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CONVERGING CYLINDRICAL DETONATION WAVES IN A NON-IDEAL GAS WITH AN AXIAL MAGNETIC FIELD

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Abstract

The convergence of cylindrical detonation waves in a non-ideal gas with an axial magnetic field is analysed. The Chester-Chisnell-Whitham (CCW) method is used to solve the problem. The front velocity and the other flow variables just behind the shock are determined in the cases when (i) the gas is weakly ionized before and behind the detonation front, (ii) the gas is strongly ionized before and behind the detonation front and (iii) non-ionized (or weakly ionized) gas undergoes intense ionization as a result of the passage of the detonation front. It is investigated that in case (i) an increase in the value of ratio of specific heats of gas γ accelerates the convergence of the front and decreases the pressure behind it, while a change in the value of the parameter of non-idealness of the gas δ shows small effects on these variables. In case (ii) the front velocity and the pressure show similar behaviour as in the case (i). In both the cases, the front velocity increases very fast as the axis is approached. In the case (iii) magnetic field has damping effect on the convergence of the gas-ionizing detonation front and there is slow increase of front velocity near the axis, which is in contrast with the cases (i) and (ii).

AMS Subject Classification -76 L : Shock Waves and Blast Waves.

Key Words : Detonation Wave, Non-Ideal Gas, Axial Magnetic Field, CCW Method.

1. Introduction

Converging shock and detonation waves offer interesting possibilities of attaining extremely high tempreature, pressure and density. In fact, even the applications to thermonuclear fusion, synthesizing of materials, phenomenon of sonoluminescence and treatment of stones in the human body (lithotripsy) were considered (Glass and Sagie [1], Glass and Sharma [2], Roberts and Wu [3, 4], Takayama [5], Delius [6]). The problem of contracting cylindrical or spherical shock front propagating into a uniform gas at rest was investigated by Guderley [7] and Stanyukovich [8] by using the method of self-similarity. Nigmatulin [9], Welsh [10] and Teipel [11] replaced the shock front by a contracting detonation front propagating into a uniform combustible gas. These studies show that the similarly solution can not be obtained for a general energy release, but is can be used for studying the flow-field only if the detonation front is governed by the Chapman-Jouguet condition (Helliwell [12]). Lee and Lee [13] described the method of generation of cylindrical detonation waves in acetylene-oxygen mixture, and discussed the possibility of theoretical explanation of the process of convergence of detonation waves by means of Chester-Chisnell-Whitham (CCW) method [14, 15, 16]. The CCW method is a very simple and effective method for the analysis of imploding shocks and detonation waves. Although this method is approximate one, it agrees well with exact solutions and with experimental results (Lee and Lee [13], Lee [17] Jumper [18]).

Tyl and Wlodarczyk [19] studied cylindrical and spherical detonation waves converging in gaseous explosive mixtures by CCW method. They applied the Chapman-Jouguet condition on the detonation wave in the initial position only, and obtained analytical solution describing its propagation in the absence of magnetic field. Their solutions agreed very well with the experimental results. Vishwakarma and Vishwakarma [20] extended the case of converging detonation waves of Tyl and Wlodarczyk [19] to include the effects of the presence of an azimuthal magnetic field. They studied both the cases (i) when the gas is strongly ionized before and behind the detonation front and (ii) when the non-ionized gas undergoes intense ionization as a result of passage of the detonation front. The combustible gas was assumed to obey the equation of state of a perfect gas.

When the flow takes place in extreme conditions, the assumption that the gas is ideal is no more valid. Anisimov and Spiner [21] have taken an equation of state for low density non-ideal gases in a simplified form, and investigated the effect of parameter for non-idealness on the problem of a strong point explosion. Roberts and Wu [3, 4] ahve used an equivalent equation of state to discuss the shock wave theory of sonoluminescence. In the present work, we analyse the convergence of a strong cylindrical detonation wave in a non-ideal gas (combustible) in the presence of an axial magnetic field. The initial density is taken to be constant. It is assumed that the detonation wave is initially Chapman-Jouguet, i.e., initially it travels with the velocity of propagation of small disturbances relative to the burnt gas (Helliwell [12]). The effects of the non-idealness of the gas and the axial magnetic field are investigated. To our knowledge, the problem of converging detonation wave in a non-ideal gas, which takes into account the effects of magnetic field, has not been studied previously.

During the experiments involving the implosion of a detonation wave in a gas, the following states may occur:

(i) The gas is weakly ionized before and behind the detonation front, i.e, $R_m << 1$, where R_m is the magentic Reynolds number.

(ii) The gas is strongly ionized before and behind the detonation front, i.e., $R_m >> 1$ or $\sigma \to \infty$, where σ is the electrical conductivity.

(iii) Non-ionized (or weakly ionized) gas undergoes intense ionization as a result of the passage of the detonation front, i.e., σ increases in a jump like manner from 0 to ∞.

In our study, we analyse all the three cases when the initial magnetic field is axial and constant. CCW method is employed to determine the shock velocity and the other flow variables just behind the shock.

2. Fundamental Equations and Boundary Conditions

The equation of state for a non-ideal gas is borrowed from the statistical physics (Landau and Lifshitz [22]) which has been simplified by Anisimov and Spiner [21] in the form

$$p = \bar{R}\rho\, T\left(1+\bar{b}\rho\right), \tag{1}$$

where $\bar{b}$ (<< 1) is internal volume of the molecules, $\bar{R}$ is the gas constant, and p, ρ and T are pressure, density and temperature of the gas, respectively.

The internal energy e per unit mass is given by (Ojha [23], Roberts and Wu [3,4])

$$e = \frac{p}{(\gamma-1)\rho\left(1+\bar{b}\rho\right)} \simeq \frac{p\left(1-\bar{b}\rho\right)}{(\gamma-1)\rho} \tag{2}$$

which implies that

$$C_p - C_v = \bar{R}\left(1+\frac{\bar{b}^2\rho^2}{1+2\bar{b}\rho}\right) \simeq \bar{R}, \tag{3}$$

neglecting the term $\bar{b}^2\rho^2$. Here C_p, C_v are the specific heats of the gas at constant pressure and constant volume processes, respectively, and $\gamma = C_p/C_v$.

The basic equations governing the unsteady and cylindrically symmetric motion of a weakly conducting non-ideal gas (case I, R_m <<1) are given by (Tyl [24], Sakurai [25])

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0, \tag{4}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r}\right) + \frac{\partial p}{\partial r} = -\sigma B_0^2 u, \tag{5}$$

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-a^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=(\gamma-1)\sigma B_0^2u^2, \tag{6}$$

$$\frac{\partial B}{\partial r}=\mu\sigma B_0u, \tag{7}$$

where u, B are velocity and axial magnetic induction at distance r from the axis of symmetry, γ is the ratio of specific heats, μ is the magnetic permeability, B_0 is the initial magnetic induction and 'a' the speed of sound in the non-ideal gas, is given by

$$a^2=\frac{\gamma p}{\rho}\left(\frac{1+2\bar{b}\rho}{1+\bar{b}\rho}\right)\simeq\frac{\gamma\, p}{\rho\left(1-\bar{b}\rho\right)} \tag{8}$$

Equations (4) to (7) can be combined to form the characteristic equation (Whitham [16], Tyl [20]),

$$dp-\rho a du+\frac{\rho a^2u}{u-a}\frac{dr}{r}=\frac{\left[(\gamma-1)\left(1+\bar{b}\rho\right)u^2+ua\right]\sigma B_0^2\,dr}{(u-a)} \tag{9}$$

along the negative characteristic

$$\frac{dr}{dt}=u-a \tag{10}$$

The fundamental equations governing the unsteady flow behind a cylindrical magnetogasdynamic (case II, $R_m >> 1$) or gas-ionizing (case III, $\sigma: 0 \rightarrow \infty$) detonation front are given by (Whitham [16], Vishwakarma and Yadav [26])

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\rho u}{r}=0 \tag{11}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right)+\frac{\partial p}{\partial r}+\frac{B}{\mu}\frac{\partial B}{\partial r}=0 \tag{12}$$

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-a^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0 \tag{13}$$

$$\frac{\partial B}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rBu)=0 \tag{14}$$

Equations (11) to (14) can be combined to obtained the characteristic equation (Whitham [16])

$$dp+\mu h dh-\rho c du+\frac{\rho c^2 u}{u-c}\frac{dr}{r}=0 \tag{15}$$

along the negative characteristic

$$\frac{dr}{dt}=u-c, \tag{16}$$

where $h(=B/\mu)$ is the axial magnetic field and c is the effective speed of sound given by

$$c^2 = a^2 + b^2$$

$$\text{and} \quad b^2=\frac{B^2}{\mu\rho}$$

Since σ is small in the case I, and σ is zero ahead of the detonation front in the case III, the magnetic induction may be taken continuous in these cases (Sakurai [25], Ranga Rao and Ramana [27]). The conditions across the detonation front in the cases I and III are, therefore (Tyl and Wlodarczyk [16], Vishwakarma and Viswakarma [17]),

$$\rho_1(D-u_1)=\rho_0 D,$$

$$p_1 = p_0 + \rho_0 D u_1,$$

$$e_1 = e_0 + \frac{1}{2}(p_1 + p_0)\left(\frac{1}{\rho_0} - \frac{1}{\rho_1}\right) + Q, \tag{17}$$

where D and Q denote the velocity of the detonation wave and heat energy release per unit mass, respectively. The indices '1' and '0' refer to the states jsut behind and just ahead of the detonation front.

In the pure magnetogasdynamic case (the case II), the gas is strongly ionized, i.e., highly conducting, before and behind the detonation front, upon which the magnetic induction may be discontinuous at the front resulting from a sheet current there (Sakurai [25]). The conditions across the detonation front, in this case, may be written in the form (c.f. Whitham [16]).

$$h_1(D - u_1) = h_0 D,$$

$$\rho_1(D - u_1) = \rho_0 D,$$

$$p_1 + \frac{\mu h_1^2}{2} + \rho_1(D - u_1)^2 = p_0 + \frac{\mu h_0^2}{2} + \rho_0 D^2,$$

$$\frac{1}{2}(D - u_1)^2 + e_1 + \frac{p_1}{\rho_1} + \frac{\mu h_1^2}{\rho_1} = \frac{1}{2}D^2 + e_0 + \frac{p_0}{\rho_0} + \frac{\mu h_0^2}{\rho_0} + Q. \tag{18}$$

The detonation front is assumed to be strong, i.e., $p_0 << p_1$, therefore we take $p_0 = e_0 = 0$ in the relations (17) and (18).

3. Solution of the Problem

The denotation front is assumed to be initially in the Chapman-Jouguet state. The Chapman-Jouguet condition requires that the down stream flow will be sonic in the shock fixed co-ordinates, i.e.,

$$|D_{cj} - u_{cj}| = a_{cj}, \tag{19}$$

where 'cj' referes to the Chapman-Jouguet sate. Therefore, the conditions across the strong Chapman-Jouguet front, in the cases I and III, are expressed as

$$p_{cj} = \frac{1-\delta}{\gamma+1}\rho_0 D_{cj}^2, \tag{20}$$

$$u_{cj} = \frac{1-\delta}{\gamma+1} D_{cj}, \tag{21}$$

$$\rho_{cj} = \frac{\gamma+1}{\gamma+\delta}\rho_0, \tag{22}$$

$$h_{cj} = h_0, \tag{23}$$

$$a_{cj} = \frac{\gamma+\delta}{\gamma+1}\left|D_{cj}\right|, \tag{24}$$

$$\left|D_{cj}\right| = \frac{\sqrt{2Q(\gamma^2-1)}}{1-\delta}, \tag{25}$$

where $\delta = \bar{b}\rho_0$ is the parameter of non-idealness of the gas. In the case II, the magnetic field is also discontinuous across the front and therefore, the condition (23) is replaced by

$$h_{cj} = \frac{\gamma+1}{\gamma+\delta} h_0. \tag{26}$$

Making use of relations (17) and (20) to (26), the conditions across the strong detonation front can be expressed in terms of the velocity of the detonation products (burnt gas), in the case I, by the equations

$$\frac{D}{D_{cj}} = \frac{1}{2}\left[q + q^{-1}\right], \tag{27}$$

$$\frac{p_1}{p_{cj}} = \frac{1}{2}\left[q^2 + 1\right] \tag{28}$$

$$\frac{\rho_1}{\rho_{cj}} = \frac{(\gamma + \delta)\left[q^2 + 1\right]}{(\gamma + 1) + (2\delta + \gamma - 1)q^2}, \tag{29}$$

$$\frac{a_1}{a_{cj}} = \left[\frac{\{3\delta + \gamma(1 + \delta) - 1\}\{q^2 + 1\} + (2 - 2\delta)}{2(\gamma + 2\delta + \gamma\delta)}\right]^{\frac{1}{2}}, \tag{30}$$

where $q = \dfrac{u_1}{u_{cj}}$.

Using relations (18) and (20) to (26), the conditions across the strong magnetogasdynamic detonation front (case II) can be expressed by the equations

$$\frac{D}{D_{cj}} = \frac{1}{2}\left[q + q^{-1}\right], \tag{31}$$

$$\frac{p_1}{p_{cj}} = \frac{1}{2}\left[q^2 + 1\right], \tag{32}$$

$$\frac{\rho_1}{\rho_{cj}} = \frac{(\gamma + \delta)}{S_1}\left[q^2 + 1\right], \tag{33}$$

$$\frac{h_1}{h_{cj}} = \frac{(\gamma + \delta)}{S_1}\left[q^2 + 1\right], \tag{34}$$

$$\frac{c_1}{a_{cj}} = \frac{1}{(\gamma + \delta)}\left[\frac{\gamma S_1^2}{2S_2} + \frac{(\gamma + 1)^2(\gamma + \delta)(q^2 + 1)M_{cj}^{-2}}{S_1}\right]^{\frac{1}{2}}, \tag{35}$$

where $S_1 = (\gamma + 1) + (2\delta + \gamma - 1)q^2, \quad S_2 = (\gamma + 1) + (\gamma - 1)q^2,$

and the Alfven Mach number M_{cj} of the detonation front in the Chapman-Jouguet state is given by

$$M_{cj}^2 = D_{cj}^2 / \left(\mu h_{cj}^2 / \rho_{cj}\right).$$

In the case III (strong gas-ionizing detonation front), the equations (34) and (35) are replaced by

$$h_1 = h_{cj}, \tag{36}$$

$$\frac{c_1}{a_{cj}} = \frac{1}{(\gamma + \delta)} \left[\frac{\gamma S_1^2}{2S_2} + \frac{(\gamma + 1)^2 S_1 M_{cj}^{-2}}{(\gamma + \delta)(q^2 + 1)} \right]^{\frac{1}{2}} . \tag{37}$$

Now, we shall use CCW method [16] to obtain the speed of detonation front and the other flow variables just behind the front in all the three cases. For converging fronts, the method is to apply the characteristic equation (valid along a negative characteristic) to the flow quantities just behind the front.

Case I $\quad R_m << 1$

(Detonation Wave in a Weakly Conducting Non-Ideal Gas)

Using the flow variables just behind the detonation front, into the characteristic equation (9) (keeping in the mind that u_1 is negative), we obtain

$$\frac{d\left(\dfrac{p_1}{p_{cj}}\right)}{\dfrac{\rho_1}{\rho_{cj}} \dfrac{a_1}{a_{cj}}} + dq + \frac{q \dfrac{a_1}{a_{cj}} x^{-1}}{\left(\dfrac{1-\delta}{\gamma + \delta}\right) q + \dfrac{a_1}{a_{cj}}} dx$$

$$= -\frac{\left[\left(\frac{\gamma^2-1}{\gamma+\delta}\right)q^2\left\{1+\delta\frac{(\gamma+1)}{(\gamma+\delta)}\frac{\rho_1}{\rho_{cj}}\right\} - q\frac{a_1}{a_{cj}}\left(\frac{1+\gamma}{\gamma-\delta}\right)\right]R_m M_{cj}^{-2}dx}{\left[q+\frac{a_1}{a_{cj}}\left(\frac{\gamma+\delta}{1-\delta}\right)\right]\frac{\rho_1}{\rho_{cj}}\frac{a_1}{a_{cj}}}, \quad (38)$$

where $x = R/R_1$, R being the radius of detonation front and R_1 is the value of R at the Chapman –Jouguet position (initial position).

The magnetic Reynolds number R_m is given by

$$R_m = \sigma\mu / D_{cj} / R_1.$$

Using the values of flow variables, given by equations (27) to (30), into the characteristic equation (38), we obtain the differential equation between the velocity of the motion of the detonation products and the location of the wave front as

$$\frac{dq}{dx} = -\frac{\left[\left(\frac{\gamma^2-1}{\gamma+\delta}\right)q^2\left\{2G_1(\gamma+2\delta+\gamma\delta) - qS_1\sqrt{G_1}\left(\frac{1+\gamma}{1-\delta}\right)\right\}\right]}{\left\{(1-\delta)q+\sqrt{G_1}(\gamma+\delta)\right\}\left\{S_1 q+(\gamma+\delta)(q^2+1)\sqrt{G_1}\right\}}$$

$$\times R_m M_{cj}^{-2}(1-\delta) + q(q^2+1)x^{-1}(\gamma+\delta)^2 G_1, \quad (39)$$

where

$$G_1 = \left\{\frac{S_1+\delta(\gamma+1)(1+q^2)}{2(\gamma+2\delta+\gamma\delta)}\right\}.$$

Numerical integration of the differential equation (39), along with the equations (27, 28, 29), with initial conditions $q = 1, x = 1$ gives the values of q, $\frac{D}{D_{cj}}$, $\frac{p_1}{p_{cj}}$, $\frac{\rho_1}{\rho_{cj}}$ as x decreases from 1 to zero.

Case II $R_m >> 1$ (Pure Magnetogasdynamic Detonation Wave)

Using the flow variables just behind the detonation front into the characteristic equation (15) (keeping in the mind that u_1 is negative), we obtain

$$\frac{d\left(\dfrac{p_1}{p_{cj}}\right)}{\dfrac{\rho_1}{\rho_{cj}}\dfrac{c_1}{a_{cj}}}+\frac{\dfrac{(1+\gamma)^2}{(1-\delta)(\gamma+\delta)}\left(\dfrac{h_1}{h_{cj}}\right)M_{cj}^{-2}d\left(\dfrac{h_1}{h_{cj}}\right)}{\dfrac{\rho_1}{\rho_{cj}}\dfrac{c_1}{a_{cj}}}+dq-\frac{\left(\dfrac{\gamma+\delta}{1-\delta}\right)\dfrac{c_1}{a_{cj}}qx^{-1}dx}{q+\left(\dfrac{\gamma+\delta}{1-\delta}\right)\dfrac{c_1}{a_{cj}}}=0. \quad (40)$$

On using the equations (32, 33, 34, 35) in the equation (40), we obtain the following differential equations

$$\frac{dq}{dx}=\frac{\left[qG_2^2R(1-\delta)x^{-1}\right]\left(q^2+1\right)S_1\left[(1-\delta)q+G_2\right]^{-1}}{\left[S_1^3q(1-\delta)+G_2S_1(1-\delta)\left(q^2+1\right)+2\left(q^2+1\right)qM_{cj}^{-2}(1+\gamma)^2(\gamma+\delta)\left\{S_1\left(q^2+1\right)(2\delta+\gamma-1)\right\}\right]}, \quad (41)$$

where

$$G_2=\sqrt{\frac{\gamma R^3+2S(1+\gamma)^2\left(q^2+1\right)(\gamma+\delta)M_{cj}^{-2}x^{2\alpha_1}}{2S_1S_2}}$$

Integrating the differential equation (41), numerically, and using (31) to (37), we can obtain q, $\dfrac{D}{D_{cj}}$, $\dfrac{p_1}{p_{cj}}$, $\dfrac{\rho_1}{\rho_{cj}}$ and $\dfrac{h_1}{h_{cj}}$ in terms of x.

Case III $\sigma: 0 \to \infty$ (Gas-Ionizing Detonation Wave)

On using the equations (32, 33, 36, 37) into the characteristic equation (15) and simplifying we obtain the differential equation

$$\frac{dq}{dx} = -\frac{qG_3^2 x^{-1}\left(q^2+1\right)}{\{(1-\delta)q+G_3\}\{S_1 q+G_3\left(q^2+1\right)\}} \tag{42}$$

$$-\frac{\left(1+\gamma^2\right)x^{-1}S_1 M_{cj}^{-2}}{(1-\delta)(\gamma+\delta)\{S_1 q+G_3\left(q^2+1\right)\}}$$

where $$G_3 = \sqrt{\frac{\gamma S_1^2(\gamma+\delta)\left(q^2+1\right)+S_1(1+\gamma)^2 2S_2 M_{cj}^{-2}}{2S_2(\gamma+\delta)\left(q^2+1\right)}}.$$

Numerical integration of the differential equation (3.9) and use of equations (31, 32, 33) give the variation of q, $\frac{D}{D_{cj}}$, $\frac{p_1}{p_{cj}}$, and $\frac{\rho_1}{\rho_{cj}}$ with x.

4. Results and Discussion

For the purpose of numerical calcualtions, we used the values of R_m, γ, δ, and M_{cj}^{-2} given by $R_m = 0.001$ (in the case I only); $\gamma = 1.4, 3.0$; $\delta = 0, 0.1$; and $M_{cj}^{-2} = 0, 0.1$. The value $\delta = 0$ corresponds to the case of a perfect gas, and $M_{cj}^{-2} = 0$ to the non-magnetic case.

In the case I, the front velocity D/D_{cj} and the pressure behind it p_1/p_{cj} are plotted against x in figures 1 and 2. It is found that an increase in the value of the ratio of specific heats of the gas γ, accelerates the convergence of the front (figure 1) and decreases the pressure behind it (figure 2). A change in the value of the parameter δ characterizing the non-idealness of the gas shows small effects on these variables.

In the case II, the front velocity $\frac{D}{D_{cj}}$ (figure 3) and the pressure $\frac{p_1}{p_{cj}}$ (figure 4) show the similar behaviour as in the case I. In both the cases, the front velocity increases very fast as the axis is approached. The fast reduction of frontal areas and the assumption that the detonation front is strong (i.e. the neglect of the pressure ahead of the front) cause this behaviour of the front velocity near the axis. An increase in the value of M_{cj}^{-2} (a measure of the initial magnetic field) shows small effects on the front velocity and the pressure in the case II.

In the case III, $\frac{D}{D_{cj}}$ and $\frac{p_1}{p_{cj}}$ are plotted in figures 5 and 6. These figures show that for $M_{cj}^{-2} = 0$, the front velocity increases rapidly and the pressure behind the front tends to zero as the axis is approached; and for $M_{cj}^{-2} = 0.1$, the increase of front velocity is very slow and the pressure does not tend to zero. This shows that the magnetic field has damping effect on the convergence of the gas-ionizing detonation front. The phenomenon of slow increase of front velocity near the axis, which is in contrast with the cases I and II, may be physically interpreted as follows :

In the case III, where a non-conducting (or weakly conducting) gas becomes highly conducting due to passage of a strong detonation front, and there is no jump of the magnetic induction across the front, the medium behind the front acts as a piston compressing the magnetic flux and pushing it into the region ahead of the front. In fact the speed of the front is higher than the speed of the conducting medium behind the front, therefore, the magenetic flux is 'transported by convection' from the compression region even if the medium behind the front is an ideal conductor. The convection of the magnetic flux leads to the formation of a current carrying layer of considerable thickness behind the front, and this fact increases the compression of the magnetic flux and, therefore, it is pushed into the region ahead of the front (Nagayama [28], Tyl [24]). Thus, the magnetic pressure in the region ahead of the front increases very fast which causes the decay of the front.

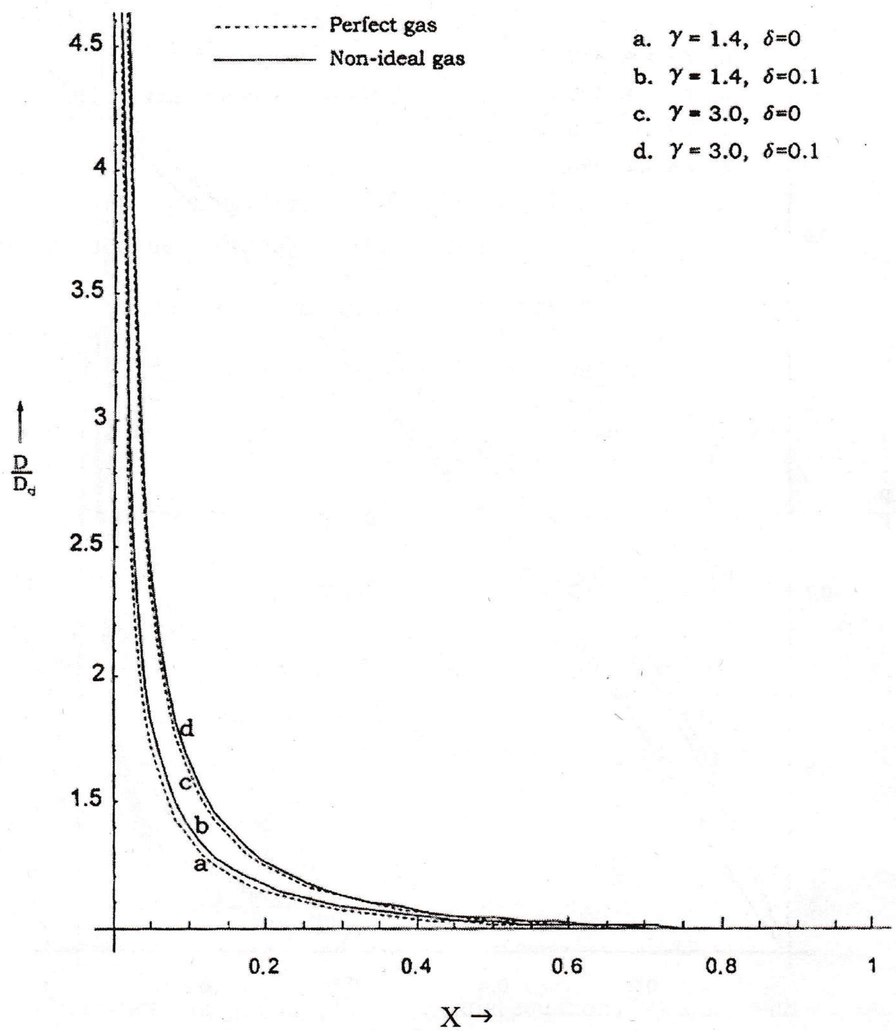

Fig. 1: Variation of velocity of detonation front with its radius in the case I ($R_m << 1$) for $R_m = 0.001$ and $M_{cj}^{-2} = 0.1$)

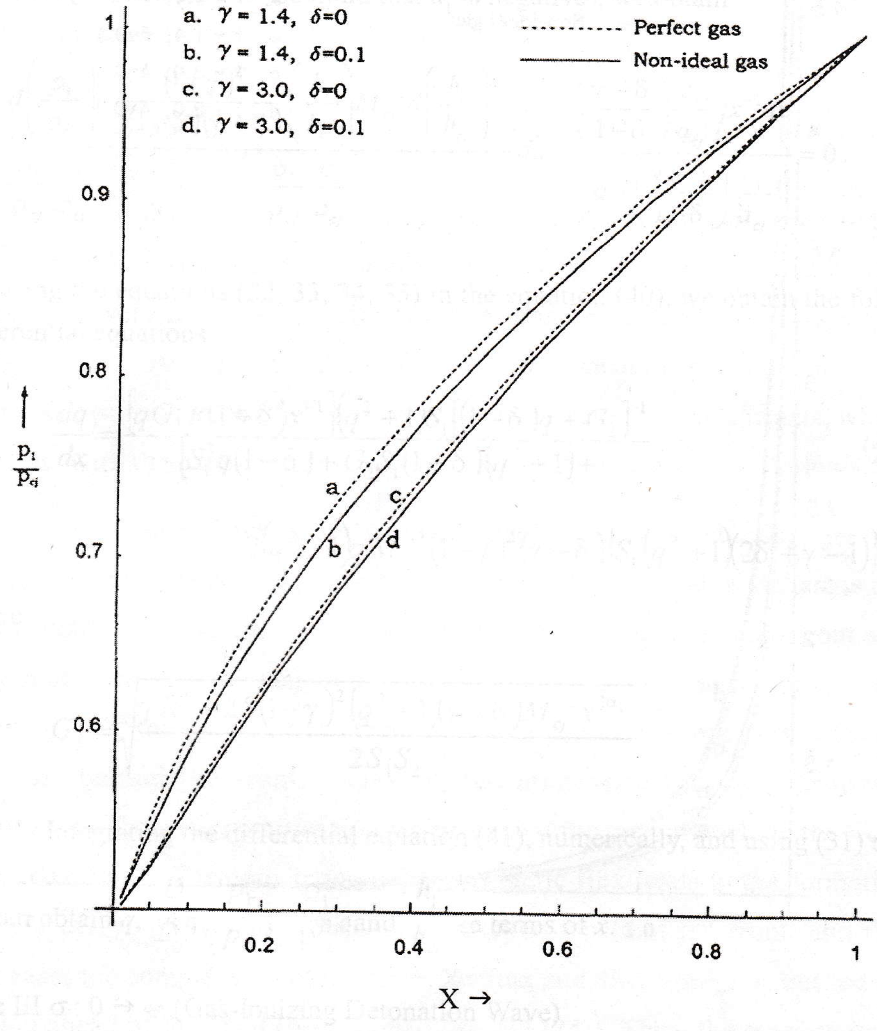

Fig. 2: Variation of pressure behind the detonation front with its radius in the case I ($R_m << 1$) for $R_m = 0.001$ and $M_{cj}^{-2} = 0.1$)

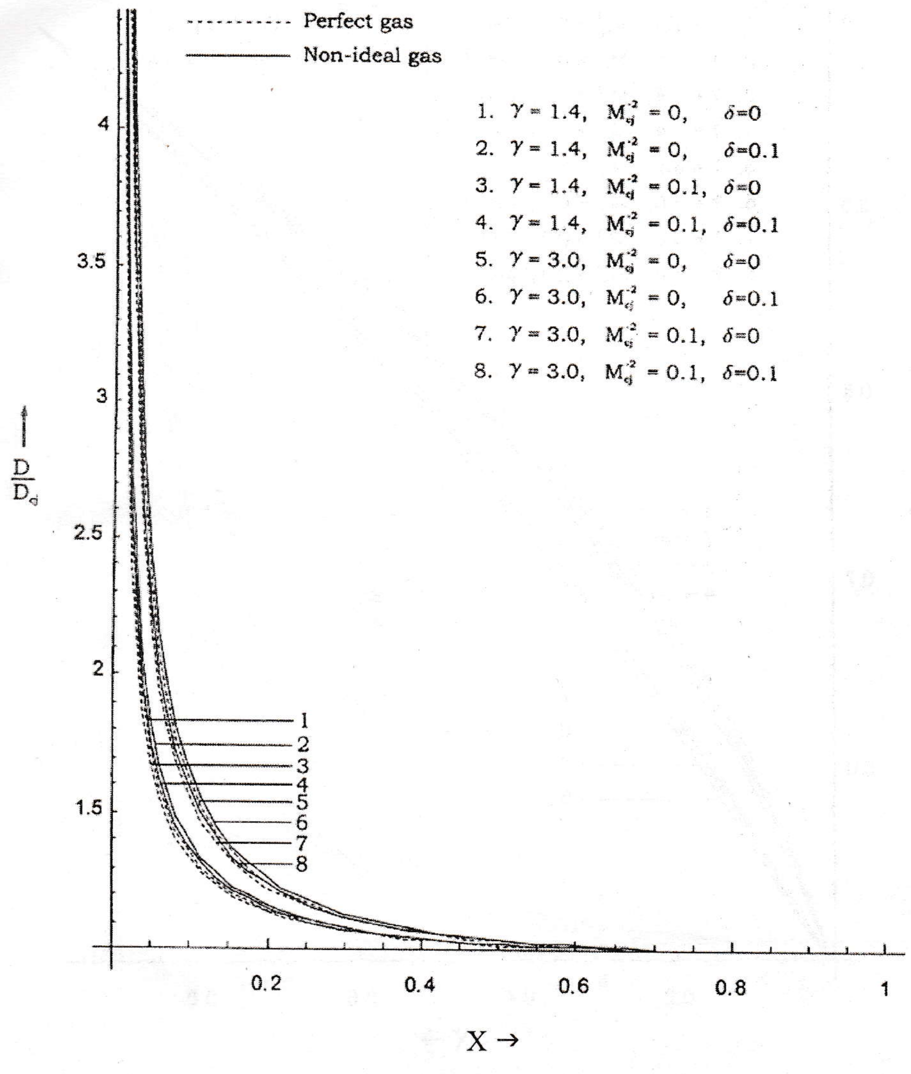

Fig. 3: Variation of velocity of detonation front with its radius in the case II ($R_m << 1$)

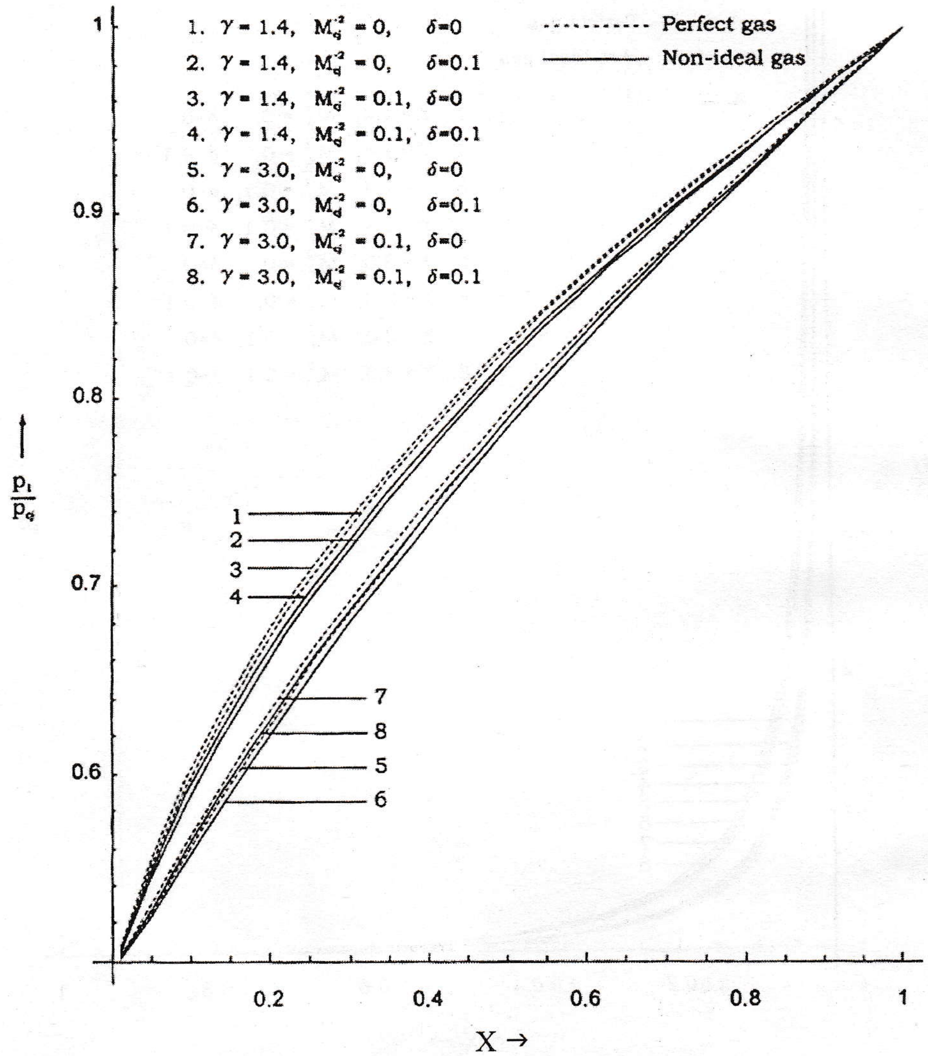

Fig. 4: Variation of pressure behind the detonation front with its radius in the case II ($R_m << 1$)

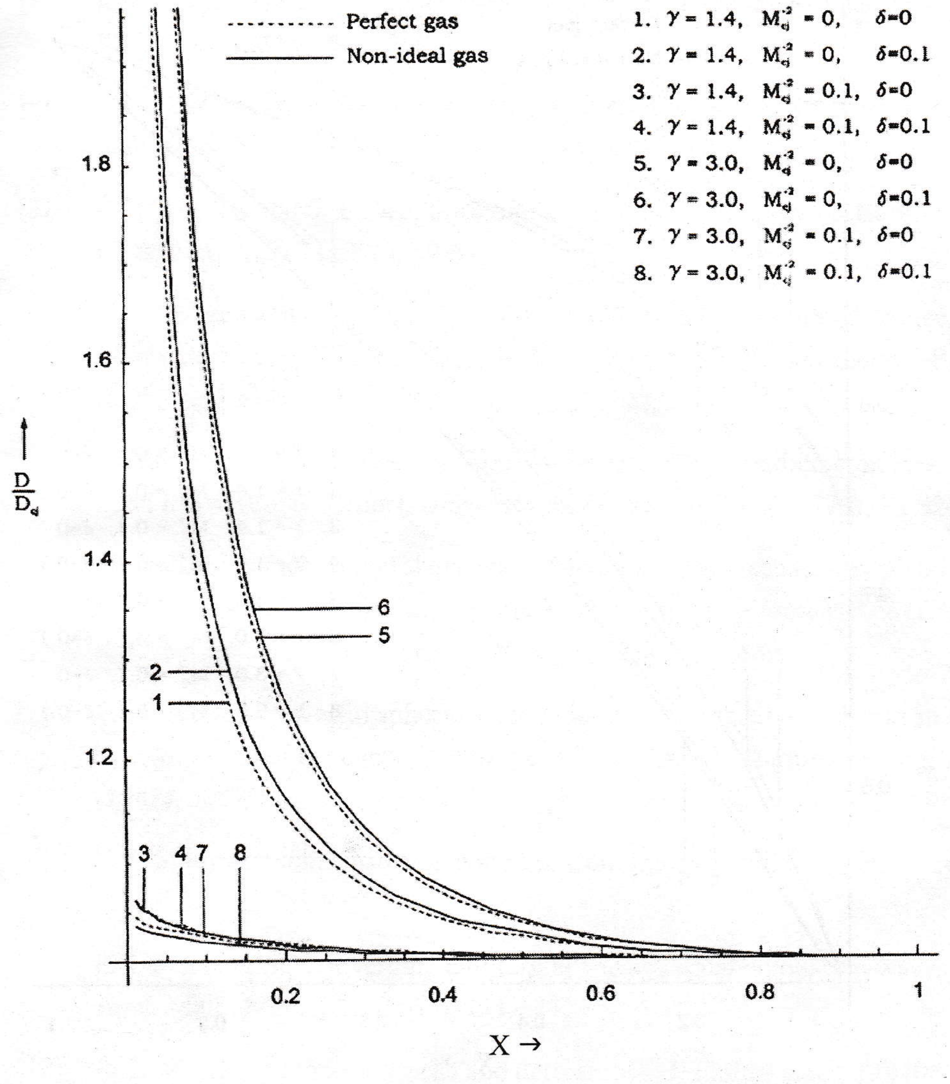

Fig. 5: Variation of velocity of detonation front with its radius in the case III ($\sigma : 0 \rightarrow \infty$)

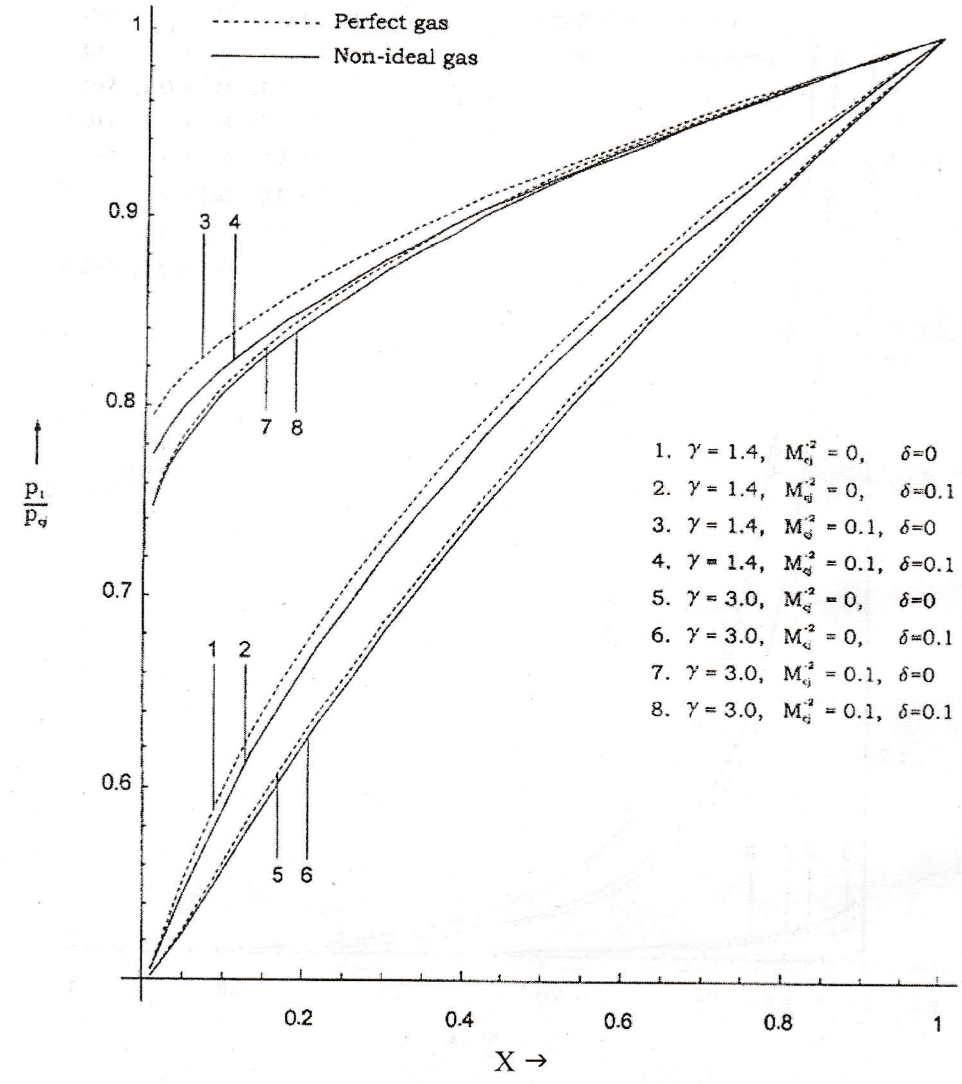

Fig. 6: Variation of pressure behind the detonation front with its radius in the case III ($\sigma : 0 \rightarrow \infty$)

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