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ON BIANCHI-I VACUUM SOLUTIONS IN BIMETRIC RELATIVIT

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Abstract:

For Bianchi - I space-time

$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2$

where A, B, and C are functions of time 't'.

The solution of the field equations in bimetric relativity $N_1^{\ j} = 0$ and $N_1^{\ j} = \lambda g_1^{\ j}$ are $A = e^{mt} = B = C$ and $A = \exp\{-(\lambda/2)\, t^2 + nt\} = B = C$ respectively, where λ is cosmologic constant and m, n are constants of integration. And it is observed that the resulting space-time can be reduced to the conformal one.

Key words: Binachi - I, Vacuum, Bimetric relativity.

AMS sub code 83C05 (General Relativity).

Introduction : Rosen[3] (1973) proposed the bimetric theory of relativity to remo some of the unsatisfactory features of the general theory of relativity in which the exist two metric tensors at each point of space-time g_{ij}, which describes gravitati and the background metric γ_{ij}, which enters into the field equations and interacts wi g_{ij} but does not interact directly with matter.

One can regard γ_{ij} as describing the geometry that exists if no matter were present.

Accordingly, at each space-time point one has two line elements

$$ds^2 = g_{ij}\, dx^i\, dx^j$$

and $$d\sigma^2 = \gamma_{ij}\, dx^i\, dx^j$$

Deo[1] has studied the Bianchi - I cosmological model does not accommodate perfect fluid as well as Maxwell fields in bimetric relativity.

Further Mahurpawar and Deo[2] have studied Bianchi - I space-time with the source Maxwell fields coupled with cosmic cloud strings in bimetric relativity and obtained nil contribution of Maxwell fields as well as cosmic cloud strings in this theory.

Here we continue the above study with the another set of field equations $N_1^{\,j} = 0$ and $N_1^{\,j} = \lambda g_1^{\,j}$, where λ is the cosmological constant, as the empty space-time field equations of the bimetric relativity.

Bianchi - I vacuum solutions

The line-elements describing Bianchi type - I space-time is taken in the form

$$ds^2 = A^2 (dx^2 - dt^2) + B^2dy^2 + C^2dz^2 \tag{1}$$

where, the metric potentials A, B and C are functions of time 't'

The background metric of flat space-time corresponding to equation (1) is

$$d\sigma^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{2}$$

An empty space-time field equations of bimetric relativity assume the form

$$N_1^{\,j} = 0 \tag{3}$$

where, $N_1^{\,j} = \frac{1}{2} \gamma^{\alpha\beta} (g^{hj} g_{hi} |_{\alpha}) |_{\beta}$ (4)

and a vertical bar (|) denotes the γ - covariant differentiation.

Using the equations (1) to (4) the field equations $N_1^{\,j} = 0$ are

$$\frac{(A^{\bullet})^2}{A^2} - \frac{A^{\bullet\bullet}}{A} = 0 \tag{5}$$

$$\frac{(B^{\bullet})^2}{B^2} - \frac{B^{\bullet\bullet}}{B} = 0 \tag{6}$$

$$\frac{(C^{\bullet})^2}{C^2} - \frac{C^{\bullet\bullet}}{C} = 0 \tag{7}$$

where $A^{\bullet} = \frac{\partial A}{\partial t}$, & $A^{\bullet\bullet} = \frac{\partial^2 A}{\partial t^2}$ etc.

From equations (5), (6) and (7) w e have,

$$A = e^{mt+n},\ B = e^{pt+q} \text{ and } C = e^{rt+s} \tag{8}$$

Where m, n, p, q, r and s are arbitrary constants of integration

Absorbing the constants n, q and s in differentials line-element (1) with (8) takes the form

$$ds^2 = e^{2mt}(-dt^2 + dx^2) + e^{2pt}dy^2 + e^{2rt}dz^2 \tag{9}$$

For m = p = r = k = constant, it reduce to the conformal space-time form

$$ds^2 = e^{2kt}(-dt^2 + dx^2 + dy^2 + dz^2) \tag{10}$$

It is an interesting note that equation (10) is free from singularity at t = 0.

If we introduced the cosmological constant λ in the field equations of bimetric relativity and write $N_1^{\,j} = \lambda g_1^{\,j}$ (11)

Similar to the Einstein field equations $R_1^{\,j} = \lambda g_1^{\,j}$ of general relativity, as its empty space-time field equation (11)

Then using equations (1), (2), (4) and (11) we have

$$A = B = C = \exp\{-(\lambda/2)t^2 + ut + w\} \tag{12}$$

where u and w are constants of integrations.

Absorbing the constant w in the differentials the line-element (1) takes the form

$$ds^2 = \exp\{-(\lambda/2)t^2 + ut\}[-dt^2 + dx^2 + dy^2 + dz^2] \tag{13}$$

It is observed that the conformal metric (13) has no singularity at t = 0.

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[1] Deo, S. D. and Thengane, K. D.; "Fizika B" (Zagreb) 11, (2002), 155-160

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[3] Rosen, N.; Genl. Rela Grav. 4 (1973), 435-447.