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ON SEMI-(γ,γ')-OPEN SETS AND BI-OPERATION IN TOPOLOGICAL SPACES

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Abstract

In this paper we introduce the comcept of semi - (γ,γ') open sets in a topologica space together with its $SO(X)_{(\gamma,\gamma')}$- closure, semi - (γ,γ') - closure and $SO(X)_{(\gamma,\gamma')}$ - interio operators and study the relationship between them. Also we introduce semi -(γ,γ')- T spaces (i = 0, $^1/_2$, 1,2) and study the topological properties on them.

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Key words : semi-(γ,γ')-open set, semi-(γ,γ')-closure,SO $(X)_{(\gamma,\gamma')}$ - semi closure, semi (γ,γ')-interior, semi-(γ,γ')-T_i (i = 0, $^1/_2$, 1,2)

1. Introduction

Kasahara [1] unfied several unknown characterization of compact space and H-closed spaces by introducting cretain operation on a topology. Based on thos operations Ogata [4] defined the notion of γ-open sets and introduced some separatio axioms. Umehara etal. [7] defined the notion of (γ,γ')-open sets and introduced som new separation axioms and investigated their relationships. Sai Sundara Krishna etal [6] introduced the concept of semi-γ-open sets by introducing certain operation on semi-open sets [3].

In this paper, in Section 2 we introduce the concept of semi-(γ,γ')-ope sets and study some of its properties. Further we introduce the notion of semi-(γ,γ') closure, SO $(X)_{(\gamma,\gamma')}$-closure and SO $(X)_{(\gamma,\gamma')}$-interior operators and study the relationshi between them.

In Section 3, 4 we introduce the notion of semi-(γ,γ')-T_i spaces ($i = 0, {}^1/_2, 1, 2$) and characterize them using the notion of semi-(γ,γ')-open or semi-(γ,γ')-closed sets.

2. Semi-(γ,γ')-Open set

A subset S of a topological space X is called a semi open set if S is contained in Cl (Int (S)). The set of all semi open subsets in X is denoted by SO(X).

Definition 2.1 [6]: (i) Let (X,τ) be a topological space and γ be an operation defined on SO (X). Then a subset A of X is said to be a semi -γ-open set if for each $x \in A$ there exists a semi-open set U such that $x \in U$ and $U^\gamma \subseteq A$. The family of all semi -γ-open sets in (X,τ) is denoted by $SO(X)_\gamma$.

(ii) Let (X,τ) be a topological space, A be a subset of X and γ be an operation on SO(X). Then $SO(X)_\gamma\text{-cl}(A) = \cap\{F: A \subseteq F, X - F \in SO(X)_\gamma\}$

(iii) Let (X,τ) be a topological space, A be a subset of X and γ be an operation on SO(X). Then a point $x \in X$ is in semi -γ-closure of A if $U^\gamma \cap A \neq \phi$ for each semi-open set U containing x. The semi -γ-closure of a set A is denoted by $scl_\gamma(A)$.

(iv) Let (X,τ) be a topological space, A be a subset of X and γ be an operation on SO(X). Then A is said to be semi -γ-generalized closed if $scl_\gamma(A) \subseteq U$ whenever $A \subseteq U$ and U is a semi -γ-open set in (X,τ). It is denoted by semi -γ-g.closed.

(v) Let (X,τ) be a topological space and (γ/τ) be a restriction map defined on τ. Then a subset A of X is said to be (γ/τ)-open set if for each $x \in A$ there exists an open set U such that $x \in U$ and $U^{\gamma/\tau} \subseteq A$. The family of all (γ/τ)-open sets in (X,τ) is denoted by $\tau_{(\gamma/\tau)}$.

Definition 2.2[6]: A topological space (X,τ) is said to be a

(i) semi-γ-T_0 space, if for each distinct points $x,y \in X$ there exists semi-open sets U such that either $x \in U$ and $y \notin U^\gamma$ or $y \in U$ and $x \notin U^\gamma$.

(ii) semi-γ-$T_{1/2}$ space if every semi-γ-g.closed set of (X,τ) is semi-γ-closed.

(iii) semi-γ-T_1 space, if for each distinct points $x,y \in X$ there exists two semi-open sets U and V containing x,y respectively such that $y \notin U^\gamma$ and $x \notin U^\gamma$.

(iv) semi-γ-T_2 space, if for each distinct points, x,y of X there exists semi-open sets, U,V such that $x\in U$, $y\in V$ and $U^{\gamma}\cap V^{\gamma}=\phi$.

Definition 2.3 : Let (X,τ) be a topological space and γ,γ' are the operations on SO(X). Then a subset A of X is said to be a semi-(γ,γ')-open set if for each $x\in A$ there exist semi-open sets U and V containing x such that $U^{\gamma}\cup V^{\gamma'}\subseteq A$. The family of all semi-(γ,γ')-open sets in (X,τ) is denoted by $SO(X)_{(\gamma,\gamma')}$.

Example 2.4 : Let $X=\{a,b,c\}$, $\tau=\{\phi,X,\{a\},\{b\},\{a,b\},\{a,c\}\}$ and γ,γ' are the operations defined on SO(X) such that $A^{\gamma}=\begin{cases} A & \text{if } b\notin A \\ cl(A) & \text{if } b\in A\end{cases}$ and $A^{\gamma'}=\begin{cases} cl(A) & \text{if } b\notin A \\ A & \text{if } b\in A\end{cases}$, Then the semi-$(\gamma,\gamma')$-open sets of (X,τ) are $\{\phi,X,\{a,b\},\{a,c\}\}$.

Theorem 2.5 : Let (X,τ) be a topological space, A be a subset of X and γ,γ' are the operations defined on SO(X).

(i) A is semi-(γ,γ')-open set if and only A is semi-γ-open and semi-γ'-open.

(ii) If A is semi-(γ,γ')-open set, then A is semi-open.

(iii) If A_i is semi-(γ,γ')-open set for each $i\in J$ (the index set), then $\bigcup_{i\in J} A_i$ is a semi-(γ,γ')-open set.

(iv) The following statements are equivalent.

(a) A is semi-(γ,γ')-open set.

(b) A is semi-γ-open.

(c) A is (γ,id)-semi-open where id: $SO(X)\to P(X)$ is an identity operation such that is $A^{id}=A$ for every $A\in SO(X)$.

Proof. (i) Given A is semi-(γ,γ')-open set. Let $x\in A$. Then there exist semi-open sets U and V containing x such that $U^{\gamma}\cup V^{\gamma'}\subseteq A$. This implies that $U^{\gamma}\subseteq A$ and $V^{\gamma'}\subseteq A$. Hence A is semi-γ-open and semi-γ'-open set (X,τ).

Conversely, Let $x\in A$. Then it follows that there exits semi-open sets U and V containing x such that $U^{\gamma}\subseteq A$ and $V^{\gamma'}\subseteq A$. Therefore $U^{\gamma}\cup V^{\gamma'}\subseteq A$. Hence A is semi-$(\gamma,\gamma')$-open set.

(ii) Proof is straightforward from Remark 2.7[6] and (i)

(iii) Let $x\in \bigcup_{i\in J} A_i$. Then $x\in A_i$ for some $i\in J$. Since each A_i is a semi-(γ,γ')-open set, so there exist semi-open sets W and S containing x such that $W^{\gamma}\cup S^{\gamma'}\subseteq A_i$.

This implies that $W^{\gamma} \cup S^{\gamma'} \subseteq \bigcup_{i\in J} A_i$. Hence $\bigcup_{i\in J} A_i$ is a semi-(γ,γ')-open set.

(iv) (a) if and only if (b) is shown by setting in $\gamma = \gamma'$ in (i).

(b) if and only if (c) shown by Definitions.

Remark 2.6 : By Theorem 2.5(i) and Remark 2.7[6] we have the following relations:

$$SO(X)_{(\gamma,\gamma')} = SO(X)_{\gamma} \cap SO(X)_{\gamma'} \subseteq SO(X).$$

Remark 2.7 : Let (X, τ) be a topological space and γ,γ' are the operations defined on $SO(X)$. If A and B are semi -(γ,γ')-open sets in $((X, \tau)$ then $A \cap B$ need not be a semi-(γ,γ')-open set in (X, τ) .

In example 2.4, $A = \{a,b\}$ and $B=\{a,c\}$ are semi (γ,γ')-open sets but $A\cap B=\{a\}$ is not a semi-(γ,γ')-open set in (X, τ) .

Definition 2.8 : A space (X, τ) is said to be semi-(γ,γ')-regular if for each $x \in X$ and every semi-open set U containing x there exist semi-open sets W and S such that $W^{\gamma} \cup S^{\gamma'} \subseteq U$.

Theorem 2.9 : Let (X, τ)be a topological space and γ,γ' are the operations defined on $SO(X)$. Then.

(i) (X, τ) is semi-(γ,γ')-regular if and only if $SO(X)_{(\gamma,\gamma')}=SO(X)$ holds.

(ii) If (X, τ) is semi-(γ,γ')-regular if and only if it is semi-γ–regular and semi-γ'-

(iii) The following statements are equivalent

(a) (X, τ) is semi-(γ,γ')-regular.

(b) (X, τ) is semi-γ–regular.

(c) (X, τ) is semi-$(\gamma$,id)-regular.

Proof : (i) By the Remark 2.6 we already have $SO(X)_{(\gamma,\gamma')} \subseteq SO(X)$. Now to prove that $SO(X) \subseteq SO(X)_{(\gamma,\gamma')}$. Let $A \in SO(X)$ and $x \in A$. Then there exist semi-open sets W and S containing x such that $W^{\gamma} \cup S^{\gamma'} \subseteq A$. This implies that A is a semi-(γ,γ')-open set and so $SO(X) \subseteq SO(X)_{(\gamma,\gamma')}$.

Convesely, Let $x \in X$ and U be a semi-open set containing X. Since U is semi-(γ,γ')-open set, there exist a semi open sets W and S containing x such that $W^{\gamma} \cup S^{\gamma'} \subseteq U$. This implies that (X, τ) is semi-(γ,γ')-regular.

(ii) Proof is straightforward from Remark 2.6 and (i).

(iii) Since $SO(X)_{(\gamma,\gamma)} = SO(X)_{\gamma} = SO(X)_{(\gamma,id)} \subseteq SO(X)$ holds, the equivalences are proved by (i).

Theorem 2.10 : Let (X, τ) be a topological space and γ and γ' are semi-regular operations defined on SO(X). If A and B are semi-(γ,γ')-open sets, then $A \cap B$ is also semi-(γ,γ')-open sets.

Proof : Let $x \in X$ $A \cap B$. Then there exist semi-open set U,V, W, S such that $(U^{\gamma} \cup V^{\gamma'}) \cap (W^{\gamma} \cup S^{\gamma'}) \subseteq A \cap B$. This implies that $(U^{\gamma} \cap W^{\gamma}) \cup (V^{\gamma'} \cap S^{\gamma'}) \subseteq A \cap B$. By semi-regularity of γ and γ', there exist semi-open sets E and F containing x such that $E^{\gamma} \subseteq (U^{\gamma} \cap W^{\gamma})$ and $F^{\gamma'} \subseteq (V^{\gamma'} \cap S^{\gamma'})$. This implies that $E^{\gamma} \cup F^{\gamma'} \subseteq A \cap B$. Therefore $A \cap B$ is semi-(γ,γ')-open set.

Remark 2.11 : If γ and γ' are semi-regular operations on SO(X), then $SO(X)_{(\gamma,\gamma')}$ from a topology on X.

Proof : Proof is straightforward from the Theorem 2.11, 2.5 (iii).

Definition 2.12 : Let (X, τ) be a topological space and γ, γ' are the operations defined on SO(X). Then a subset F of X is said to be semi-(γ,γ')-closed set if its complement X-F is semi-(γ,γ')-open set.

Definition 2.13 : Let (X, τ) be a topological space, A be subset of X and γ γ' are the operations defined on SO(X). Then $SO(X)_{(\gamma,\gamma')}$ - cl (A) denotes the intersection of all semi-(γ,γ')-closed set containing A.

That is $SO(X)_{(\gamma,\gamma')} - cl(A) = \cap\{F : A \subseteq F \in SO(X)_{(\gamma,\gamma')}\}$

Remark 2.14 : Let (X, τ) be a topological space and A and B are two subsets of X

(i) $A \subseteq SO(X)_{(\gamma,\gamma')}$ - cl(A)

(ii) If $A \subseteq B$ the $SO(X)_{(\gamma,\gamma')}$ - cl(A)$\subseteq SO(X)_{(\gamma,\gamma')}$ - cl(B)

(iii) $SO(X)_{(\gamma,\gamma')}$ - cl(A) is a semi-(γ,γ') closed set containing A.

Proof : Proof of (i) and (ii) straightforward from the Definition 2.13.

Proof of (iii) is straightforward from Definition 2.13 and Theorem 2.5(iii).

Theorem 2.15 : Let (X, τ) be a topological space, A be subset of X and γ, γ' are the operations defined on SO(X). Then

(i) For a point $x \in X$, $x \in SO(X)_{(\gamma,\gamma')}$ - cl(A) if and only if $V \cap A \neq \phi$ for every semi-(γ,γ')-open set V containing x.

(ii) A is semi-(γ,γ')-closed if and only if $SO(X)_{(\gamma,\gamma')}$ - cl(A)=A.

Proof : (i) Let Fo be the set of all $y \in X$ such that $V \cap A \neq \phi$ for $y \in SO(X)_{(\gamma,\gamma')}$ and $y \in V$. To prove the Theorem it is enough to prove that Fo = $SO(X)_{(\gamma,\gamma')}$ - cl(A). It is easy to see that X-Fo is a semi-(γ,γ')-open set and $A \subseteq$ Fo.This means that $SO(X)_{(\gamma,\gamma')}$ -cl(A)$\subseteq$ Fo. Conversely, Let F be a set such that $A \subseteq F$ and $X - F \in SO(X)_{(\gamma,\gamma')}$. If $x \notin F$, then we have $x \in X-F$ and $(X-F) \cap A=\phi$. This means $x \notin$ Fo, hence we have Fo$\subseteq$ F. Therefore Fo$\subseteq SO(X)_{(\gamma,\gamma')}$ - cl(A).

(ii) If A = $SO(X)_{(\gamma,\gamma')}$ - cl(A), then to prove that A is semi-(γ,γ')-closed. Let $x \in X-A$ then $x \notin SO(X)_{(\gamma,\gamma')}$ - cl(A) and so there exists a semi-(γ,γ')-closed set F containing A such that $x \notin F$. This implies that X - F is a semi(γ,γ')-open set containig x such that $X - F \subseteq X-A$. This is true for every $x \in X-A$, hence X-A is semi-(γ,γ')-open set. Therefore A is semi-(γ,γ')-closed.

Conversely, If A is semi-(γ,γ')-closed, then from the Definition 2.13 it is follows that A = $SO(X)_{(\gamma,\gamma')}$ - cl(A).

Definition 2.16 : Let (X, τ) be a topological space, A be subset of X and γ,

γ' are the operations defined on SO(X). Then we define $scl_{(\gamma,\gamma')}$ as follows :

$scl_{(\gamma,\gamma')}(A) = \{x \in X : (U^{\gamma} \cup V^{\gamma'}) \cap A \neq \phi$, holds for every semi-open sets U and V containing x$\}$.

Theorem 2.17 : Let (X, τ) be a topological space, A be subset of X and γ, γ' are the operations defined on SO(X). Then $scl_{(\gamma,\gamma')}(A) = scl_{\gamma}(A) \cup scl_{\gamma'}(A)$ holds where $scl_{\gamma}(A), scl_{\gamma'}(A)$ are the semi-γ-closure and semi-γ'-closure of A respectively.

Proof : From the Definition 2.16 and Definition 2.23 [6] it is shown that the following statements (a) - (e) are equivalent :

(a) $x \notin scl_{(\gamma,\gamma')}(A)$.

(b) There exists semi-open sets U and W containing x such that $(U^{\gamma} \cup W^{\gamma'}) \cap A = \phi$.

(c) There exists a semi-open set U and W containing $U^{\gamma} \cap A \neq \phi$ and $W^{\gamma'} \cap A = \phi$.

(d) $x \notin scl_{\gamma}(A)$ and $x \notin scl_{\gamma'}(A)$.

(e) $x \notin scl_{\gamma}(A) \cup scl_{\gamma'}(A)$.

Theorem 2.18 : Let (X,τ) be a topological space, A be subset of X and γ,γ' are the operations on SO(X).

(i) A is semi-(γ,γ')-closed if and only of $scl_{(\gamma,\gamma')}(A) = A$. (in the sense of Janokovic

(ii) $SO(X)_{(\gamma,\gamma')}$- $cl(A) = A$ if and only if $scl_{(\gamma,\gamma')}(A) = A$.

(iii) A is semi-(γ,γ')-open if and only if $scl_{(\gamma,\gamma')}(X\text{-}A) = X\text{-}A$.

Proof : (i) (Necessity) It is enough to prove that $scl_{(\gamma,\gamma')}(A) \subseteq A$. Let $x \notin A$. Then, i complement X-A is a semi-(γ,γ')-open set containing x. This implies that there exi semi-open sets U and W containing x such that $U^{\gamma} \cup W^{\gamma'} \subseteq X\text{-}A$, ar so $(U^{\gamma} \cup W^{\gamma'}) \cap A = \phi$. This shows that $x \notin scl_{(\gamma,\gamma')}(A)$. Hence $scl_{(\gamma,\gamma')}(A) = A$.

(Sufficiency) Let $x \in X\text{-}A$. By assumption $x \notin scl_{(\gamma,\gamma')}(A)$, there exists a sem open set U and W containing x such that $(U^{\gamma}\cup W^{\gamma'})\cap A = \phi$, and so $U^{\gamma}\cup W^{\gamma'} \subseteq X-A$. This shows that X-A is semi-(γ,γ')-open set and so A is semi-(γ,γ')-closed set.

(ii) Proof is straightforward from (i) and Theorem 2.15 (ii)

(iii) Proof is straightforward from (i) Definition 2.12.

Remark 2.19 : Let (X,τ) be a topological space. Then it follows from Theorem 2.18 (i) that $scl_{(\gamma,\gamma')}(A) = scl_{\gamma}(A)$ for every subset A of X.

Theorem 2.20 : Let (X,τ) be a topological space and A and B are the subsets of X, Suppose γ,γ' are semi-regular operations on SO(X). Then $scl_{(\gamma,\gamma')}(A\cup B) = scl_{(\gamma,\gamma')}(A)\cup scl_{(\gamma,\gamma')}(B)$.

Proof : Proof is straightforward from the Theorem 2.17 and Theorem 2.30 ([6]).

Theorem 2.21 : Let (X,τ) be a topological space. Then for any subset A of X the following properties holds good :

(i) $A\subseteq scl(A)\subseteq scl_{(\gamma,\gamma')}(A)\subseteq SO(X)_{(\gamma,\gamma')}\text{-}cl(A)$.

(ii) If (X,τ) is semi-(γ,γ')-regular, then $scl(A) = scl_{(\gamma,\gamma')}(A) = SO(X)_{(\gamma,\gamma')}\text{-}cl(A)$.

(iii) $scl_{(\gamma,\gamma')}(A)$ is a semi-closed subset of (X,τ).

(iv) $SO(X)_{(\gamma,\gamma')}\text{-}cl\,(scl_{(\gamma,\gamma')}(A)) = SO(X)_{(\gamma,\gamma')}\text{-}cl(A) = scl_{(\gamma,\gamma')}(SO(X)_{(\gamma,\gamma')}\text{-}cl(A))$.

Proof : (i) By Theorem 2.17 and Theorem 2.30([6]) we have $scl_{(\gamma,\gamma')}(A) = scl_{\gamma}(A)\cup scl_{\gamma'}(A) \supset scl(A)$. It follows from the Definition 2.24[6], Remark 2.26[6] and Theorem 2.5 and Definition 2.15 we have that $scl_{\gamma}(A) \subseteq SO\,(X)_{\gamma}\text{-}cl(A) \subseteq SO\,(X)_{(\gamma,\gamma')}\text{-}cl(A)$ and similarly $scl_{\gamma'}(A) \subseteq SO(X)_{(\gamma,\gamma')}\text{-}cl(A)$. This shows that $scl_{(\gamma,\gamma')}(A) \subseteq SO(X)_{(\gamma,\gamma')}\text{-}cl(A)$.

(ii) By Theorem 2.19 (ii) we have that $SO(X)_{(\gamma,\gamma')} = SO(X)$ and hence $scl_{(\gamma,\gamma')}(A) = SO(X)_{(\gamma,\gamma')}\text{-}cl(A)$. By using (i), we have $scl(A) = scl_{(\gamma,\gamma')}(A) = SO(X)_{(\gamma,\gamma)}\text{-}cl(A)$.

(iii) $scl(scl_{(\gamma,\gamma')}(A)) = scl(scl_{\gamma}(A)\cup scl_{\gamma'}(A)) = scl(scl_{\gamma}(A))\cup scl(scl_{\gamma'}(A))$

$= scl_{\gamma}(A) \cup scl_{\gamma'}(A) = scl_{(\gamma,\gamma')}(A)$.

(iv) By Remark 2.14 (iii) and Theorem 2.18 (ii) we have that $SO(X)_{(\gamma,}$ $- cl(A) \subseteq scl_{(\gamma,\gamma')}(SO(X)_{(\gamma,\gamma')}- cl(A))$. It follows from (i) and Remark 2.14 (ii) tha $scl_{(\gamma,\gamma')}(A) \subseteq SO(X)_{(\gamma,\gamma')}- cl((A)) \subseteq SO(X)_{(\gamma,\gamma')}- cl(A))$. By Remark 2.14 (i) and (ii we have $SO(X)_{(\gamma,\gamma')}- cl(scl_{(\gamma,\gamma')}(A)) \subseteq SO(X)_{(\gamma,\gamma')}- cl(A) \subseteq SO(X)_{(\gamma,\gamma')}- cl(scl_{(\gamma,\gamma')}(A$ and hence $SO(X)_{(\gamma,\gamma')}- cl(A) = SO(X)_{(\gamma,\gamma')}- cl(scl_{(\gamma,\gamma')}(A))$.

Definition 2.22 : Let (X,τ) be a topological space and γ be the operatio on SO(X). Then for a subset A of X, $SO(X)_{\gamma}$-interior of A is defined as union c all semi-γ-open set contained in A.

That is $SO(X)_{\gamma}- int(A) = \cup\{G : G \in SO(X)_{\gamma} \text{ and } G \subseteq A\}$.

Theorem 2.23 : Let (X,τ) be a topological space, A be a subset of X and be an operation on SO(X). Then

(i) $SO(X)_{\gamma}- int(A)$ is a semi-γ-open set contained in A.

(ii) A is semi-γ-open if and only $SO(X)_{\gamma}$-int(A) = A.

Proof : Proof of (i) and (ii) follows from the Definition 2.4 and Theore 2.13 [6].

Theorem 2.24 : Let (X,τ) be a topological spacy, γ be an operation o SO(X). Then for any subsets A and B of X the following are holds good.

(i) If $A \subseteq B$, then $SO(X)_{\gamma}- int(A) \subseteq SO(X)_{\gamma}- int(B)$

(ii) $SO(X)_{\gamma}- int(A \cup B) = SO(X)_{\gamma}- int(A) \cup SO(X)_{\gamma}- int(B)$.

(iii) If γ is a semi-regular operations, then $SO(X)_{\gamma}- int(A \cap B) = SO(X)$ $int(A) \cap SO(X)_{\gamma}- int(B)$.

Proof : Proof of (i) follows from the Definition 2.22

Proof of (ii) follows from the Definition 2.22 and (i)

Proof of (iii) follows from the Definition 2.22 and Theorem 2.20.

Definition 2.25 : Let (X,τ) be a topological space, A be a subset of X and γ,γ' are the operations on SO(X). Then $SO(X)_{(\gamma,\gamma')}$ - interior of A is defined as union of all semi-(γ,γ')-open sets contained in A.

That is $SO(X)_{(\gamma,\gamma')}\text{-}int(A) = \cup\{G: G\in SO(X)_{(\gamma,\gamma')} \text{ and } G\subseteq A\}$.

Theorem 2.26 : Let (X,τ) be a topological space, A be a subset of X and γ,γ' are the operations defined on SO(X). Then

(i) $SO(X)_{(\gamma,\gamma')}$ - int(A) is a semi-(γ,γ')-open sets contained in A.

(ii) A is semi-(γ,γ')-open if and only if $SO(X)_{(\gamma,\gamma')}\text{-}int(A)=A$.

(iii) $SO(X)_{(\gamma,\gamma')}\text{-}int(A)= SO(X)_{\gamma}\text{-}int(A)\cap SO(X)_{\gamma'}\text{-}int(A)$.

Proof : Proof of (i) follows from the Definition 2.25.

Proof of (ii) follows from the Definition 2.25 and (i)

Proof of (iii) follows from the Definition 2.22, 2.25 and Theorem 2.5(i).

Remark 2.27 : Let (X,τ) be a topological space and γ,γ' are the operations on SO(X). Then from Theorem 2.26 and Remark 2.6 it follows that for any subset A of X the following relation holds good.

$$SO(X)_{(\gamma,\gamma')}\text{-}int(A)\subseteq SO(X)_{\gamma}\text{-}int(A)\subseteq sint(A)\subseteq A$$

3. Semi-γ,γ'-g.closed and Semi-γ,γ'-$T_{1/2}$ spaces

Definition 3.1 : Let (X,τ) be a topological space and γ,γ' are the operations on τ. Then a subset A of X is said to be semi-(γ,γ')-generalized closed set if $scl_{(\gamma,\gamma')}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-(γ,γ')-open set. It is denoted by semi-(γ,γ')-g.closed.

Remark 3.2 : Let (X,τ) be a topological space and γ,γ' are the operations on SO(X). Then a subset A of X is semi-(γ,γ')-g.closed if and only if A is semi-γ-g.closed and semi-γ'-g.closed.

Proof : Proof follows from the Theorem 2.5(i) and Theorem 2.17 and Definition 3.4[6].

Theorem 3.3 : Let (X,τ) be a topological space, A be a subset of X and γ,γ' are the operations on SO(X). Then A is semi-(γ,γ')-g.closed if and only if $A \cap SO(X)_{(\gamma,\gamma')}\text{-cl}(\{x\}) \neq \phi$ for every $x \in scl_{(\gamma,\gamma')}(A)$.

Proof : (Necessity). Suppose that there exist a point $x \in scl_{(\gamma,\gamma')}(A)$ such that $A \cap SO(X)_{(\gamma,\gamma')}\text{-cl}(\{x\}) \neq \phi$. Let $V = (X\text{-}SO(X)_{(\gamma,\gamma')}\text{-cl}(\{x\}))$. Then V is semi-$(\gamma,\gamma')$-open set such that $A \subseteq V$. Since A is semi-(γ,γ')-g.closed, implies that $scl_{(\gamma,\gamma')}(A) \subseteq V$. Hence $x \notin scl_{(\gamma,\gamma')}(A)$. This is a contradiction. Hence $A \cap SO(X)_{(\gamma,\gamma')}\text{-cl}(\{x\}) \neq \phi$ for every $x \notin scl_{(\gamma,\gamma')}(A)$.

(Sufficiency) Let U be semi -(γ,γ')-open set containing A. Let $x \in scl_{(\gamma,\gamma')}(A)$ such that $A \cap SO(X)_{(\gamma,\gamma')}\text{-cl}(\{x\})$. Since $y \in U$ and U is semi-(γ,γ')-open, then by Theorem 2.15(i) we have $U \cap \{x\} \neq \phi$ and hence $x \in U$. This implies that $scl_{(\gamma,\gamma')}(A) \subseteq U$. Therefore, A is semi-$(\gamma,\gamma')$-g. closed.

Corollary 3. 4: Let (X,τ) be a topological space and γ,γ' are the operations on SO(X). Then a subset A of X is semi -γ- g. closed if any only if $A \cap SO(X)_\gamma\text{-scl}(\{x\}) = \phi$ for every $x \in scl_\gamma(A)$.

Proof : Proof is straight forward if we put $\gamma = \gamma'$ in Theorem 3.3.

Theorem 3.5: Let (X,τ) be a topological space and γ,γ' are the operation on SO (X). If a subset A of X is semi -(γ,γ') - g. closed, then $scl_{(\gamma,\gamma')}(A)$ - A doest not contain any non empty semi - γ,γ' - closed set.

Proof : Let F be semi - (γ,γ') - closed such that $F \subseteq scl_{(\gamma,\gamma')}(A)$ -A. Then by assumption it follows that $scl_{(\gamma,\gamma')}(A) \subseteq X\text{-}F$. Hence $F \subseteq (X - scl_{(\gamma,\gamma')}(A)) \cap scl_{(\gamma,\gamma')}(A)$. Therefore $F = \phi$.

Definition 3, 6: A space (X,τ) is said to be a semi - (γ,γ') - $T\,{}^1/_2$ space if every semi- (γ,γ') -g. closed set of (X,τ) is semi - (γ,γ') - closed.

Theorem 3.7: Let (X,τ) be a topological space and γ,γ' are the operation on SO(X). Then for each $x \in X$, $\{x\}$ is semi - (γ,γ') -closed or its cimplement is semi - (γ,γ') -g. closed in (X,τ).

Proof : Suppose that $\{x\}$ is not a semi - (γ,γ') - closed. Then $X - \{x\}$ is not a semi - (γ,γ') - open set. Therefore X is the only semi - (γ,γ') -open set cotaining $X - \{x\}$. Hence $X - \{x\}$ is semi - (γ,γ') -g. closed set.

Theorem 3. 8: A space (X,τ) is semi - (γ,γ') - $T\,{}^1/_2$ space if and only if for each $x \in X$, $\{x\}$ is a semi -(γ,γ') -open or semi - (γ,γ') - closed in (X,τ).

Proof : (Necessity) Suppose that, for $x \in X$, $\{x\}$ is not a semi - (γ,γ') - closed. Then by Theorem 3.7 and assumption $X - \{x\}$ is semi - (γ,γ') - closed set and so $\{x\}$ is semi - (γ,γ') - open.

(Sufficiency) Let A be semi - (γ,γ') -g. closed set. Then, we claim that $scl_{(\gamma,\gamma')}(A) = A$ holds. Let $x \in scl_{(\gamma,\gamma')}(A)$, then by assumption $\{x\}$ is semi - (γ,γ') - open or semi - (γ,γ') - closed.

Case (i) : Suppose $\{x\}$ is semi - (γ,γ') - closed. Then by Theorem 3.5 we have $scl_{(\gamma,\gamma')}(A) - A$ does not contain $\{x\}$ Hence $scl_{(\gamma,\gamma')}(A) \subseteq A$.

Case (ii) : Suppose $\{x\}$ is semi - (γ,γ') - open. Then it follows from Theorem 2.15 (i) that $\{x\} \cap A \neq \phi$. This implies that $x \in A$

By case (i) and Case (ii) A is semi - (γ,γ') - g. closed.

Remark 3.9 : If we put $\gamma = \gamma'$ in Thereom 3.8, then we have (X,τ) is

(i) semi - (γ,γ') - $T\,{}^1/_2$ space.

(ii) semi - γ - $T\,{}^1/_2$ space.

(iii) for each $x \in X$, $\{x\}$ is semi -γ - open or semi -γ - closed.

4.Separtion Axioms

Let $X \times X$ be direct product of X and $\Delta(X) = \{(x,x) : x \in X\}$ the diagonal set and γ,γ' are the operations defined on SO(X).

Definition 4.1 : A space (X,τ) is said to be a semi - (γ,γ') - T_2 space, if for each $(x,y) \in (X\times X) - \Delta(X)$, there exist semi-open sets U and V contains x and y respectively, such that $U^{\gamma} \cap V^{\gamma'} = \phi$.

Remark 4.2 : For given two distinct points x and y, the semi- (γ,γ') - T_2 axiom, requires that there exist semi-open sets U, V, W,S such that $U^{\gamma} \cap V^{\gamma'} = \phi$, $W^{\gamma} \cap S^{\gamma'} = \phi$; $x \in U$, W and $y \in V$, S.

Definition 4.3 : A space (X,τ) is said to be a semi - (γ,γ') - T_1 space if for each $(x,y) \in (X \times X) - \Delta(X)$, there exist semi-open sets U and V contains x and y, respectively such that $y \notin U^{\gamma}$ and $x \notin V^{\gamma'}$

Definition 4.4 : A space (X,τ) is said to be a semi - (γ,γ') - T_0 space if for each $(x,y) \in (X \times X) - \Delta(X)$ there exist semi-open sets U and V such that $x \in U$ and $y \notin U^{\gamma}$ or $y \in V$ and $x \notin V^{\gamma'}$

Remark 4.5 : For given two distinct points x and y the semi - (γ,γ') - T_0 axiom requires that there exists two semi-open sets U and V such that one condition of the following (i), (ii), (iii), (iv) should be satisfied.

(i) $x \in U$, $y \notin U^{\gamma}$, $y \in V$ and $x \notin V^{\gamma}$.

(ii) $x \in U$, $y \notin U^{\gamma}$, $x \in V$ and $y \notin V^{\gamma'}$.

(iii) $y \in U$, $x \notin U^{\gamma'}$, $y \in V$ and $x \notin V^{\gamma}$.

(iv) $y \in U$, $x \notin U^{\gamma'}$, $x \in V$ and $y \notin V^{\gamma'}$.

Theorem 4.6 : A space (X,τ) is semi - (γ,γ') - T_1 space if and only if for any $x \in X$ the singleton set $\{x\}$ is semi - (γ,γ') - closed set.

Proof: Let $x \in X$ and let $y \in X - \{x\}$. Then for $(x,y) \in (X \times X) - \Delta(X)$, there exist semi-open sets U and V contains y and x respectively such that $x \notin U^{\gamma}$ and $y \notin V^{\gamma'}$. Similarly for $(y,x) \in (X \times X) - \Delta(X)$, there exist semi-open sets W and S contains x and y respectively, such that $y \notin W^{\gamma}$ and $x \notin S^{\gamma'}$. Therefore, we have $y \in U^{\gamma} \cup S^{\gamma'} \subseteq X - \{x\}$. This implies, $X - \{x\}$ is semi- (γ,γ')-open.

Conversely, Let x and y be two distinct points of X. Then $X - \{x\}$ is a semi-(γ,γ') -open set containing y and $X - \{y\}$ is also a semi-(γ,γ')- open set containing x.

This implies there exist semi-open sets U and W containing x and semi -open sets V and S containing y such that $U^{\gamma} \cup W^{\gamma'} \subseteq X - \{y\}$ and $V^{\gamma} \cup S^{\gamma'} \subseteq X - \{x\}$. This implies that $y \notin U^{\gamma}$, $x \notin S^{\gamma'}$ and $x \notin V^{\gamma'}$, $y \notin W^{\gamma'}$. Therefore, for each $(x,y) \in (X \times X) - \Delta(X)$ there exist semi-open sets A and B containing x and y respectively, such that $y \notin A^{\gamma}$ and $x \notin B^{\gamma'}$. This implies that (X,τ) is a semi - (γ,γ')- T_1 space.

Theorem 4.7 :

(i) If a space (X,τ) is a semi - (γ,γ')- T_2, then it is semi - (γ,γ')- T_1 space.

(ii) If a space (X,τ) is a semi - (γ,γ')- T_1, then it is semi - (γ,γ')- $T_{1/2}$ space.

(iii) If a space (X,τ) is a semi - (γ,γ')- $T_{1/2}$, then it is semi - (γ,γ')- T_0 space.

Proof : (i) The proof is straightward from the Definition 4.1 and 4.3

(ii) Proof is straightward from the Definition 4.3 and Theorem 4.6.

(iii) Let x and y be two distinct points of X. Then by Theorem 3.8 {x} is semi - (γ,γ') - closed or semi - (γ,γ')- open.

Case (i) : If {x} is semi - (γ,γ')-open, then there exist semi-open sets U and V contains x such that $U^{\gamma} \cup V^{\gamma'} \subseteq \{x\}$ and so $y \notin U^{\gamma}$ and $y \notin V^{\gamma'}$. Hence X is semi - (γ,γ')- T_0.

Case (ii) : If {x}is semi - (γ,γ')- closed, then X - {x} is semi - (γ,γ')- open. Therefore there exist semi-open sets W and S such that $W^{\gamma} \cup S^{\gamma'} \subseteq X - \{x\}$ and so $x \notin W^{\gamma}$ and $x \notin S^{\gamma'}$. Therefore we have X is semi - (γ,γ')- T_0 space.

Remark 4.8 : The following examples shows that the converse of (i), (ii) and (iii) of the Theorem 4.7 need not be true.

(i) Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ and γ,γ' are the operations defined on SO(X) such that $A^{\gamma} = \begin{cases} A & \text{if } b \notin A \\ cl(A) & \text{if } b \in A \end{cases}$, $A^{\gamma'} = \begin{cases} A & \text{if } b \in A \\ cl(A) & \text{if } b \notin A \end{cases}$, respectively. Then (X,τ) is a semi - (γ,γ')- T_0 space but not a semi - (γ,γ')- $T_{1/2}$ space.

(ii) Let $X = \{a,b,c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$ and γ,γ' are the operations defined on SO(X) such that $A^{\gamma} = \begin{cases} A & \text{if } b \notin A \\ cl(A) & \text{if } b \in A \end{cases}$, $A^{\gamma'} = \begin{cases} A \cup \{c\} & \text{if } A \neq \{a\} \\ A & \text{if } A = \{a\} \end{cases}$, respectively.

Then (X,τ) is a semi - (γ,γ')- $T_{1/2}$ space but not a semi - (γ,γ')- T_1 space.

(iii) Let $X = \{ a,b,c\}, \tau = \{ \phi, X, \{a\}, \{b\}, \{a,b\}\}$ and γ,γ' are the operation

defined on SO(X) such that $A^{\gamma} = \begin{cases} A & \text{if } b \in A \\ cl(A) & \text{if } b \notin A \end{cases}$, $A^{\gamma} = \begin{cases} A \cup \{c\} & \text{if } A = \{a\} \text{or} \{b\} \\ A \cup \{a\} & \text{if } A = \{c\} \\ A & \text{if } A \neq \{a\}, \{b\} \text{and} \end{cases}$

respectively. Then (X,τ) is a semi - (γ,γ')- T_1 space but not a semi - (γ,γ')- T_2 spac

Remark 4.9 : From Theorem 4.7 and Remark 4.8 we have the followi

diagram implications :

Semi - (γ,γ')-$T_2 \rightleftarrows$ semi-(γ,γ')-$T_1 \rightleftarrows$ semi - (γ,γ')- $T_{1/2} \rightleftarrows$ semi-(γ,γ')-T_0

Where $A \rightarrow B$ denotes A implies B and $A \not\rightarrow B$ denotes A does not implies

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