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δ-SEMICONTINUOUS FUNCTIONS

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ABSTRACT

In this paper, the notion of δ-semicontinuous functions is introduced. Basic properties of δ-semicontinuous functions are investigated. The class of δ-semicontinuous functions is a weaker form of the classes of perfectly continuous functions, clopen maps, Z-supercontinuous functions, D_δ-supercontinuous functions, supercontinuous functions.

Key Words: δ - semiopen sets, δ - semicontinuity, supercontinuity, Z-supercontinuity, D_δ - supercontinuity perfect continuity, complete continuity.

MSC : 54C10.

1. INTRODUCTION

General topologists have introduced and investigated many different types of continuous functions. Some of them are strong continuity [6], perfect continuity

[11], clopen maps [13], complete continuity [1], Z-supercontinuity [4], D_δ-supercontinuity [5], strong θ-continuity [8] and supercontinuity [10].

In 1968, Veličko [15] introduced δ-open sets and in 1997, Park et.al., [12] introduced a new notion called δ-semiopen sets which are weaker than δ-open sets.

The aim of this paper is to introduce a new class of continuity called δ-semicontinuity which include strong-continuity, perfect - continuity, clopen maps, complete continuity, Z-supercontinuity, D_δ-supercontinuity, strong -θ-continuity and supercontinuity. Moreover, basic properties of this new class of functions are investigated.

Firstly, in section 3, basic properties of δ-semicontinuous functions are investigated, then the relationships among δ-semicontinuity, connectedness, compactness are discussed in section 4 and finally, graphs and separation axioms are investigated in section 5.

2. PRELIMINARIES :

In this paper, spaces X and Y mean topological spaces on which no sepation axioms are assumed unless explicitly stated. Let A be a subset of a space X. For a subset A of X, $Cl(A)$ and $Int(A)$ represent the closure of A and the interior of A, respectively.

A subset A of a space X is said to be regular open (respectively regular closed) if $A = Int(Cl(A))$ (respectively $A = Cl(Int(A))$) [14].

The δ-interior [15] of a subset A of X is the union of all regular open sets of X contained in A is denoted by δ-$Int(A)$. A subset A is called δ-open [15] if $A=\delta\text{-}Int(A)$, i.e., a set is δ-open if it is the union of regular open sets.

The complement of δ-open set is called δ-closed. Alternatively, a set A of (X,τ) is called δ-closed [15] if $A = \delta\text{-}Cl(A)$, where $\delta\text{-}Cl(A) = \{x \in X: A \cap Int(Cl(U)) \neq \phi, U \in \tau \text{ and } x \in U\}$.

A subset S of a topological space X is said to be δ-semiopen [12] iff $S \subset Cl(\delta\text{-}Int(S))$. The complement of a δ-semiopen set is called a δ-semiclosed set [12].

The union (resp. intesection) of all δ-semiopen (resp. δ-semiclosed) sets, each contained in (resp. containing) a set S in a topological space X is called the δ-semiinterior (resp. δ-semiclosure) of S and it is denoted by $\delta\text{-}sInt(S)$ (resp. $\delta\text{-}sCl(S)$) [12].

For any subset S of a topological space X, $X \setminus \delta\text{-}sCl(K) = \delta\text{-}sInt(X \setminus K)$ [12].

The family of all δ-semiopen sets of X containing a point $x \in X$ is denoted by $\delta SO(X,x)$. The family of all δ-semiopen (resp., δ-semiclosed, δ-open) sets of X is denoted by $\delta SO(X)$ (resp., $\delta SC(X)$, $\delta O(X)$).

Definition 1 : *A subset H of a space X is said to be a regular G_δ-set if H is an intersection of a sequence of closed sets whose interiors contain H, i.e., if $H=\bigcap_{i=1}^{\infty} F_i=\bigcap_{i=1}^{\infty} Int(F_i)$, where each F_i is a closed subset of X. The complement of a regular G_δ-set is called a regular F_σ-set [9].*

δ-open sets in a topological space (X,τ) form a topology τ_δ weaker than τ such that the regular open sets of (X,τ) form a base for τ_δ.

Proposition 2 : *Let (X,τ_X) and (Y,τ_Y) be two topological spaces. Then $(\tau_X)_\delta \times (\tau_Y)_\delta = (\tau_X \times \tau_Y)_\delta$ [2].*

Proposition 3 : *If $U \in \delta SO(X)$ and $V \in \delta SO(Y)$, then $U \times V \in \delta SO(X \times Y)$.*

Definition 4 : *A function $f : X \to Y$ is called strongly continuous [6] if $f(Cl(A)) \subset f(A)$ for all $A \subset X$.*

Definition 5 : *A function $f : X \to Y$ is said to be perfectly continuous [11] if $f^{-1}(V)$ is clopen in X for every open set V of Y.*

Definition 6 : *A function $f : X \to Y$ is said to be clopen map [13] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a clopen set U of X containing x such that $f(U) \subset V$.*

Definition 7 : *A function $f: X \to Y$ is said to be Z-supercontinuous [4] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a cozero set U of X containing x such that $f(U) \subset V$.*

Definition 8 : *A function $f: X \to Y$ is said to be Dδ-supercontiuous [5] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists a regular F_σ-set U of X containing x such that $f(U) \subset V$.*

Definition 9 : *A function $f: X \to Y$ is said to be strongly θ-continuous [8] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U of X containing x such that $f(Cl(U)) \subset V$.*

Definition 10 : *A function $f: X \to Y$ is said to be supercontinuous [10] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U of X containing x such that $f(Int(Cl(U))) \subset V$.*

Definition 11 : *A function $f: X \to Y$ is said to be completely continuous [1] if $f^{1}(V)$ is regular open in X for every open set V of Y.*

3. δ-SEMICONTINUOUS FUNCTIONS :

Definition 12 :*A function $f: X \to Y$ is said to be δ-semicontinuous at a point $x \in X$ if for each open set V in Y containing $f(x)$, there exists a δ-semiopen set U in X containing x such that $f(U) \subset V$ and f is said to be δ-semicontinuous if it has this property at each point of X.*

Theorem 13 : *Let (X,τ) and (Y,υ) be topological spaces. The following statements are equivalent for a function $f: X \to Y$:*

(1) f is δ - semicontinuous;

(2) for every open set $V \subset Y$, $f^{1}(V)$ is δ-semiopen;

(3) for every clsoed set $V \subset Y$, $f^{1}(V)$ is δ-semiclosed.

Proof. (1)⇒(2) : Let V be a open subset of Y and let $x \in f^{1}(V)$. Since $f(x) \in V$, by (1), there exists a δ-semiopen set U_x in X containing x such that $U_x \subset f^{1}$(V). We obtain that $f^{-1}(V) = \bigcup_{x \in f^{-1}(V)} U_x$. Thus, $f^{1}(V)$ is δ-semiopen.

(2)$\Rightarrow$(1) : Let V be a open subset in Y containing $f(x)$. By (2), $f^{-1}(V)$ is δ-semiopen. Take $U=f^{-1}(V)$. Then $f(U)\subset V$. Hence, f is δ-semicontinuous.

(2)$\Rightarrow$(3) : Let V be a closed subset of Y. Then, $Y\backslash V$ is open. By (2), $f^{-1}(Y\backslash V) = X\backslash f^{-1}(V)$ is δ-semiopen. Thus, $f^{-1}(V)$ is δ-semiclosed.

(3)$\Rightarrow$(2) : It can be shown easily.

Remark 14 : *The following diagram holds :*

$$\begin{tikzcd}
\text{Strongly continuous} \arrow[d, Rightarrow] & & \\
\text{perfectly continuous} \arrow[r, Rightarrow] \arrow[d, Rightarrow] & \text{completely continuous} \arrow[d, Rightarrow] & \\
\text{clopen map} \arrow[d, Rightarrow] & \text{supercontinuous} \arrow[r, Rightarrow] & \delta\text{-semicontinuous} \\
\text{Z-supercontinuous} \arrow[d, Rightarrow] & & \\
D_{\delta}\text{- supercontinuous} \arrow[r, Rightarrow] & \text{strongly } \theta\text{-continuous} \arrow[uu, Rightarrow] &
\end{tikzcd}$$

Remarks 15 *None of the implications is reversible. The following example shows the last implication. The other examples can be seen in [3,4,5].*

Example 16 *Let* $X = \{a,b,c,d\}$ *and* $\tau = \{X, \emptyset, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}, \{a,c,d\}\}$. *If we take the identity function* $f : X\to X$, *then* f *is* δ*-semicontinuous but it is not supercontinuous.*

Theorem 17 *Let* $f : X\to Y$ *be a function and let* $g:X\to X\times Y$ *be the graph function of* f, *defined by* $g(x) = (x,f(x))$ *for every* $x\in X$. *If* g *is* δ*-semicontinuous, then* f *is* δ*-semicontinuous.*

Proof. Let V be an open subset in Y. Then $X\times V$ is an open subset in $X\times Y$. Since g is δ-semicontinuous, then $f^{-1}(V) = g^{-1}(X\times Y) \in \delta SO(X)$. Thus, f is δ-semicontinuous.

Definition 18 *A function* $f : X\to Y$ *is called* δ*-semiirresolute if for every* δ*-semiopen subset* G *of* Y, $f^{-1}(G)$ *is* δ*-semiopen in* Y.

Theorem 19 *Let $f: X \to Y$ and $g: Y \to Z$ be functions.*

(1) If f is δ-semiirresolute and g is δ-semicontinuous, then $g \circ f: X \to Z$ is δ-semicontinuous.

(2) If f is δ-semicontinuous and g is supercontinuous, then $g \circ f: X \to Z$ is δ-semicontinuous.

(3) If f is δ-semicontinuous and g is clopen map, then $g \circ f: X \to Z$ is δ-semicontinuous.

(4) If f is δ-semicontinuous and g is completely continuous, then $g \circ f: X \to Z$ is δ-semicontinuous.

(5) If f is δ-semicontinuous and g is perfectly continuous, then $g \circ f: X \to Z$ is δ-semicontinuous.

Proof. (1) Let V by any open set in Z. Since g is δ-semicontinuous, $g^{-1}(V)$ is δ-semiopen. Since f is δ-semiirresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is δ-semiopen. Therefore, $g \circ f$ is δ-semicontinuous.

The other proofs can be obtained similarly.

Definition 20 *A function $f: X \to Y$ is called δ-semiopen if for every δ-semiopen subset A of X, $f(A)$ is δ-semiopen in Y.*

Theorem 21 *Let $f: X \to Y$ and $g: Y \to Z$ be functions. If f is δ-semiopen and surjective and $g \circ f: X \to Z$ and is δ-semicontinuous, then g is δ-semicontinuous.*

Proof. Let V be any open set in Z. Since $g \circ f$ is δ-semicontinuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is δ-semiopen. Since f is δ-semiopen, then $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is δ-semiopen. Hence, g is δ-semicontinuous.

Combining the previous two theorem, we obtain the following result.

Theorem 22 *Let $f: X \to Y$ be surjective, δ-semiirresolute and δ-semiopen and $g: Y \to Z$* be a function. Then $g \circ f: X \to Z$ is δ-semicontinuous if and only if g is δ-semicontinuous.

Definition 23 *A filter base Λ is said to be δ-semiconvergent to a point x in X if for any $U \in \delta SO(X)$ containing x, there exists a $B \in \Lambda$ such that $B \subset U$.*

Definition 24 *A filter base Λ is said to be convergent to a point x in X if for any open subset U in X containing x, there exists a $B\in\Lambda$ such that $B\subset U$.*

Theorem 25 *If a function $f : X\to Y$ is δ-semicontinuous, then for each point $x\in X$ and each filter base Λ in X which is δ-semiconvergent to x, the filter base $f(\Lambda)$ is convergent to $f(x)$.*

Proof. Let $x\in X$ and Λ be any filter base in X which is δ-semiconvergent to x. Since f is δ-semicontinuous, then for any open subset V in Y containing $f(x)$, there exists a $U \in \delta SO(X)$ containing x such that $f(U)\subset V$. Since Λ is δ-semiconvergent to x, there exists a $B\in\Lambda$ such that $B\subset U$. This means that $f(B)\subset V$ and therefore the filter base $f(\Lambda)$ is convergent to $f(x)$.

Lemma 26 *Let S be an open subset of a space (X,τ)*

(1) If U is regular open set in X, then so is $U\cap S$ in the subspace $(S,\tau s)$.

(2) If B $(\subset S)$ is regular open in $(S,\tau s)$, then there is a regular open set U in (X,τ) such that $B = U\cap S$ [7].

Remark 27 *The intersection of even two δ-semiopen sets may not be δ-semiopen set [12].*

Theorem 28 *A set S in X is δ-semiopen if and only if $S\cap G\in\delta SO(X)$ for every δ-open set G of X.*

Lemma 29 *Let A and X_0 be subsets of a space (X,τ). If $A\in\delta SO(X)$ and $X_0\in\delta O(X)$, then $A\cap X_0\in\delta SO(X_0)$.*

Theorem 30 *If $f : X\to Y$ is δ-semicontinuous and $A\in\delta O(X)$, then the restriction $f|_A : A\to Y$ is δ-semicontinuous.*

Proof. Let V be a open subset of Y. We have $(f|_A)^{-1}(V) = f^{-1}(V)\cap A$. Since $f^{-1}(V)$ is δ-semiopen and A is δ-open, it follows from the previous lemma that $(f|_A)^{-1}(V)$ is δ-semiopen in the relative topology of A. Thus, $f|_A$ is δ-semicontinuous.

Lemma 31 *Let $A \subset X_0\subset X$. If $X_0\in\delta O(X)$ and $A\in\delta SO(X_0)$, then $A\in\delta SO(X)$.*

Theorem 32 *Let $f : X\to Y$ be a function and $\{U_\alpha : \alpha \in I\}$ be a cover of X such that $U_\alpha\in\delta O(X)$ for each $\alpha \in I$. If $f|_{U_\alpha}$ is δ-semicontinuous for each $\alpha \in I$, then f is a δ-semicontinuous function.*

Proof. Suppose that V is any open set of Y. Since $f|_{U_\alpha}$ is δ-semicontinuous for each $\alpha \in I$, it follows that $(f|_{U_\alpha})^{-1}(V) \in \delta SO(U_\alpha)$. We have $f^{-1}(V) = \bigcup_{\alpha\in I}(f^{-1}(V)\cap U_\alpha) = \bigcup_{\alpha\in I}(f|_{U_\alpha})^{-1}(V)$. Then, as a direct consequence of the previous lemma we obtain that $f^{-1}(V) \in \delta SO(X)$ which means that f is δ-semicontinuous.

Theorem 33 *Let $f: X\to Y$ be a function and $x\in X$. If there exists $U\in\delta O(X)$ such that $x\in U$ and the restriction of f to U is a δ-semicontinuous function at x, then f is δ-semicontinuous at x.*

Proof. Suppose that G is an open subset in Y containing $f(x)$. Since $f|_U$ is δ-semicontinuous at x, there exists $V\in\delta SO(U)$ containing x such that $f(V) = (f|_U)(V) \subset G$. Since $U\in\delta O(X)$ containing x, it follows from Lemma 31 that $V\in\delta SO(X)$ containing x. This shows clearly that f is δ-semicontinuous at x.

4. δ-SEMICONNECTED SPACES AND δ-SEMICOMPACT SPACES :

In this section, the relationships between δ-semicontinuous functions and connectedness and between δ-semicontinuous functions and compactness are investigated.

Definition 34 *A space X is called δ-semiconnected provided that X is not the union of two disjoint nonempty δ-semiopen sets.*

Theorem 35 *If $f: X\to Y$ is δ-semicontinuous surjective function and X is δ-semiconnected space, then Y is connected space.*

Proof. Suppose that Y is not connected space. Then there exists nonempty disjoint open sets U and V such that $Y=U\cup V$. Therefore, U and V are open sets in Y. Since f is δ-semicontinuous, then $f^{-1}(U)$ and $f^{-1}(V)$ are δ-semiclosed and δ-semiopen in X.

Moreover, $f^{-1}(U)$ and $f^{-1}(V)$ are nonempty disjoint and X = $f^{-1}(U) \cup f^{-1}(V)$. This shows that X is not δ-semiconnected. This is a contradiction. By contradiction, Y is connected.

Definition 36 *A space X is said to be δ-semi-compact if every δ-semiopen cover of X has a finite subcover.*

A subset A of a space X is said to be δ-semi-compact relative to X if every cover of A by δ-semiopen sets of X has a finite subcover.

A subset A of a space X is said to be δ-semi-compact if the subspace A is δ-semi-compact.

Theorem 37 *If a function* $f : X \to Y$ *is δ-semicontinuous and K is δ-semi-compact relative to X, then f(K) is compact in Y.*

Proof. Let $\{H_\alpha : \alpha \in I\}$ be any cover of $f(K)$ by open sets of the subspace $f(K)$. For each $\alpha \in I$, there exists a open set K_α of Y such that $H_\alpha = K_\alpha \cap f(K)$. For each $x \in K$, there exists $\alpha_x \in I$ such that $f(x) \in K_{\alpha_x}$ and there exists $U_x \in \delta SO(X)$ containing x such that $f(U_x) \subset K_{\alpha_x}$. Since the family $\{U_x : x \in K\}$ is a cover of K by δ-semiopen sets of K, there exists a finite subset K_0 of K such that $K \subset \{U_x : x \in K_0\}$. Therefore, we obtain $f(K) \subset \bigcup \{f(U_x) : x \in K_0\}$ which is a subset of $\bigcup \{K_{\alpha_x} : x \in K_0\}$. Thus $f(K) = \bigcup \{H_{\alpha_x} : x \in K_0\}$ and hence $f(K)$ is compact.

Corollary 38 *If* $f : X \to Y$ *is δ-semicontinuous surjection and X is δ-semicompact, then Y is compact.*

Definition 39 *A space X said to be*

(1) countably δ-semi-compact if every δ-semiopen countably cover of X has a finite subcover.

(2) δ-semi-Lindelof if every δ-semiopen cover of X has a countable subcover.

Theorem 40 *Let* $f : X \to Y$ *be a δ-semicontinuous sujection. Then the following statements hold :*

(1) if X is δ-semi-Lindelof, then Y is Lindelof.

(2) if X is countably δ-semi-conpact, then Y is countably compact.

Proof. We prove (1), the proof of (2) being entirely analogous.

Let $\{V_\alpha : a \in I\}$ be any open cover of Y. Since f is δ-semicontinuous, then $\{f^{-1}(V_\alpha) : a \in I\}$ is a δ-semiopen cover of X. Since X is δ-semi-Lindelof, there exists a countable subset I_0 of I such that $X = \bigcup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Thus, we have $Y = \bigcup \{V_\alpha : \alpha \in I_0\}$ and Y is Lindelof.

5. SEPARAZTION AXIOMS AND δ_s-GRAPHS

In this section, we investigate the relationships between δ-semicontinuous functions and graphs and between δ-semicontinuous functions and separation axioms.

Definition 41 *A space X is said to be δ-semi-T_1 if for each pair of distinct points x and y of X, there exist δ-semiopen sets U and V containing x and y respectively such that $y \notin U$ and $x \notin V$.*

Remark 42 *The following example shows that the concepts of δ-semi-T_1 and T_1 are independent.*

Example 43 *Let $X = \{x,y,z\}$ with the topology $\tau = \{X, \emptyset, \{x\}, \{y\}, \{x,y\}\}$. Then (X,τ) is δ-semi-T_1 but not T_1.*

Example 44 *Let $\mathbb{R}$ be the real numbers with the finite complements topology τ. Then $(\mathbb{R},\tau)$ is T_1 but not δ-semi-T_1.*

Theorem 45 *If $f : X \to Y$ is δ-semicontinuous injection and Y is T_1, then X is δ-semi-T_1.*

Proof. Suppose that Y is T_1. For any distinct points x and y in X, there exist open subsets V, W in Y such that $f(x) \in V$, $f(y) \notin V$, $f(x) \notin W$ and $f(y) \in W$. Since f is δ-semicontinuous, $f^1(V)$ and $f^1(W)$ are δ-semiopen subset of X such that $x \in f^1(V)$, $y \notin f^1(V)$, $x \notin f^1(W)$ and $y \in f^1(W)$. This shows that X is δ-semi-T_1.

Definition 46 *A space X is said to be δ-semi-T_2 if for each pair of distinct points x and y in X, there exist disjoint δ-semiopen sets U and V in X such that $x \in U$ and $y \in V$.*

Theorem 47 *If $f : X \to Y$ is a δ-semicontinuous injection and Y is T_2, then X is δ-semi-T_2.*

Proof. For any pair of distinct points and y in X, there exist disjoint open sets U and V in Y such that $f(x) \in U$ and $f(y) \in V$. Since f is δ-semicontinuous, $f^1(U)$ and $f^1(V)$ is δ-semiopen in X containing x and y respectively. We have $f^1(U) \cap f^1(V) = \emptyset$. This shows that X is δ-semi-T_2.

Theorem 48 *If $f : X \to Y$ is supercontinuous function and $g : X \to Y$ is δ-semicontinuous function and Y is Hausdorff, then $E = \{x \in X : f(x) = g(x)\}$ is δ-semiclosed in X.*

Proof. If $x \in X \setminus E$, then it follows that $f(x) \neq g(x)$. Since Y is Hausdorff, there exist open sets V and W in Y containing $f(x)$ and $g(x)$, respectively, such that $V \cap W = \emptyset$. Since f is supercontinuous, and g is δ-semicontinuous, then $f^{-1}(V)$ is δ-open and

$g^{-1}(W)$ is δ-semiopen in X with $x \in f^{-1}(V)$ and $x \in g^{-1}(W)$. Set $0 = f^{-1}(V) \cap g^{-1}(W)$. Then, by Theorem 28, O is δ-semiopen. Therefore, $f(O) \cap g(O) = \emptyset$ and it follows that $x \notin \delta\text{-}scl(E)$. This shows that E is δ-semiclosed in X.

Theorem 49 : *If $f : X \to Y$ is δ-semicontinuous function and Y is Hausdorff, then $E = \{(x,y) \in X \times X : f(x) = f(y)\}$ is δ-semiclosed in $X \times X$.*

Proof. Let $(x, y) \in (X \times X) \setminus E$. It follows that $f(x) \neq f(y)$. Since Y is Hausdorff, there exist open sets V and W containing $f(x)$ and $f(y)$, respectively, such that $V \cap W = \emptyset$. Since f is δ-semicontinuous, there exist δ-semiopen sets U and G in X containing x and y, respectively, such that $f(U) \subset V$ and $f(G) \subset W$. Hence, $(U \times G) \cap E = \emptyset$. We have $U \times G$ is δ-semiopen in $X \times X$ containing (x,y). This means that E is δ-semiclosed in $X \times X$.

Definition 50 *A space is called δ-semi-regular if for each δ-semiclosed set F and each point $x \notin F$, there exist disjoint open sets U and V such that $F \subset U$ and $x \in V$.*

Definition 51 *A space is said to be δ-semi-normal if for every pair of disjoint δ-semiclosed subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.*

Theorem 52 *If f is δ-semicontinuous injective open function from a δ-semi-regular space X onto a space Y, then Y is regular.*

Proof. Let F be closed set in Y and be $y \notin F$. Take $y = f(x)$. Since f is δ-semicontinuous, $f^{-1}(F)$ is a δ-semiclosed set. Take $G = f^{-1}(F)$. We have $x \notin G$. Since X is δ-semi-regular, there exist disjoint open sets U and V such that $G \subset U$ and $x \in V$. We obtain that $F = f(G) \subset f(U)$ and $y = f(x) \in f(V)$ such that $f(U)$ and $f(V)$ are disjoint open sets. This shows that Y is regular.

Theorem 53 *If f is δ-semicontinuous injective open function from a δ-semi-normal space X onto a space Y, then Y is normal.*

Proof. Let F_1 and F_2 be disjoint closed subsets of Y. Since f is δ-semicontinuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are δ-semiclosed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \emptyset$. Since X is δ-semi-normal, there exist disjoint open sets A and B such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint open sets. Thus, Y is normal.

Recall that for a function $f: X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subset X \times Y$ is called the graph of f and is denoted by $G(f)$.

Definition 54 *A graph $G(f)$ of a function $f: X \to Y$ is said to be δ_s-graph if for each $(x,y) \in (X \times Y) \setminus G(f)$, there exist a $U \in \delta SO(X)$ containing x and an open subset V in Y containing y such that $(U \times Int(Cl(V))) \cap G(f) = \emptyset$.*

Lemma 55 *A graph $G(f)$ of a function $f: X \to Y$ is δ_s-graph if and only if for each $(x,y) \in (X \times Y) \setminus G(f)$, there exist a $U \in \delta SO(X)$ containing x and an open subset V in Y containing y such that $f(U) \cap Int(Cl(V)) = \emptyset$.*

Theorem 56 *If $f: X \to Y$ is δ-semicontinuous and Y is T_2, then $G(f)$ is δ_s-graph.*

Proof. Let $(x,y) \in (X \times Y) \setminus G(f)$, then $f(x) \neq y$ and there exists disjoint open subsets U and V of Y such that $f(x) \in U$ and $y \in V$. Since f is δ-semicontinuous, then there exists a δ-semiopen set G containing x such that $f(G) \subset U$. Therefore, we obtain that $f(G) \cap Int(Cl(V)) = \emptyset$ and $G(f)$ is a δ_s-graph.

Theorem 57 *Let $f: X \to Y$ has a δ_s-graph $G(f)$. If f is injective, then X is δ-semi-T_1.*

Proof. Let x and y be any two distinct points of X. Then, we have $(x, f(y)) \in (X \times Y) \setminus G(f)$. By definition of δ_s-graph, there exist a δ-semiopen set U of X and an open set V of Y such that $(x, f(y)) \in U \times V$ and $f(U) \cap Int(Cl(V)) = \emptyset$; hence $U \cap f^{-1}(Int(Cl(V))) = \emptyset$. Therefore, we have $y \notin U$. This implies that X is δ-semi-T_1.

Theorem 58 *Let $f: X \to Y$ has a δ_s-graph $G(f)$. If f is surjective δ-semiopen function, then Y is δ-semi-T_2.*

Proof. Let y_1 and y_2 be any distinct points of Y. Since f is surjective $f(x) = y_1$ for some $x \in X$ and $(x, y_2) \in (X \times Y) \setminus G(f)$. By the definition of δ_s-graph $G(f)$, there exist a δ-semiopen set U of X and an open set V of Y such that $(x, y_2) x \in X \in U \times V$ and $(U \times Int(Cl(V))) \cap G(f) = \emptyset$. Then, we have $f(U) \cap Int(Cl(V)) = \emptyset$. Since f is δ-semiopen, then $f(U)$ is δ-semiopen such that $f(x) = y_1 \in f(U)$. This implies that Y is δ-semi-T_2.

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