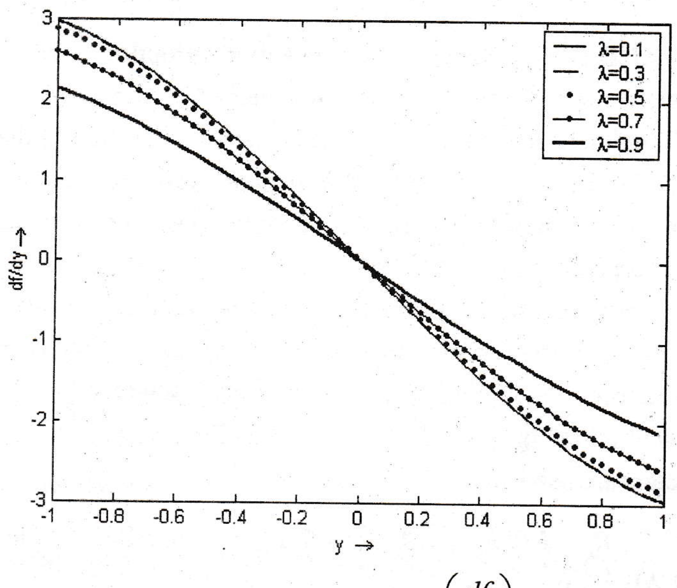

Fig.-3(b): Variation of friction factor $\left(\frac{df}{dy}\right)$ for M_0=1.0

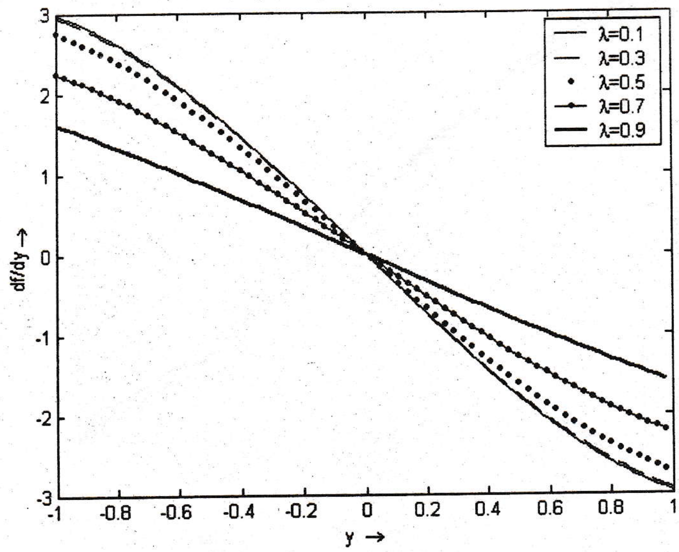

Fig.-3(c): Variation of friction factor $\left(\frac{df}{dy}\right)$ for M_0=1.5

5. RESULT AND DISCUSSION: The velocity distribution f against the distan from fixed plates y are plotted for different values of λ ($\lambda = \cos\theta$) i.e. for differe angle of inclination of magnetic field with horizontal direction in the Figs 1.a, 1 and 1.c. In the Fig 1.a considering four different values of inclination graphs of f a plotted for $M_0 = .5$, where M_0 is the Hartmann number when a imposed magne field is taken perpendicular to the flow direction as considered in original Hartma flow problem. In the present problem the Hartmann number of flow is $M = M_0\lambda$. Fig 1.b plotings of f are presented for same variations of λ with $M_0 = 1$. In Fig 1 plotings of f are presented for same set of variations of λ with $M_0 = 1.5$. On the ba of same consideration Fig. 2(a) - 2(c) and Fig. 3.a - Fig 3.c are plotted. The followi fluid parameters are used:

$P_r = 0.71$, $R_e = 1.0$, $R_a = 1.0$, $n = 1.0$, $R_m = 10.0$, All these plotting are do using MATLAB 5.2.0.

From the Fig 1(a) - 1(c) it is observed that with the increase of angle of inclination (decrease of value of λ) velocity increases in all the three cases of different magnetic field strength. Further it is also observed that the with the bigger strength of magnetic field variation of velocity with angle of inclination is bigger. So we can conclude that with large magnetic field strength if inclination of field is slightly changed we can expect large change in fluid velocity.

In Fig 2(a) - 2(c) the induced magnetic field strength are plotted against distance from the plates at point equal distance from the plates and at points on the plates. It is observed that the induced field strength is zero at the points on the plates. The field strength attains maximum value at points at distances $h/2$ and $3h/2$ from any of the plates if plates are considered to be separated by distance $2h$. When the strength of magnetic field is small ($M_0 < 1$), the fluctuation increase with increase of value of λ (i.e. decrease of angle of inclination of imposed field with direction of flow. But with $M_0 \geq 1$ this order is not maintained with increase field strength ($M_0 > 1$) rate of fluctuation is faster when angle of inclination is big and fluctuation is slow when λ is small.

From Fig 3(a) - 3(b) it is observed that in the case of strong imposed field ($M_0 > 1$) frictional factor will decrease with increase value of λ. In the other words, if imposed field inclination with direction of flow decreases, smaller frictional factor variation become distinctive as field strength are made stronger.

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