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ON SOME GENERALIZATIONS OF FUZZY G_δ- CONTINUOUS MAPPINGS

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ABSTRACT

In this paper, making use of fuzzy G_δ - set and fuzzy F_σ - set various generalizations of fuzzy G_δ - functions are introduced and studied. Further, the interrelations among the concepts introduced are also discussed in detail.

Key Words

Fuzzy G_δ - continuous, M - fuzzy G_δ - continuous, σ - closure, σ - interior fuzzy almost G_δ - continuous, strong fuzzy G_δ - continuous, perfectly fuzzy G_δ - continuous.

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1. INTRODUCTION :

The fuzzy concept has invaded almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [9]. Fuzzy sets have applications in many fields such as information [7] and control [8]. The theory of fuzzy topological spaces was introduced and developed by Chang [5] and since then various notions in classical topology have been extended to fuzzy topological spaces [1 to 4]. The concept of fuzzy G_δ - set was introduced by Balasubramanian [2]. In this paper making use of fuzzy F_σ - sets and fuzzy G_δ-sets, we have introduced the concepts of fuzzy σ - closure and fuzzy σ - interior and investigated their properties. Further we have introduced and studied various types of fuzzy continuous functions involving fuzzy G_δ - sets. Further we have given complete discussion about the interrelations among functions introduced.

2. PRELEMINARIES

Definition 1. Let (X,T) be a fuzzy topological space and λ be fuzzy set in X. λ is called a fuzzy G_δ - set [2] if $\lambda = \wedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$, $i \in I$.

Definition 2. Let (X,T) be a fuzzy topological space and λ be a fuzzy set in X. λ is called a fuzzy F_σ - set [2] if $\lambda = \vee_{i=1}^{\infty} \lambda_i$ where each $\lambda_i' \in T$, $i \in I$.

Definition 3. Let $f:(X,T) \rightarrow (Y,S)$ be a mapping from a fuzzy topological space X to another fuzzy topological space Y. f is called (i) a fuzzy continuous mapping [1] if $f^{-1}(\lambda) \in T$ for each $\lambda \in S$. (ii) a fuzzy open/closed mapping [1] if $f(\lambda)$ is open/closed set of Y for each open/closed set λ of X.

Definition 4. Let (X,T) be fuzzy topological space and Y be an ordinary subset of X. Then $T_Y = (\lambda/Y/ \lambda \in T)$ is a fuzzy topology on Y and is called the induced or relative fuzzy topology [3]. The pair (Y,T_Y) is called a fuzzy subspace of (X,T).

Definition 5. Let $\{(X_i, T_i) : i \in I\}$ be a family of fuzzy topological spaces and let $X = \prod_i X_i$ be the cartesian product of X_i's. The fuzzy product topology [6]$\prod T_i$ on

X is the smallest fuzzy topology on X (if exists) which makes each projection $p_i:X \rightarrow (X_i,T_i)$, $i\in I$, fuzzy continuous.

3. FUZZY G_δ - CONTINUOUS MAPPINGS :

Definition 6. Let $f:(X,T) \rightarrow (Y,S)$ be a mapping from a fuzzy topological space X to another fuzzy topological space Y.

1. f is said to be fuzzy G_δ - continuous, if $f^{-1}(\lambda)$ is a fuzzy G_δ - set in (X,T) for any fuzzy open set λ in (Y,S).
2. f is said to be fuzzy G_δ - if $f(\lambda)$ is a fuzzy G_δ - set in (Y,S) for any fuzzy G_δ - set λ in (X,T).
3. f is said to be fuzzy F_σ, if $f(\lambda)$ is a fuzzy F_σ-set in (Y,S) for any fuzzy F_σ-set λ in (X,T).
4. f is said to be M-fuzzy G_δ - continuous, if $f^{-1}(\lambda)$ is a fuzzy G_δ - set in (X,T) for any fuzzy G_δ-set λ in (Y,S).
5. f is said to be fuzzy G_δ-homeomorphism, if f is one-to-one, onto, M-fuzzy G_δ-continuous and fuzzy G_δ.
6. f is said to be strongly fuzzy G_δ-continuous, if $f^{-1}(\lambda)$ is both fuzzy G_δ and fuzzy F_σ for any fuzzy set λ in (Y,S).
7. f is said to be perfectly fuzzy G_δ-continuous, if $f^{-1}(\lambda)$ is both fuzzy open and fuzzy closed in (X,T) for any fuzzy G_δ-set λ in (Y,S).
8. f is said to be weakly perfectly fuzzy G_δ- continuous, if $f^{-1}(\lambda)$ is fuzzy open in (X,T) for any fuzzy G_δ-set λ in (Y,S).

Note : The identity function $i : (X,T) \rightarrow (X,T)$ is fuzzy G_δ-continuous

Remark : Since every fuzzy open set is fuzzy G_δ, every fuzzy continuous mapping is fuzzy G_δ-continuous. The converse need not be true See Example 1.

Example 1. Fuzzy G_δ-continuity fuzzy ≠> fuzzy continuity.

Let $X = [0,1]$. Define $T = \{0_X, 1_X, \lambda_n\}$ where $\lambda_n(x) = (n-1)/2n$ ($n = 2,3,4,\ldots\ldots$), $\forall\, x \in X$. Let $Y = \{p\}$. Define $S = \{0_Y, 1_Y, \mu)$ where $\mu : Y \to [0,1]$ is such that $\mu(p) = \inf\{\lambda_n(x)\}_{n=2}^{\infty}$. Define

$$f : (X,T) \to (Y,S) \text{ as } f(x) = p,\ \forall\, x \in X. \text{ Now,}$$

$f^{-1}(\mu)(x) = \mu f(x) = \mu(p) = \inf\ \{\lambda_n(x)\}_{n=2}^{\infty} = \bigwedge_{n=2}^{\infty} \lambda_n(x)$. That is, $f^{-1}(\mu) = \bigwedge_{n=2}^{\infty} \lambda_n$ which is fuzzy G_δ. But $f^{-1}(\mu) = \lim_{n\to\infty} \lambda_n = 1/2$, is not fuzzy open. Therefore, **f is fuzzy G_δ-continuous but not fuzzy continuous.**

Definition 7. For any fuzzy set λ in (X,T) define the σ-closure of λ, denoted by $cl_\sigma\lambda$, to be the intersection of all fuzzy F_σ-sets containing λ. That is $cl_\sigma\lambda = \wedge\{\mu \in I^X/ \mu$ is fuzzy F_σ and $\mu \geq \lambda\}$.

Remarks if λ is a fuzzy F_σ-set, then $cl_\sigma\lambda = \lambda$. Also $\lambda \leq cl_\sigma\lambda \leq cl\lambda$.

Definition 8. A Fuzzy set λ in X is called fuzzy σ-closed if $\lambda = cl_\sigma\lambda$.

Note A fuzzy F_σ-set is fuzzy σ-closed.

Definition 9. For any fuzzy set λ in (X,T) define the σ - interior of λ denoted by $int_\sigma(\lambda)$ to be the union of all fuzzy G_σ-sets contained in λ. That is $int_\sigma(\lambda) = \vee(\mu \in I^X/ \mu$ is fuzzy G_δ and $\mu \leq \lambda)$.

3. PROPERTIES

1. For any two fuzzy sets λ and μ of (X,T) we have

(i) $cl_\sigma 0 = 0$

(ii) $\lambda \leq \mu \Rightarrow cl_\sigma\lambda \leq cl_\sigma\lambda$.

(iii) $cl_\sigma[cl_\sigma(\lambda)] = cl_\sigma\lambda$.

(iv) $cl_\sigma(\lambda \vee \mu) = cl_\sigma\lambda \vee cl_\sigma\mu$.

(v) $int_\sigma(\lambda \wedge \mu) = int_\sigma\lambda \wedge int_\sigma\mu$.

(vi) For a fuzzy set λ of a fuzzy space X, $1\text{-}Int_\sigma(\lambda) = cl_\sigma(1\text{-}\lambda)$ and $1\text{-}cl_\sigma\lambda = int_\sigma(1\text{-}\lambda)$.

2. Let f : (X,T) → (Y,S) be fuzzy G_δ- continuous function. Then the following statements are valid.

(i) For a fuzzy set λ in X, $f[cl_\sigma \lambda] \le clf(\lambda)$.

(ii) For any fuzzy set μ in Y, $cl_\sigma f^{-1}(\mu) \le f^{-1}(cl\mu)$

(iii) If $\lambda \in I^Y$ is a fuzzy closed set, $f^{-1}(\lambda) = cl_\sigma f^{-1}(\lambda)$.

(iv) Let A⊂X be a fuzzy subspace of X. Then the restriction f/A :(A,T/A) → (Y,S) is fuzzy G_δ- continuous.

(v) If f(X) ⊂ Z ⊂ Y, then the mapping g:(X,T) → (Z,R) restricting the range of f is fuzzy G_δ-continuous.

3. Let A ⊂ (X,T) be any non-empty subset of X. The inclusion map j ; (A,T/A) → (X,T) is fuzzy G_δ-continuous.

4. Let (X,T) and (Z,R) be fuzzy topological spaces and Y⊂ Z be a fuzzy subspace of Z. The mapping h:(X,T) → (Z,R) obtained by expanding the range of the fuzzy G_δ-continuous mapping f : (X,T) → (Y,S) is fuzzy G_δ-continuous.

5. An injective fuzzy open map is fuzzy G_δ.

6. Any fuzzy closed map is fuzzy F_σ but not conversely.

Example 2. Let X = [0,1]. Define T = $\{0_X, 1_X, \lambda\}$ where λ(x) = 1/2, ∀ x ∈ X. Let Y = [0,1]. Define S = $\{0_Y, 1_Y, \mu_n\}$ where $\mu_n(x) = (n + 1) / 2n$ (n = 2,3,...),∀ x ∈ X. Define f : (X,T) → (Y,S) as f (x) = x, ∀ x ∈ X. Now. f(1-λ) (x) = 1/2, ∀ x ∈ X. This shows that **f is fuzzy F_σ but not fuzzy closed in (Y,S)**.

7. If g of is fuzzy G_δ and f is bijective fuzzy G_δ- continuous map. Then g is fuzzy G_δ.

8. If (X,T) and (Y,S) are fuzzy topological spaces, and f is a mapping from (X,T) to (Y,S), then the conditions below are related as follows.

(a) and (b) are equivalent; (a) and (c) are equivalent.

(a) f is M-fuzzy G_δ-continuous

(b) For every fuzzy set λ of X, $f(cl_\sigma\lambda) \leq cl_\sigma f(\lambda)$.

(c) For all fuzzy sets λ of Y, $cl(f^{-1}(\lambda)) \leq f^{-1}(cl_\sigma\lambda)$.

9. Let $f:(X,T)\rightarrow(Y,S)$ be a mapping from a fuzzy space X to another fuzzy space Y. Then if the graph $g:X\rightarrow X\times Y$ of f is strongly fuzzy G_δ- continuous, f is also strongly fuzzy G_δ-continuous.

10. Let $f:(X,T)\rightarrow(Y,S)$ be a mapping from a fuzzy space X to another fuzzy space Y. Then if the graph $g : X\rightarrow X\times Y$ of *f* is perfectly fuzzy G_δ- continuous, f is also perfectly fuzzy G_δ-continuous.

The converse need not be true. See example 3.

Example 3.

Let μ_1 and μ_2 be fuzzy set of I defined as

$$\mu_1(x) = 0,\ 0 \leq x \leq 1/2$$
$$= 2x-1,\ 1/2 \leq x \leq 1$$

and
$$\mu_2(x) = 1,\ 0 \leq x \leq 1/4$$
$$= -4x + 2,\ 1/4 \leq x \leq 1/2$$
$$= 0,\ 1/2 \leq x \leq 1$$

Clearly $T = \{0, \mu_1, \mu_2, \mu_1 \vee \mu_2, 1\}$ is a fuzzy topology on I. Let $S = \{0,\mu_1,1\}$. Clearly (I,S) is also a fuzzy topological space and $\mu_2\times \mu_1$ is a fuzzy G_δ-set of the fuzzy product space (I,T,) × (I,S).

Let $f : (I,T) \rightarrow (I,S)$ be defined by $f(x) = 1(x)$ for each $x\in I$. Clearly $f\text{-}1(\mu_1)=1$ is both fuzzy open and fuzzy closed in (I,T). We have $g^{-1}(\mu_2\times\mu_1) = \mu_2 \wedge f^{-1}(\mu_1) = \mu_2$. Now μ_2 is not fuzzy closed in (I,T) and hence g is not perfectly fuzzy G_δ-continuous.

11. Let X, X_1 and X_2 be fuzzy topological spaces and $p_i : X_1 \times X_2 \to X_i$ (i=1,2) be the projection of $X_1 \times X_2$ onto X_i. Then if $f: X \to X_1 \times X_2$ is a fuzzy G_δ-continuous mapping, p_i f is also fuzzy G_δ-continuous.

12. If $f:(X,T) \to (Y,S)$ is fuzzy G_δ- homeomorphism, then

(i) $cl_\sigma[f^{-1}(\lambda)] = f^{-1}[cl_\sigma(\lambda)]$ where λ is a fuzzy set in Y.

(ii) $cl_\sigma[f(\lambda)] = f[cl_\sigma(\lambda)]$ where λ is a fuzzy set in X.

(iii) $f(int_\sigma(\lambda)] = int_\sigma(f(\lambda)]$ where λ is a fuzzy set in X.

(iv) $f^{-1}[int_\sigma(\lambda)] = int_\sigma[f^{-1}(\lambda)]$ where λ is a fuzzy set in Y.

Definition 10. Let (X,T) be a fuzzy topological space. A fuzzy set λ in X is called generalized fuzzy F_σ if $cl_\sigma(\lambda) \leq \mu$ when ever $\lambda \leq \mu$ and μ is fuzzy G_δ. It is denoted by gf F_σ. A fuzzy set λ is called gf$G_\delta \Leftrightarrow 1-\lambda$ is gf F_σ.

Remark. For any fuzzy F_σ-set λ, $cl_\sigma\lambda=\lambda$. So that a fuzzy F_σ set is gf F_σ.

13. If λ_1 and λ_2 are gf F_σ-sets. Then $\lambda_1 \vee \lambda_2$ is a gf F_σ-set.

14. If λ is gf F_σ and if $\lambda \leq \mu \leq cl_\sigma\lambda$, then μ is gf F_σ.

Remark. However, the intersections of two gfF_σ-sets need not be gfF_σ-set as the following example shows.

Example 4. The intersection of two gfF_σ-sets not generally a gfF_σ-set. Let $X = \{x_1,x_2,x_3\}$. Define $\lambda_1,\lambda_2,\lambda_3 : X \to [0,1]$ as follows $\lambda_1 = 0_x$, $\lambda_2 = 1_x$,

$\lambda_3(x) = 0$ if $x = x_2, x_3$

$= 1$ if $x = x_1$.

Clearly $T = \{\lambda_1, \lambda_2, \lambda_3\}$ is a fuzzy topology on X. Define $\mu_1, \mu_2 : X \to [0,1]$ as follows

$\mu_1(x) = 0$ if $x = x_3$,

$= 1$ if $x = x_1, x_2$.

$\mu_2(x) = 0$ if $x = x_2$.

$= 1$ if $x = x_1, x_3$. It is easy to verify that μ_1 and μ_2 are gfF_σ- sets but $\mu_1 \wedge \mu_2$ is not gfF_σ.

15. If λ is a gfF_σ-set in X and if f:(X,T) → (Y,S) is M-fuzzy G_δ-continuous and fuzzy F_σ then f(λ) is gfF_σ in Y.

Example 5. Under fuzzy closed, fuzzy continuous maps gfG_δ-sets are generally not taken into gfG_δ-sets. Let X = {a}, y = {b,c}. Define T = {0_X, 1_X}, S = {0_Y, 1_Y, λ_1} where λ_1 : Y → [0,1] such that $\lambda_1(x) = 1$ if x = b, $\lambda_1(x) = 0$ if x = c. Clearly T and S are fuzzy topologies on X and Y respectively. Define f:(X,T) → (Y,S) as follows f(a) = c,$f^{-1}(\lambda_1(a)) = \lambda_1 f(a)) = \lambda_1(c) = 0$. f is fuzzy continuous and fuzzy closed. Now we shall show that f does not take gfG_δ sets to gfG_δ-sets. Clearly 1x is gfG_δ in X. But f (1x) = 1-λ_1 which is not gfG_δ in Y.

16. In a fuzzy topological space (X,T), $T_{G\delta}$ (the family of all fuzzy G_δ sets) = $K_{F\sigma}$ (the family of all fuzzy F_σ-sets) ⇔ every fuzzy subset of X is a gfF_σ-set.

17. A fuzzy set λ is gfG_δ⇔ $\mu \leq int_\sigma(\lambda)$ wherever μ is fuzzy F_σ and $\mu \leq \lambda$.

18. If $int_\sigma(\lambda) \leq \mu \leq \lambda$ and λ is gfG_δ, then μ is gfG_δ.

Definition 11.

Let f : (X,T) → (Y,S) be a mapping from a fuzzy topological space. X to another space Y.

1. f is said to generalized fuzzy G_δ-continuous, if $f^{-1}(\lambda)$ is a gfF_σ-set in (X,T) for any fuzzy F_σ-set λ in (Y,S).

2. f is said to be fuzzy gc-G_δ- irresolute (f-gc-G_δ-irresolute), if $f^{-1}(\lambda)$ is a gfF_σ-set in (X,T) for any gfF_σ-set λ in (Y,S).

3. f is said to be strongly gfG_δ-continuous, if $f^{-1}(\lambda)$ is both fuzzy G_δ and gf F_σ in (X,T) for any gfG_δ-set λ in (Y,S).

4. f is said to be perfectly fuzzy gfG_δ-continuous, if $f^{-1}(\lambda)$ is both fuzzy G_δ, and fuzzy F_σ in (X,T) for any gfG_δ-set λ in (Y,S).

20. Let (X,T), (Y,S) and (Z,R) be fuzzy topological spaces and f:(X,T) → (Y,S), g: (Y,S)→(Z,R).

Then the following statement are valid.

1. If f and g are fuzzy G_δ then g.f is too.
2. If f is M-fuzzy G_δ-continuous and g is fuzzy G_δ-continuous then g. f is fuzzy G_δ-continuous.
3. If f and g are M-fuzzy G_δ-continuous, then g. f is too.
4. If f is fuzzy gc-G_δ-irresolute and g is gf G_δ-continuous then g.f is gf G_δ-continuous.
5. If f and g are fuzzy gc-G_δ-irresolute, then g.f is too.
6. If f and g are fuzzy G_δ-continuous, then g.f is too.
7. If f is perfectly fuzzy G_δ-continuous and g is strongly fuzzy G_δ-continuous, then g.f is perfectly fuzzy G_δ-continuous.
8. If f is perfectly gfG_δ-continuous and g is strongly gfG_δ-continuous, then g.f is perfectly fuzzy gc G_δ- irresolute.
9. If f is M-fuzzy G_δ-continuous, and g is strongly gfG_δ-continuous, then g.f is strongly gfG_δ-continuous.
10. If f is gfG_δ-continuous and g is strongly gfG_δ-continuous, then g.f is fuzzy gc-G_δ-irresolute.
11. If f is strongly gfG_δ-continuous and g is fuzzy gc-G_δ- irresolute, then g.f is strongly gfG_δ- continuous.
12. If f is fuzzy G_δ-continuous and g is perfectly fuzzy G_δ-continuous, then g.f is M-fuzzy G_δ- continuous.

4. INTERRELATIONS

For the mapping f:(X,T)→(Y,S), the following statements are valid.

1. f, strongly fuzzy G_δ-continuous $\Rightarrow$ f, fuzzy G_δ-continuous,
   f, strongly gfG_δ-continuous,

			f, gfG_δ-continuous,
			f, M-fuzzy G_δ-continuous,
			f, f-gc-G_δ-irresolute,
2.	f, strongly gf G_δ-continuous	$\Rightarrow$	f, fuzzy G_δ-continuous,
			f, f-gc-G_δ-irresolute,
			f, M-fuzzy G_δ-continuous,
			f, gfG_δ-continuous,
3.	f, perfectly fuzzy G_δ-continuous	$\Rightarrow$	f, weakly perfectly fuzzy G_δ-continuous
			f, fuzzy G_δ-continuous,
			f, gfG_δ-continuous,
			f, M-fuzzy G_δ-continuous,
4.	f, perfectly gf G_δ-continuous	$\Rightarrow$	f, strongly gfG_δ-continuous,
			f, gfG_δ-continuous,
			f, M-fuzzy G_δ-continuous,
			f, f-gc-G_δ-irresolute,
			f, fuzzy G_δ-continuous,
5.	f, M-fuzzy G_δ-continuous	$\Rightarrow$	f, gfG_δ-continuous,
			f, fuzzy G_δ-continuous,
6.	f, weakly perfectly fuzzy G_δ-continuous	$\Rightarrow$	f, M-fuzzy G_δ-continuous,
			f, fuzzy G_δ-continuous,
			f, gfG_δ-continuous,
7.	f, f-gc-G_δ-irresolute	$\Rightarrow$	f, gfG_δ-continuous.

Example 6.

Fuzzy G_δ-continuity/M-fuzzy G_δ-continuous ≠>

a. Strongly fuzzy G_δ-continuity

b. f-gc-G_δ-irresolute

c. Strongly gf G_δ-continuity

d. Perfectly gf G_δ-continuity

e. Perfectly fuzzy G_δ-continuity

f. Weakly perfectly fuzzy G_δ-continuity.

Let X = {a,b,c,}. Define T = {0x, 1x, λ} where $\lambda : X \rightarrow [0,1]$ is such that λ(a) = 1, λ(b) = λ(c)=0 and S={0x, 1x}. Define f : (X,T)→(X,S) as f(a) = b, f(b) = a, f(c) = c. Now $f^{-1}(1x)$, $f^{-1}(0x)$ = 0x. Therefore, **f is fuzzy G_δ-continuous/M-fuzzy G_δ-continuous.**

Define a fuzzy set μ : X [0,1] such that μ(a) = μ(c) = 0 μ(b) = 1.

(a) f is not strongly fuzzy fuzzy G_δ-continuous. For the set μ, $f^{-1}(\mu)(a)=1$, $f^{-1}(\mu)(b) = 0$, $f^{-1}(\mu)(c)=0$. Therefore $f^{-1}(\mu) = \lambda$ is fuzzy G_δ but not fuzzy F_σ in (X,T). Therefore, **f is not strongly fuzzy G_δ-continuous.**

(b) f is not f - gc - G_δ - irresolute.

$cl_\sigma \mu = 1$. Therefore μ is gfF_σ in (X,S) $f^{-1}(\mu)(a) = 1$, $f^{-1}(\mu)(b) = 0$, $f^{-1}(\mu)(c) = 0$, $f^{-1}(\mu) = \lambda$. $cl_\sigma f^{-1}(\mu) = cl_\sigma\lambda = 1 \nleq \lambda$. But $f^{-1}(\mu)$ is not gfF_σ. Hence **f is not f - gc - G_δ-irresolute.**

(c) f is not strongly gf G_δ-continuous.

$cl_\sigma \mu = 1$. Therefore, μ is gfF_σ in (X,S). But $f^{-1}(\mu) = \lambda$ which is fuzzy G_δ but not gfF_σ in (X,T). Therefore **f is not strongly gfG_δ-continuous.**

(d) f is not perfectly gfG_δ- continuous. For μ, $f^{-1}(\mu) = \lambda$ is fuzzy G_δ in (X,T) but not fuzzy F_σ in (X,T). Therefore, **f is not perfectly gfG_δ-continuous.**

(e) f is not perfectly fuzzy G_δ-continuous.

Let X = {a,b,c}. Define T = {0x, 1x, λ} where λ : X → [0,1] is such that λ(a) = 1, λ(b) = λ(c) = 0. S={0x,1x,λ} where μ : X→[0,1] is such that μ(a) = 0, μ(b) = 1, μ(c) = 0. Define f : (X,T)→(X,S) as f(a) = b, f(b) =a, f(c) = c.f^{-1}(1x) = 1x, f^{-1}(0x) = 0x, f^{-1}(μ)(a) = 1, f^{-1}(μ)(b) = 0 f^{-1}(μ)(c) = 0, f^{-1}(μ) = λ which is fuzzy G_δ in (X,T). Therefore, **f is fuzzy G_δ-continuous/M-fuzzy G_δ-continuous.** But for G_δ set μ in (X,S), f^{-1}(μ) is not fuzzy closed. Therefore **f is not perfectly fuzzy G_δ-continuous.**

(f) f is not weakly perfectly fuzzy G_δ-continuous.

Let X = [0,1]. Define T = {0x, 1x, λ_n} where λ_n = (n-1)/2n for n = 2,3,... S = {0x, 1x, λ_n}. Define f : (X,T) → (X,S) as f(x) = x ∀x∈X. $f^{-1}(\lambda_n) = \lambda_n$ $\forall_n$. f^{-1}(0x) = 0x, f^{-1}(1x) = 1x. Therefore **f is fuzzy G_δ-continuous/M-fuzzy G_δ-continuous.** But for fuzzy G_δ set $\overset{\infty}{\underset{n=2}{\wedge}} \lambda_n = 1/2$ in (X,S), $f^{-1}\left(\overset{\infty}{\underset{n=2}{\wedge}} \lambda_n\right) = 1/2$ is not fuzzy open in (X,T). Therefore **f is not weakly perfectly fuzzy G_δ-continuous.**

Example 7.

gfG_δ- continuity ≠>

(a) fuzzy G_δ-continuity

(b) M-fuzzy G_δ-continuity

Let X = {a, b, c}. Define T = {0x, 1x, λ} where λ : X→[0,1] is such that λ(a)=0, λ(b)= λ(c)=0. S={0x, 1x, μ} where μ : X→[0,1] is such that μ(a)=0, μ(c)=1, μ(b)=0. f : (X,T)→(X,S) as f(a) = a, f(b) = c, and f(c) = a, f^{-1}(1-μ)(a) = 1, f^{-1}(1-μ)(b) = 0, f^{-1}(1-μ)(c)=1 and $cl_\sigma f^{-1}$(1-μ)=1. f^{-1}(1-μ) is gfF_δ in (X ,T). Therefore, **f is gf G_δ-continuous.**

a. f is not fuzzy G_δ-continuous

For the fuzzy open set μ, f^{-1}(μ) = (0,1,0) is not fuzzy G_δ. Therefore **f is not fuzzy G_δ-continuous.**

b. f is not M-fuzzy G_δ-continuous.

For the fuzzy G_δ set μ in (X,S), $f^{-1}(\mu)$ is not fuzzy G_δ. Therefore **f is not M-fuzzy G_δ-continuous.**

Example 8.

Perfectly fuzzy G_δ-continuous ≠>

(a) f-gc-G_δ-irresolute

(b) Strongly fuzzy G_δ-continuity

(c) Strongly gfG_δ-continuity

(d) Perfectly gfG_δ-continuity

Let X = {a,b,c}. Define T = {0x, 1x, λ_1,λ_2} where λ_1: X→(0,1) is such that $\lambda_1(a)=1$, $\lambda_1(b)=1$ and $\lambda_1(c)=0$; λ_2 : X →[0,1] is such that $\lambda_2(a) = 1$, $\lambda_2(b)=0$ and $\lambda_2(c)=0$. Define S = {0x, 1x, λ_1} λ_1 : X→[0,1] is such that $\lambda_1(a) = 1$, $\lambda_1(b)=1$ and $\lambda_1(c) = 0$. Also define f : (X,T) →(X,T) as f(a)=a, f(b) = a and f(c) = b. $f^{-1}(\lambda_1)(a)=1$, $f^{-1}(\lambda_1)(b)=1$, $f^{-1}(\lambda_1)(c) = 1$, and $f^{-1}(\lambda_1) = 1x$ which is both fuzzy open and fuzzy closed in (X,T). Therefore **f is perfectly fuzzy G_δ-continuous.** Define μ : X →[0,1] as $\mu(a)=1$, $\mu(b)=0$, and $\mu(c)=1$.

(a) f is not f-gc-G_δ-irresolute.

$cl_\sigma(\mu) = 1$. Since μ is not less than any fuzzy G_δ set other than 1, μ is gfF_σ in (X,S). $f^{-1}(\mu)(a)=1$, $f^{-1}(\mu)(b)=1$, $f^{-1}(\mu)(c)= 0$, and $cl_\sigma f^{-1}(\mu) \neq 1 \nless \lambda_1$. This shows that $f^{-1}(\mu)$ is not gfF_σ. Therefore **f is not f-gc-G_δ-irresolute.**

(b) f is not strongly fuzzy G_δ-continuous.

In the above example, $f^{-1}(\mu) = \lambda_1$ which is fuzzy G_δ in (X,T) but not fuzzy F_σ in (X,T). Therefore **f is not strongly fuzzy G_δ-continuous.**

(c) f is not strongly gfG_δ- continuous.

1- μ is gfG_δ in (X,S) but $f^{-1}(1-\mu)$ is not fuzzy G_δ in (X,T). Therefore **f is not strongly gfG_δ-continuous.**

(d) f is not perfectly gfG_δ - continuous

1- μ is gfG_δ in (X,S) but $f^{-1}(1-\mu)$ is not fuzzy F_σ and not fuzzy G_δ in (X,T). Therefore **f is not strongly gfG_δ-continuous.**

Example 9.

Weakly perfectly fuzzy G_δ-continuity $\not\Rightarrow$

(a) f-gc-G_δ-irresolute

(b) Strongly fuzzy G_δ-continuity

(c) Perfectly gf G_δ-continuity

(d) Strongly gf G_δ-continuity

Let X = {a,b,c}. Define T={0x, 1x, λ} where $\lambda : X \to [0,1]$ is such that $\lambda(a) = 1$, $\lambda(b) = \lambda(c) = 0$. Define S = {0x, 1x}. Also define f : (X,T) $\to$ (X,S) as f(a) = b, f(b) = a, f(c) = c. $f^{-1}(0x) = 0x$ and $f^{-1}(1x) = 1x$. Therefore **f is weakly perfectly fuzzy G_δ-continuous.**

a. f is not f-gc-G_δ-irresolute

Define $\mu : X \to [0,1]$ such that $\mu(a) = 0$, $\mu(b) = 1$ and $\mu(c) = 0$. $cl_\sigma \mu = 1$. Hence μ is gf F_σ in (X,S). But $f^{-1}(\mu) = \lambda$, $cl_\sigma f^{-1}(\mu) = 1 > \lambda_1$. This shows that $f^{-1}(\mu)$ is not gfF_σ in (X,T). Therefore **f is not f-gc-G_δ-irresolute.**

b. f is not strongly fuzzy G_δ-continuous.

For the set μ in (X, S), $f^{-1}(\mu)$ is fuzzy G_δ but not fuzzy F_σ in (X, T). Therefore **f is not strongly fuzzy G_δ- continuous.**

c. f is not perfectly gfG_δ-continuous.

$f^{-1}(\mu) = \lambda$ is G_δ but not fuzzy F_σ in (X,T). Therefore **f is not perfectly gfG_δ-continuous.**

d. f is not strongly gf G_δ-continuous.

$f^{-1}(\mu) = \lambda$ is not gfF_σ in (X,T). Therefore, **f is not strongly gfG_δ-continuous.**

e. Weakly perfectly fuzzy G_δ-continuity ≠> perfectly fuzzy G_δ-continuity.

Let $X = \{a,b,c\}$. Define $T = \{0x, 1x, \lambda\}$ where $\lambda : X \to [0,1]$, is such that $\lambda(a)=1$, $\lambda(b)=\lambda(c)=0$. Define $S = \{0x, 1x, \mu\}$ where $\mu : X \to [0,1]$, is such that $\mu(a) = 0$, $\mu(b) = 1$ and $\mu(c) = 0$, Define $f: (X,T) \to (X,S)$ as $f(a) = b$, $f(b) = a$ and $f(c) = c$, $f^{-1}(1x) = 1x$, $f^{-1}(0x) = 0x$ and $f^{-1}(\mu) = \lambda$. Therefore, **f is weakly perfectly fuzzy G_δ-continuous.** For the fuzzy G_δ set μ in (X,S), $f^{-1}(\mu)$ is fuzzy open but it is not fuzzy closed in (X,T). Therefore **f is not perfectly fuzzy G_δ-cintinuous.**

Remarks

From the results proved so far, we have the following table of implication. In the following table a,b,c,d,e,f,g,h and i denote fuzzy G_δ-continuity, gfG_δ-continuity, fuzzy gc-G_δ-irresolute, strongly fuzzy G_δ-continuity, perfectly fuzzy G_δ-continuity, strongly gfG_δ-continuity, perfectly gfG_δ-continuity, M-fuzzy G_δ-continuity and weakly perfectly fuzzy G_δ-continuity respectively. Also 1 denotes 'implies', 0 denotes 'does not imply' and - denotes 'not known'.

⇒	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**
a	1	-	0	0	0	0	0	-	0
b	0	1	-	-	-	-	-	0	-
c	-	1	1	-	-	-	-	-	-
d	1	1	1	1	-	1	-	1	-
e	1	1	0	0	1	0	0	1	1
f	1	1	1	-	-	1	-	1	-
g	1	1	1	-	-	1	1	1	-
h	1	1	0	0	0	0	0	1	0
i	1	1	0	0	-	0	0	1	1

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