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ON STRONGLY *T*-COMPACTNESS PRE-SEPARATED FUZZY TOPOLOGICAL SPACES

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ABSTRACT

By utilizing triangular conorm (briefly *t*-norm) *T* and the notion of preopen fuzzy covering, we define a new type of compactness in the field of fuzzy topology. We also generalize the Lowens notion of a fuzzy compact fuzzy topological space. Moreover, we introduce, among others, the notions of *PS*-open and *PS*-closed fuzzy sets and investigate the *PS*-closedness of strongly *T*-compact fuzzy sets.

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1. Introduction and Preliminaries

In 1965, Zadeh [9] introduced the notion fuzzy set. Also the following notions used in the sequel are due to Zadeh : $C(A(x))$ (=conplement of A) is $1 - A(x)$, $\bigcup_{i\in I} A_i(x) = \sup_{i\in I} A_i(x)$ and $\bigcap_{i\in I} A_i(x) = \inf_{i\in I} A_i(x)$ for any fuzzy set A and any $(A_i)_{i\in I}$ of fuzzy sets in a universe X. We denote the class of all fuzzy sets in X by $F(X)$. In what follows, crisps are identified with their characteristics function. A fuzzy set A of a fuzzy topological space (X, τ) is called fuzzy *preopen* ([3], [1], [8]) if $A \leq Int(Cl(A))$ where *Int* and *Cl* denote the interior and the *closure* operators. The complement of a preopen fuzzy set is called *preclosed* [3], equivalently, A is preclosed if $Cl(Int(A)) \leq A$. We denote the family of fuzzy preopen sets of (X,τ) by $PO(X,\tau)$. A fuzzy set A in a fuzzy topological space (X, τ) is called a fuzzy *pre-neigbourhood* of a fuzzy set B in X if and only if there exists a $G \in PO(X, \tau)$ such that $B \subset G \subset A$. Clearly, a fuzzy set in a fuzzy topological space is preopen if and only if it is a fuzzy pre-neighbourhood of all of the fuzzy singleton contained in it. Throughout this paper, an arbitrary preopen fuzzy neighbourhood of a fuzzy set A in X is denoted by $N(A)$ or $H(A)$. Similarly an arbitrary preopen fuzzy neighbourhood of a fuzzy singleton x_δ in X is denoted by $N(x_\delta)$ or $H(x_\delta)$.

Definition 1 : *A triangular norm (brief t-norm) [7] is a function* $T : [0,1]^2 \to [0, 1]$ *such that :*

$(T1)$	$T(a, b) = T(b, a)$	*(commutativity)*
$(T2)$	$T(T(a, b), d) = T(a, T(b, d))$,	*(associativity)*
$(T3)$	$a \leq b \to T(a, d) \leq T(b, d)$,	*(monotonocity)*
$(T4)$	$T(a, 1) = a$.	*(boundary condition)*

Definition 2 : *A traingular conorm (briefly t-conorm) [7] is a function* $S : [0,1]^2 \to [0, 1]$ *such that* :

$(S1)$	$S(a, b) = S(b, a)$	*(commutativity)*
$(S2)$	$S(S(a, b), d) = S(a, S(b, d))$,	*(associativity)*
$(S3)$	$a \leq b \to S(a, d) \leq S(b, d)$,	*(monotonocity)*
$(S4)$	$S(a, 0) = a$.	*(boundary condition)*

2. Some Separation Axioms

Definition 3 : A fuzzy topological space (X, τ) is called :

(1) pre-separated if and only if for any two fuzzy singletons x_δ *and* y_γ *in* X *with different supports, there exist two preopen fuzzy neighbourhoods* $N(x_\delta)$ *and* $H(y_\gamma)$ *such that* $N(x_\delta) \subseteq C(H(y_\gamma))$;

(2) quasi-p-separated if and only if for any two fuzzy singletons x_δ *and* y_γ *in* X *with different supports, there exist two preopen fuzzy neighbourhoods* $N(x_\delta)$ *and* $H(y_\gamma)$ *such that* $N(x_\delta) \subseteq C(y_\gamma)$ *and* $H(y_\gamma) \subseteq C(x_\delta)$.

(3) weakly quasi-p-separated if and only if for any two fuzzy singletons x_δ *and* y_γ *in* X *with different supports, there exist a preopen fuzzy neighbourhoods* $N(x_\delta)$ *such that* $N(x_\delta) \subseteq C(y_\gamma)$ *or there exists a preopen fuzzy neighbourhood* $H(y_\gamma)$ *such that* $H(y_\gamma) \subseteq C(x_\delta)$.

The proofs of the following results can be shown easily and therefore omitted.

Theorem 2.1 : *A fuzzy topological space* (X, τ) *is quasi-p-separated if and only if every (crisp) singleton in* X *is preclosed.*

Theorem 2.2 : *For any strongly compact crisp set* K *in a pre-separated fuzzy topological space* (X,τ) *and any fuzzy singleton* x_δ *in* X *such that* $x_\delta \subseteq C(K)$, *there exist two preopen fuzzy neighbourhoods* $N(x_\delta)$ *and* $H(K)$ *such that* $N(x_\delta) \subseteq C(H(K))$.

Theorem 2.3 : *A strongly compact crisp set in a pre-separated fuzzy topological space is preclosed.*

But we are also interested to prove the same results for fuzzy sets. Therefore, we improve the above definitions by relaxation of the "different supports" requirement. For doing this, we use the classical equivalences :

$$x \neq y \Leftrightarrow \{x\} \subseteq C(\{y\}) \Leftrightarrow \{y\} \subseteq C(\{x\}).$$

Now we have the following equivalences for any two fuzzy singletons x_δ and y_γ:

$$x_\delta \subseteq C(\{y_\gamma\}) \Leftrightarrow y_\gamma \subseteq C(\{x_\delta\}) \Leftrightarrow (x \neq y) V((x = y) \Lambda (\delta + \gamma \leq 1)).$$

Definition 4 : *A fuzzy topological space* (X,τ) *is called* :

(1) pre-separated if and only if for any two fuzzy singletons x_δ *and* y_γ *in* X *such that* $x_\delta \subseteq C(y_\gamma)$, *there exist two preopen fuzzy neighbourhoods* $N(x_\delta)$ *and* $H(y_\gamma)$ *such that* $N(x_\delta) \subseteq C(H(y_\gamma))$;

(2) quasi-p-separated if and only if for any two fuzzy singletons x_δ *and* y_γ *in* X *such that* $x_\delta \subseteq C(y_\gamma)$, *there exist two preopen fuzzy neighbourhoods* $N(x_\delta)$ *and* $H(y_\gamma)$ *such that* $N(x_\delta) \subseteq C(y_\gamma)$ *and* $H(y_\gamma) \subseteq C(x_\delta)$.

(3) weakly quasi-p-separated if and only if for any two fuzzy singletons x_δ *and* y_γ *in* X *such that* $x_\delta \subseteq C(y_\gamma)$, *there exists two preopen fuzzy neighbourhood* $N(x_\delta)$ *such that* $N(x_\delta) \subseteq C(y_\gamma)$ *or there exists a preopen fuzzy neighbourhood* $H(y_\gamma)$ *such that* $H(y_\gamma) \subseteq C(x_\delta)$.

Remark 2.4 : *Observe that any pre-separated fuzzy topological space is quasi-p-separated and any quasi-p-separated fuzzy topological space is weakly quasi-p-separated.*

Proposition 2.5 : *A fuzzy topological space* (X,τ) *is quasi-p-separated if and only if every fuzzy singleton in* X *is preclosed.*

Proof: Let (X, τ) be quasi-p-separated and x_δ be a fuzzy singleton in X. For any fuzzy singleton y_γ such that $y_\gamma \subseteq C(x_\delta)$, there exists a preopen fuzzy neighbourhood $H(y_\gamma)$ such that $H(y_\gamma) \subseteq C(x_\delta)$. This means that $C(x_\delta)$ is a fuzzy pre-neighbourhood of all the fuzzy singletons contained in it. Therefore, $C(x_\delta)$ is preopen and x_δ is preclosed. Conversely, assume that every fuzzy singleton in X is preclosed. Let x_δ and y_γ be two fuzzy singletons in X such that $x_\delta \subseteq C(y_\gamma)$. Then it is also true that $y_\gamma \subseteq C(x_\delta)$. By hiptesis, the fuzzy sets $C(x_\delta)$ and $C(y_\gamma)$ are preopen. This gives us the possibility to set $N(x_\delta) = C(y_\gamma)$ and $H(y_\gamma) = C(x_\delta)$. It is true that $N(x_\delta) \subseteq C(y_\gamma)$ and $H(y_\gamma) \subseteq C(x_\delta)$. Therefore, (X,τ) is quasi-p-separated.

3. Pre-regular and Pre-normal Fuzzy Topological Spaces

Definition 5 : *A fuzzy topological space (X, τ) is called pre-regular if and only if for any fuzzy singleton x_δ in X and any preclosed fuzzy set F in X such that $x_\delta \subseteq C(F)$, there exist two preopen fuzzy neighbourhoods $N(x_\delta)$ and $H(F)$ such that $N(x_\delta) \subseteq C(H(F))$.*

Proposition 3.1 : *Any weakly quasi-p-separated and pre-regular fuzzy topological space is pre-separated.*

Proof: Suppose that (X,τ) is weakly quasi-p-separated and pre-regular fuzzy topological space. Let x_δ and y_γ be two fuzzy singletons in X such that $x_\delta \subseteq C(y_\gamma)$. Since (X,τ) is weakly quasi-p-separated, we can assume that there exists a preopen fuzzy neighbourhood $N(x_\delta)$ such that $N(x_\delta) \subseteq C(y_\gamma)$. Set $F = C(N(x_\delta))$. Since F is preclosed and $x_\delta \subseteq C(F)$, the pre-regularity of (X,τ) implies that there exist two preopen fuzzy neighbourhoods $D(x_\delta)$ and E(F) such that $D(x_\delta) \subseteq C(E(F))$. Moreover, we have that $y_\gamma \subseteq C(N(x_\delta)) = F = E(F)$ Now this implies that $E(F)$ is a preopen fuzzy neighbourhood of y_δ. Therefore, (X,τ) is pre-separated.

Definition 6 : *A fuzzy topological space (X,τ) is called pre-normal if and only if for any two preclosed fuzzy sets A and B in X such that $A \subseteq C(B)$, there exist two preopen fuzzy neighbourhoods $N(A)$ and $H(B)$ such that $N(A) \subseteq C(H(B))$.*

Proposition 3.2 : *Any quasi-p-separated and pre-normal fuzzy topological space is pre-regular.*

Proof : Suppose that (X,τ) is quasi-p-separated and pre-normal fuzzy topological space. Let x_δ be a fuzzy singleton in X and F be a preclosed fuzzy in X such that $x_\delta \subseteq C(F)$. By Proposition 2.5, we have that x_δ is preclosed. By the pre-normality of (X,τ), there exist two preopen fuzzy neighbourhoods $N(x_\delta)$ and $H(F)$ such that $N(x_\delta) \subseteq C(H(F))$. Therefore, (X,τ) is pre-regular.

Remark 3.3 : *It follows from Proposition 3.1 and 3.2 that any quasi-p-separated and pre-normal fuzzy topological space is pre-separated.*

4. Strongly Compact Fuzzy Sets in Pre-separated Fuzzy Topological Spaces

Definition 7 : *A family of preopen fuzzy sets* $(P_i)_{i\in I}$ *in a fuzzy topological spaces* (X,τ) *is called a preopen fuzzy covering of a fuzzy set A in X if and only if* $A \subseteq \bigcup_{i\in I} P_i$.

Definition 8 : *A subfamily of preopen fuzzy covering of a fuzzy set A in X that is still a preopen fuzzy covering of A is called a subcovering.*

Definition 9 : *A fuzzy set A in a fuzzy topological space* (X,τ) *is called strongly compact if and only if any preopen fuzzy covering of A has a finite subcovering.*

Lemma 4.1 : *For any fuzzy A in X and any singleton* x_δ *in X such that* $x_\delta \subseteq C(A)$, *the following holds :* $A \subseteq \bigcup_{y_\gamma \in C(x_\delta)} y_\gamma$.

Theorem 4.2 : *For any strongly compact fuzzy set A in a pre-separated fuzzy topological space* (X,τ) *and any fuzzy singleton* x_δ *in X such that* $x_\delta \subseteq C(A)$, *there exist two preopen fuzzy neighbourhoods* $N(x_\delta)$ *and* $H(A)$ *such that* $N(x_\delta) \subseteq C(H(A))$.

Proof : Suppose that a strongly compact fuzzy set A in a pre-separated fuzzy topological space (X,τ) and a singleton x_δ in X such that $x_\delta \subseteq C(A)$. By Lemma 4.1, we have $A \subseteq \bigcup_{y_\gamma \in C(x_\delta)} y_\gamma \subseteq \bigcup_{x_\delta \in C(y_\gamma)} y_\gamma$. For any fuzzy singleton y_γ in X such that $x_\delta \subseteq C(y_\gamma)$, it follows by Definition 4 that there exist two preopen fuzzy neighbourhoods $N_{y_\gamma}(x_\delta)$ and $H(y_\gamma)$ such that $N_{y_\gamma}(x_\delta) \subseteq C(H(y_\gamma))$. Notice that $A \subseteq \bigcup_{y_\gamma \in C(x_\delta)} H(y_\gamma)$. By the fact that A is strongly compact, there exists a finite family $(y_{i_\vartheta})_{i=1}^{n}$ such that $A \subseteq \bigcup_{i=1}^{n} H(y_{i_\vartheta})$. Suppose that $H(A)=\bigcup_{i=1}^{n} H(y_{i_\vartheta})$ and $N(x_\delta)=\bigcap_{i=1}^{n} N^{y_{i_\vartheta}}(x_\delta)$. Then we have that

$$N(x_\delta)=\bigcap_{i=1}^{n} N^{y_{i_\vartheta}}(x_\delta) \subseteq \bigcap_{i=1}^{n} C(H(y_{i_\vartheta}))=C\left(\bigcup_{i=1}^{n} H(y_{i_\vartheta})\right)=C(H(A)).$$

Corollary 4.3 : *Any strongly compact fuzzy set in a pre-separated fuzzy topological space is preclosed.*

Proof : Assume that A is a strongly compact fuzzy set in a pre-separated fuzzy topological space (X,τ). For any $x_\delta \subseteq C(A)$. By Theorem 4.2, we have that there exist two preopen fuzzy neighbourhoods $N(x_\delta)$ and $H(A)$ such that $x_\delta \subseteq N(x_\delta) \subseteq C(H(A)) \subseteq C(A)$. Hence, we have shown that $C(A)$ is a fuzzy pre-neighbourhood of all of the fuzzy singletons contained in it. Therefore, $C(A)$ is preopen and thus A is preclosed.

5. Strongly T-Compact Fuzzy Sets in Pre-separated Fuzzy Topological Spaces

In 1976, Lowen [5] offered a way to associate a fuzzy topology with a given topology and also to associate a topology with a given fuzzy topology. He defined two functors ι, ϖ and the notion of a topologically generated fuzzy topological space. Let $\tau(X)$ be the class of all topologies on X and $\sigma(X)$ be the class of all fuzzy topologies on X, By I_r we denote to topological space (I, T_r) with $I = [0,1]$ and $T_r = \{[\alpha,1] : \alpha \in [0,1]\} \cup \{1\}$. The functor $\varpi : \tau(X) \rightarrow \sigma(X)$ is defined as an association of $\varpi(X,\tau) = (X, \varpi, (\tau))$ with $\varpi(\tau) = C_0((X,\tau), I_r)$ the set of all continuous mappings from (X,τ) into I_r.

If we change open sets with preopen sets in *Lowen's* construction mentioned above under the condition that (X,τ) is submaximal, we have the following notion which is equivalent with *Lowen's* notion of topologically generated fuzzy topological spaces [5].

Definition 10 : *A fuzzy topological space* (X,τ) *is called topologically pre-generated fuzzy topological space if there exists a submaximal preopen topology* τ *such that* $(X,\tau) = (X,\varpi(\tau))$.

One can observe readily a gap in *Nanda's* definition [6] of a strongly compact fuzzy topological space since one does not have that if (X,τ) is strongly compact topological space then $(X,\varpi(\tau))$ is a strongly compact fuzzy topological space. Now we introduce an alternative form of strongly compact called strong compactness in order ot remove the mentioned gap.

Definition 11 : *A fuzzy set A in a fuzzy topological space* (X,τ) *is called fuzzy strongly compact if and only if for any preopen fuzzy covering* $(P_i)_{i\in I}$ *of A and any fuzzy constant* 1_μ *in X, with* $\mu \in [0,1]$ *there exists a finite subfamily* $(P_i)_{i=1}^{n_\mu}$ *such that*

$$A \dot{-} 1_\mu \subseteq \bigcup_{i=1}^{n_\mu} P_i \text{ where } A \dot{-} 1_\mu \text{ is the fuzzy set defined by } A \dot{-} 1_\mu\,(x) = max(A(x) - \mu, 0).$$

Notice that 1_μ *is a fuzzy set with constant degree of membership* μ.

As a generalization of the classical notion strongly compact topological space, we have introduced the notion of fuzzy strong compactness, i.e., a fuzzy topological space (X,τ) is fuzzy strongly compact if and only if all fuzzy constants in X are fuzzy strongly compact.

We have also obtained the following alternative definition for fuzzy strongly compactness.

Definition 12 : *A fuzzy set A in a fuzzy topological space* (X,τ) *is called fuzzy strongly compact if and only if for any preopen fuzzy covering* $(P_i)_{i\in I}$ *of A and any* *fuzzy constant* 1_μ *in X, with* $\mu \in [0,1]$ *there exists a finite subfamily* $(P_i)_{i=1}^{n_\mu}$ *such that* $A - 1_\mu \subseteq \bigcup_{i=1}^{n_\mu} P_i$, *where* $A - 1_\mu$ *is the fuzzy set defined by* $A - 1_\mu(x) = A \cap C(1_\mu(x)) = \min(A(x), 1-\mu)$.

It should be noted that the T-intersection with T as a t-norm [7] of two fuzzy sets A and B in X is a fuzzy set $A \cap_\tau B$ in X defined by $A \cap_\tau B(x) = T(A(x), B(x))$. The inclusions $A \dot{-} 1_\mu \subseteq \bigcup_{i=1}^{n_\mu} P_i$ and $A - 1_\mu \subseteq \bigcup_{i=1}^{n_\mu} P_i$ in Definitions 11 and 12 can be written as follows, $A \cap_T 1_{1-\mu} \subseteq \bigcup_{i=1}^{n\mu} P_i$, where T is Lukasiewicz t-norm W, defined by $W(x, y) = \max(x + y - 1, 0)$ in $A \dot{-} 1_\mu \subseteq \bigcup_{i=1}^{n_\mu} P_i$ and the minimum operator M in $A - 1_\mu \subseteq \bigcup_{i=1}^{n_\mu} P_i$. All these motivate to give the following definition which is not a categorical approach.

Definition 13 : *Let T be a t-norm. A fuzzy set A in a fuzzy topological space* (X, τ) *is called strongly T-compact if and only if for any preopen fuzzy covering* $(P_i)_{i \in I}$ *of A and any fuzzy constant* 1_μ *in X, with* $\mu \in [0,1]$ *there exists a finite subfamily* $(P_i)_{i=1}^{n_\mu}$ *such that* $A \cap_T 1_\mu \subseteq \bigcup_{i=1}^{n\mu} P_i$.

Clearly, any strongly compact fuzzy set is strongly T-compact with respect to any t-norm T.

Now we give a generalization of a fuzzy strongly compact fuzzy topological space as follows.

Definition 14 : *Let T be a t-norm. A fuzzy topological space* (X,τ) *is called strongly T-compact if and if all fuzzy constains in X are strongly T-compact.*

6. PS-open and PS-closed Fuzzy Sets

Definition 15 : *Let S be a t-conorm* [7]. *A fuzzy set A in a fuzzy topological space* (X,τ) *is called PS-open if and only if for any fuzzy singleton* $x_\delta \subseteq A$ *and any fuzzy element* 1_μ *in X, with* $\mu \in [0,1]$, *there exists a preopen fuzzy neighbourhood* $N_\mu(x_\delta)$ *such that* $N_\mu(x_\delta) \subseteq A \cup_s 1_\mu$. *A is called PS-closed if and only if C(A) is PS-open.*

Observe that any preopen (resp. preclosed) fuzzy set is *PS*-open (resp.*PS*-closed) with respect to any *t*-conorm *S*.

Proposition 6.1 : [4] *Any fuzzy set A in X can be decomposed in terms of the fuzzy singletons contained in it i.e.,* $A = \bigcup_{x_\delta \subseteq A} x_\delta$.

Proposition 6.2 : *Let S be a t-conorm. A fuzzy set A in a fuzzy topological space* (X,τ) *is PS-open if and only if for any fuzzy constant* 1_μ *in X, with* $\mu \in [0,1]$, *there exists a preopen fuzzy neighbourhood* $N_\mu(A)$ *such that* $N_\mu(A)A \subseteq A \cup 1_\mu$.

Proof: Let *A* be a *PS*-open and 1_μ be any fuzzy constant in *X*, with $\mu \in [0,1]$. For any fuzzy singleton $x_\delta \subseteq A$, there exists a preopen fuzzy neighbourhood $N_\mu(x_\delta)$ such that $N_\mu(x_\delta) \subseteq A \cup_s 1_\mu$. By Proposition 6.1, A can be decomposed in terms of the contained fuzzy singleton as follows :

$$A = \bigcup_{x_\delta \subseteq A} x_\delta \subseteq \bigcup_{x_\delta \subseteq A} N_\mu(x_\delta) \subseteq A \cup_s 1_\mu$$

This shows that we have obtained a preopen fuzzy neighbourhood $N_\mu(A)=\bigcup_{x_\delta \subseteq A} N_\mu(x_\delta)$ such that $N_\mu(x_\delta)\subseteq A\cup_s 1_\mu$. The converse can be proved by the same token.

Theorem 6.3 : *Let T be a t-norm. For any strongly T-compact fuzzy set A in a pre-separated fuzzy topological space* (X,τ) *and any fuzzy singletons* x_δ *in X such that* $x_\delta \subseteq C(A)$, *and any fuzzy constant* 1_μ *in X, with* $\mu\in[0,1]$, *there exist two preopen fuzzy neighbourhoods* $N_\mu(x_\delta)$ *and* $H(A\cap_T 1_\mu)$ *such that* $N_\mu(x_\delta)\subseteq C(H(A\cap_T 1_\mu))$.

Proof : Let A be a strongly T-compact fuzzy set in a pre-separated fuzzy topological space (X,τ) and x_δ be any fuzzy singleton in X such that $x_\delta\subseteq C(A)$. Assume that 1_μ is a fuzzy constant in X, with $\mu\in\,]0,1]$. By Lemma 4.1, we have $A\subseteq \bigcup_{y_\gamma\subseteq C(x_\delta)} y_\gamma = \bigcup_{x_\delta\subseteq C(x_\delta)} y_\gamma$. For any fuzzy singleton y_γ in X such that $x_\delta\subseteq C(y_\gamma)$, by Definition 4 there exist two preopen fuzzy neighbourhoods $N_\mu(x_\delta)$ and $H(y_\gamma)$ such that $N_\mu(x_\delta)\subseteq C(H(y_\gamma))$. Observe that $A\subseteq \bigcup_{y_\gamma\subseteq C(x_\delta)} H(y_\gamma)$. By the fact that A is strongly T-compact, there exists a finite family $(y_{i\vartheta})_{i-1}^{n}$ such that $A\cap_T 1_\mu\subseteq\bigcup_{i=1}^{n} H(y_{i\vartheta})$. Let $H(A\cap_T 1_\mu)=\bigcup_{i=1}^{n} H(y_{i\vartheta})$ and $N_\mu(x_\delta)=\bigcap_{i=1}^{n} N_{y_{i\vartheta}}(x_\delta)$, then we have

$$N_\mu(x_\delta)=\bigcap_{i=1}^{n} N_{y_{i\vartheta}}(x_\delta)\subseteq\bigcap_{i=1}^{n} C(H(y_{i\vartheta}))=C\left(\bigcup_{i=1}^{n} H(y_{i\vartheta})=C(H(A\cap_T 1_\mu))\right).$$

Recall that the dual t-conorm $T^*$of a T-norm T is defined by $T^*(x,y)=1-T(1-X,1-y)$ [7].

Corollary 6.4 : *Let T be a t-norm and its dual t-conorm T*. Any strongly T-compact fuzzy set in pre-separated fuzzy topological space is T*-preclosed.*

Proof : Let A be a strongly T-compact fuzzy set in a pre-separated fuzzy topological space (X,τ). For any fuzzy singleton x_δ in X such that $x_\delta \subseteq C(A)$ and any fuzzy constant 1_μ in X, with $\mu \in]0,1]$, by Theorem 6.3 there exists two fuzzy neighbourhoods $N_{1-\mu}(x_\delta)$ and $H(A \cap_T 1_{1-\mu})$ such that

$$N_{1-\mu}(x_\delta) \subseteq C(H(A \cap_T 1_{1-\mu})) \subseteq C(A \cap_T 1_{1-\mu}) = C(A) \bigcup_{T^*} 1_\mu$$

So we have proved that $C(A)$ is T^* preopen and therefore A is T^*-preclosed.

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