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ASYMPTOTIC STABILITY OF SOLUTIONS OF NONLINEAR INTEGRAL EQUATION OF MIXED TYPE

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Abstract

In this paper, we study the solvability of a nonlinear quadratic integral equation of mixed type. Here we use the technique of measure of noncompactness to prove that this equation has solutions on an unbounded interval. Moreover, the asymtotic behaviour of the solution is also studied.

Key words: Measures of noncomnpactness, Quadratic integral equation of mixed type, Fixed point theorem, Asymptotic stability.

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1. Introduction

Measures of noncompactness play very important role in nonlinear analysis. They are often applied to the theories of differential and integral equations as well as to the operator theory and gemetry of Banach spaces [2,3]. The concept of a measure of noncompactness was initiated by the fundamental papers of Kurtawoski and Darbo. Starting from 1970 there have appeared a lot of papers concerning that concept and its applications [5-11, 16,20].

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Integral equations create a very important and significant part mathematical analysis and their applications to real world proble [1,4,14,15,17,21]. The theory of integral equations is now well developed w help of various tools of functional analysis, topology and fixed point theo Moreover, such integral equations are also applied in the theory of neutron transpo as well in the kinetic theory of gases [12, 13, 15, 18, 19]. In this paper, we w investigate the existence and asymptotic behaviour of solutions of a nonlin quadratic integral equation of mixed type by using the technique of measure noncompactness and the Darbō fixed point theorem.

2. Preliminaries

Let $(E, \|.\|)$ be an infinite dimensional Banach space with zero elemen Denote by $B(x, r)$ the closed ball in E centered at x and with radius r. Let B_r sta for the ball $B(\theta,r)$. If X is a subset of E then $\bar{X}$, Conv X denotes the closure convex closure of X, respectively. Moreover, the symbol M_E denotes the famil all nonempty and bounded subsets of E while N_E stands for its subfamily consis of all relatively compact sets.

We will take the following definition of the concept of measur noncompactness [3].

Definition 2.1 : A mapping $\mu : M_E \to R_+ = [0, +\infty)$ is said to be a measur noncompactness in E if the following conditions are statisfied:

(i) The family $\ker\mu = \{X \in M_E : \mu(X) = 0\}$ is nonempty and $\ker\mu \subset N$

(ii) $X \subset Y \Rightarrow \mu(X) \leq \mu(Y)$;

(iii) $\mu(\mathrm{Conv}\, X) = \mu(X)$;

(iv) $\mu(\bar{X}) = \mu(X)$;

(v) $\mu(\lambda X + (1-\lambda)) \leq \lambda\mu(X) + (1-\lambda)\mu(Y)$ for $\lambda \in [0,1]$;

(vi) If (X_n) is a sequence of sets from M_E such that $X_{n+1} \subset X_n$, $\bar{X}_n = X_n (n=1,2,3\ldots)$

and if $\lim_{n\to\infty} \mu(X_n) = 0$, then the intersection $X_\infty = \bigcap_{n=1}^{\infty} X_n$ is nonempty.

The family $\ker\mu$ described in (i) is called the kernel of the measure of noncompactness μ.

A measure μ is said to be sublinear if it satisfies the following two conditions:

(vii) $\mu(\lambda X) = |\lambda| \mu(X)$ for $\lambda \in R$;

(viii) $\mu(X+Y) \le \mu(X) + \mu(Y)$.

Moreover, a measure μ satisfying the condition

(ix) $\mu(X \cup Y) = \max\{\mu(X), \mu(Y)\}$ will be referred to as a measure with maximum property.

Other facts concerning measures of noncompactness and their properties may be found [3]. For our purpose we will need the following fixed point theorem due to Darbo.

Theorem 2.1: [3]. Let Q be nonempty bounded closed convex subset of the space E and let $G : Q \to Q$ be a continuous operator such that $\mu(GX) \le k\mu(X)$ for any nonempty subset X of Q where k is a constant, $k \in [0, 1)$. Then G has a fixed point in the set Q.

Remark 2.1 : Under the assumptions of the above theorem it can be shown that the set Fix G of fixed points of G belonging to Q is a member of the family $\ker\mu$.

This observation allows us to characterize solutions of the equation investigated.

In the sequel we will work in the Banach space $BC(R_+)$ consisting of all real functions defined, bounded and continuous on R_+. The space $BC(R_+)$ is equipped with the standard norm $\|x\| = \sup\{|x(t)| : t \ge 0\}$.

Fix a nonempty bounded subset X of $BC(R_+)$ and a positive number $T > 0$. For $x \in X$ and $\varepsilon \ge 0$ let us denote by $\omega^T(x,\varepsilon)$ the modulus of continuity of the function

x on the interval $[0,T]$, i.e., $\omega^T(x,\varepsilon) = \sup\{|x(t)-x(s)| : t,s \in [0,T], |t-s| \le \varepsilon\}$

Further, let us put

$$\omega^T(X,\varepsilon) = \sup\{\omega^T(x,\varepsilon) : x \in X\}$$

$$\omega_0^T = \lim_{\varepsilon \to 0} \omega^T(x,\varepsilon), \quad \omega_0(X) = \lim_{T \to 0} \omega_0^T(X)$$

For a fixed number $t \ge 0$ we denote

$$X(t) = \{x(t) : x \in X\}$$

and

$$\operatorname{diam} X(t) = \sup\{|x(t)-y(t)| : x, y \in X\}.$$

Finally, let us define the function μ on the family $M_{BC(R+)}$ by the formu

$$\mu(X) = \omega_0(X) + \lim_{t \to \infty} \sup \operatorname{diam} X(t).$$

It can be shown [3] that the function μ is a sublinear measure noncompactness with the maximum property in the space $BC(R_+)$. The kerne ke of this measure contains nonempty and bounded sets X such that functions from are locally equicontunuous on R_+ and the thickness of the bundle formed by functic from X tends to zero at infinity. This property of the kernel kerμ allows us characterize (in terms of asymptotic behavior) solutions of the following integ equation and will be used in the rest of the paper.

3. Main Result

Consider the following nonlinear functional-integral equation of mixed ty

$$x(t) = f(t,x(t)) + F\left(t, x(t), \int_0^t g(t,s,x(s))ds, \int_0^a h(t,s,x(s))ds\right), \quad t \ge 0 \qquad \ldots (1)$$

We will assume that the functions involved in (1) satisfy the following conditio

(H1) $f: R_+ \times R \to R$ is a continuous function $f(t,0) \in BC(R_+)$;

(H2) There exists a continuous function $m(t) : R_+ \to R_+$ such that $|f(t,x) - f(t,y)| \leq m(t)|x - y|$ for all $x, y \in R$, $t \in R_+$;

(H3) $F: R_+ \times R \times R \times R \to R$ is a continuous function $F(t,0,0,0) \in BC(R_+)$;

(H4) There exists continuous functions $n_1(t)$, $n_2(t)$, $n_3(t) : R_+ \to R_+$ such that $|F(t,x_1,y_1,z_1) - F(t,x_2,y_2,z_2)| \leq n_1(t)|x_1 - x_2| + n_2(t)|y_1 - y_2| + n_3(t)|z_1 - z_2|$ for $x_i, y_i, z_i \in R$, i = 1 to 2, $t \in R_+$;

(H5) $g : R_+ \times R_+ \times R \to R$ is a continuous function such that $\lim_{t \to \infty} n_2(t)\int_0^t |g(t,s,x(s))|\, ds = 0$ uniformly with respect to $x \in BC(R_+)$;

(H6) $h : R_+ \times R_+ \times R \to R$ is a continuous function and there exists continuous functions $p, q, u, v : R_+ \to R_+$ such that

$$|h(t,s,x) - h(t,s,y)| \leq p(t)q(s)|x - y|,$$

$$|h(t,s,0)| \leq u(t)v(s),$$

$$\lim_{t \to \infty} n_3(t)u(t) = 0$$

for all $t \in R_+$, $x \in R$;

Take

$$Q = \int_0^a q(s)ds,$$

$$V = \int_0^a v(s)ds,$$

(H7) Let

$$k = \sup_{t \in R_+} \{m(t) + n_1(t) + n_3(t)\, p\, (t) Q\} \text{ be such } 0 \le k < 1.$$

Remark 3.1: The concept of the asymptotic stability of a solution is understood i the following sense [5,6].

For any $\varepsilon > 0$ there exist $T > 0$ and $r > 0$ such that if $x, y \in B_r$ and $x = x(t$ $y = y(t)$ are solutions of (1) then $|x(t) - y(t)| \le \varepsilon$ for $t \ge T$.

Theorem 3.1: Suppose the hypotheses (H1) - (H7) hold. Then (1) has atleast or solution $x(t)$ which belongs to the space $BC(R_+)$ and is asymptotically stable on th interval R_+.

Proof:

Consider the operator P defined on the space $BC(R_+)$ by the formula

$$(Px)(t) = f(t, x(t)) + F(t, x(t), \int_0^t g(t,s,x(s))ds, \int_0^a h(t,s,x(s))ds),\ t \ge 0,$$

From the assumptions it is clear that, the function P_x is continuous on the interv R_+ for any function $x \in BC(R_+)$.

From our assumptions, we get:

$$|(Px)(t)| \le |f(t,x(t))| + |F(t,x(t), \int_0^t g(t,s,x(s))ds, \int_0^a h(t,s,x(s))ds)|$$

$$\le |f(t,x(t)) - f(t,0)| + |F(t,x(t), \int_0^t g(t,s,x(s))ds, \int_0^a h(t,s,x(s))ds)$$

$$- F(t,0,0,0)| + |f(t,0)| + |F(t,0,0,0)|$$

$$\leq \left[m(t) + n_1(t)\right] x(t) + n_2(t)\int_0^t g(t,s,x,(s))\, ds$$

$$+ n_3(t)\int_0^a h(t,s,x(s))\, ds + f(t,0) + F(t,0,0,0)$$

$$\leq \left[m(t) + n_1(t)\right] |x(t)| + n_2(t)\int_0^t |g(t,s,x,(s))|\, ds$$

$$+ n_3(t)\int_0^a |h(t,s,x(s)) - h(t,s,0)| ds + n_3(t)\int_0^a |h(t,s,0)| ds$$

$$+ |f(t,0)| + |F(t,0,0,0)|$$

$$\leq \left[m(t) + n_1(t)\right] |x(t)| + n_2(t)\int_0^t |g(t,s,x,(s))|\, ds$$

$$n_3(t)p(t)\int_0^a q(s)\, |x| ds + n_3(t)u(t)\int_0^a v(s) ds$$

$$+ |f(t,0)| + |F(t,0,0,0)|$$

The above estimate allows us to infer that the function Px is bounded on the interval R_+. Thus $Px \in BC(R_+)$.

Moreover, we obtain

$$\|Px\| \leq k\|x\| + A \qquad \ldots (2)$$

where

$$A = \sup\left\{ n_2(t)\int_0^t |\, g(t,s,x(s))\,|\, ds + n_3(t)u(t)V + |\, f(t,0)\,| + |\, F(t,0,0,0)\,| : t \geq 0 \right\}.$$

Obviously $A < \infty$ by virtue of the assumptions (H1), (H3), (H5) and (H6). Sinc $k < 1$, from (2), the operator P transforms B_r into itself for $r = A/(1-k)$.

Next we show that P is continuous on the ball B_r. For this let us fix $\varepsilon >$ and take $x, y \in B_r$ such that $\|x - y\| \leq \varepsilon$. Then for arbitrarily fixed $t \in R_+$, we get

$$
\begin{aligned}
|(Px)(t) - (Py)(t)| \leq & \, |f(t,x(t)) - f(t,y(t))| \\
& + |F(t,x(t), \int_0^t g(t,s,x(s))ds, \int_0^a h(t,s,x(s))ds) \\
& - F(t,y(t), \int_0^t g(t,s,y(s))ds, \int_0^a h(t,s,y(s))ds)| \qquad \ldots (3) \\
& \leq [m(t) + n_1(t)]|x(t) - y(t)| \\
& + n_2(t)\int_0^t |g(t,s,x(s)) - g(t,s,y(s))|ds \\
& + n_3(t)\int_0^a |h(t,s,x(s)) - h(t,s,y(s))|ds \\
& \leq [m(t) + n_1(t)]|x(t) - y(t)| \\
& + n_2(t)\int_0^t |g(t,s,x(s)) - g(t,s,y(s))|ds \\
& + n_3(t)p(t)\int_0^a q(s)|x - y|ds \qquad \ldots (4)
\end{aligned}
$$

Now denote by T a real positive number such that

$$
+ n_2(t)\int_0^t |g(t,s,x(s))|ds \leq (1-k)\varepsilon/2 \qquad \ldots (5)
$$

for $t \geq T$.

Consider two cases:

(α) $t \geq T$. Then from (4) and (5), we obtain

$$\left|(Px)(t)-(Py)(t)\right| \leq k\varepsilon+(1-k)\varepsilon/2+(1-k)\varepsilon/2=\varepsilon$$

(β) $t < T$. Then let $\omega = \omega(\varepsilon)$ be the function defined by the formula

$$\omega(\varepsilon)=\sup\{|\, g(t,s,x)-g(t,s,y)\,|: t,s \in [0.T], x,y \in [-r,r], |\, x-y\,| \leq \varepsilon\}.$$

Taking into account that the function g = g(t,s,x) is uniformly continuous on the set $[0,T] \times [0,T] \times [-r, r]$, we deduce that $\omega(\varepsilon) \to 0$ as $\varepsilon \to 0$.

Thus virtue of (4), we get

$$\left|(Px)(t)-(Py)(t)\right| \leq k\varepsilon+\sup\{n_2(t): t \in [0,T]\}\, T\omega(\varepsilon)$$

Now, linking cases (α) and (β), we can deduce the operator P is continuous on the ball B_r.

Take a nonempty set $X \subset B_r$. Then, for x, $y \in X$, and for a fixed $t \geq 0$, calculating in the same way as in the proof of estimate (3), we obtain

$$\left|(Px)(t)-(Py)(t)\right| \leq [m(t)+n_1(t)]\left|x(t)-y(t)\right|$$

$$+n_2(t)\int_0^t \left|g(t,s,x(s))-g(t,s,y(s))\right| ds$$

$$+n_3(t)p(t)\int_0^a q(s)\left|x-y\right| ds$$

Hence we can easily deduce,

$$\operatorname{diam}(PX)(t) \leq k \operatorname{diam} X(t)$$

$$+\sup_{x,y \in X}\{n_2(t)[\int_0^t |\, g(t,s,x(s))\,|\, ds+\int_0^t |\, g(t,s,y(s))\,|\, ds]\}$$

Now by our assumptions, we get

$$\limsup_{t \to \infty} \operatorname{diam} (PX)(t) \le k \limsup_{t \to \infty} \operatorname{diam} X(t). \qquad \ldots (6)$$

Further, fix arbitrarily $T > 0$ and $\varepsilon > 0$. Choose a function $x \in X$ and take $t, s \in [0,T$ such that $|t - s| \le \varepsilon$. Without loss of generality, we may assume that $s < t$. Then

$$\begin{aligned}
|(Px)(t) - (Px)(s)| \le{}& |f(t,x(t)) - f(s,x(s))| \\
&+ \Big| F\Big(t,x(t),\int_0^t g(t,\tau,x(\tau))d\tau, \int_0^a h(t,\tau,x(\tau))d\tau\Big) \\
&- F\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau, \int_0^a h(s,\tau,x(\tau))d\tau\Big)\Big| \\
\le{}& |f(t,x,(t)) - f(t,x(s))| + |f(t,x(s)) - f(s,x(s))| \\
&+ \Big| F\Big(t,x(t),\int_0^t g(t,\tau,x(\tau))d\tau, \int_0^a h(t,\tau,x(\tau))d\tau\Big) \\
&- F\Big(t,x(s),\int_0^s g(s,\tau,x(\tau))d\tau, \int_0^a h(s,\tau,x(\tau))d\tau\Big)\Big| \\
&+ \Big| F\Big(t,x(s),\int_0^s g(s,\tau,x(\tau))d\tau, \int_0^a h(s,\tau,x(\tau))d\tau\Big) \\
&- F\Big(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau, \int_0^a h(s,\tau,x(\tau))d\tau\Big)\Big|
\end{aligned}$$

$$\leq [m(t)+n_1(t)]\left|x(t)-x(s)\right| + \left|f(t,x(s))-f(s,x(s))\right|$$

$$+n_2(t)\left|\int_0^t g(t,\tau,x(\tau))d\tau - \int_0^s g(s,\tau,x(\tau))d\tau\right|$$

$$+n_3(t)\int_0^a \left|h(t,\tau,x(\tau))-h(s,\tau,x(\tau))\right| d\tau$$

$$+\left|F(t,x(s),\int_0^s g(s,\tau,x,(\tau))d\tau,\int_0^a h(s,\tau,x(\tau))d\tau)\right.$$

$$\left.-F(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau,\int_0^a h(s,\tau,x(\tau))d\tau\right|$$

$$\leq [m(t)+n_1(t)+n_3(t)p(t)Q]\left|x(t)-x(s)\right|$$

$$+\left|f(t,x(s))-f(s,x(s))\right| + n_2(t)\int_s^t \left|g(t,\tau,x(\tau))\right| d\tau$$

$$+n_2(t)\int_0^s \left|g(t,\tau,x(t))-g(s,\tau,x(\tau))\right| d\tau$$

$$+n_3(t)\int_0^a \left|h(t,\tau,x(t))-h(s,\tau,x(\tau))\right| d\tau$$

$$+\left|F(t,x(s),\int_0^s g(s,\tau,x(\tau))d\tau,\int_0^a h(s,\tau,x(\tau))d\tau)\right.$$

$$\left.-F(s,x(s),\int_0^s g(s,\tau,x(\tau))d\tau,\int_0^a h(s,\tau,x(\tau))d\tau)\right|$$

Taking supremum, we get

$$\omega^T(Px,\varepsilon)\leq k\omega^T(x,\varepsilon)+\omega_r^T(f,\varepsilon)+\varepsilon\, n_2(t)\sup\{|\,g(t,\tau,x(\tau))\,|:t,\tau\in[0,T],|\,x\,|\leq$$

$$+T\,n_2(t)\,\omega_r^T(g,\varepsilon)+a\,n_3(t)\,\omega_r^T(h,\varepsilon)+\omega_r^T(F,\varepsilon)$$

where

$$\omega_r^T(g,\varepsilon)=\sup\{|\,g(t,\tau,x)-g(s,\tau,x)\,|:t,s,\tau\in[0,T],|\,t-s\,|\leq\varepsilon,|\,x\,|\leq r\}$$

$$\omega_r^T(h,\varepsilon)=\sup\{|\,h(t,\tau,x)-h(s,\tau,x)\,|:t,s,\tau\in[0,T],|\,t-s\,|\leq\varepsilon,|\,x\,|\leq r\}$$

$$\omega_r^T(F,\varepsilon)=\sup\{|\,F(t,x,y,z)-F(s,x,y,z)\,|:t,s,\tau\in[0,T],|\,t-s\,|\leq\varepsilon,$$

$$|\,x\,|\leq r,|\,y\,|\leq M,|\,z\,|\leq N\}$$

$$M=\sup\{\int_0^s\ |\,g(s,\tau,x(\tau))\,|\,d\tau:s,\tau\in[0,T],|\,x\,|\leq r\}$$

$$N=\sup\{\int_0^a\ |\,h(s,\tau,x(\tau))\,|\,d\tau:s,\tau\in[0,T],|\,x\,|\leq r\}$$

Applying the assumptions, we infer easily that the functions $g = g(t,\tau$ and $h = h(t,\tau,x)$ are uniformly continuous on the set $[0,T] \times [0,T] \times [-r, r]$, while functions $f=f(t,x)$ and $F = F(t,x,y,z)$ are uniformly continuous on $[0,T] \times [-r,r]$ a $[0,T] \times [-r,r] \times [-M,M] \times [-N,N]$ respectively.

Hence we deduce that $\omega_r^T(g,\varepsilon)\to 0, \omega_r^T(h,\varepsilon)\to 0$, $\omega_r^T(f,\varepsilon)\to 0$ a $\omega_r^T(F,\varepsilon)\to 0$ as $\varepsilon \to 0$.

Now, from the above estimate, we get

$$\omega_0^T(PX)\leq k\omega_0^T(X)$$

and, further

$$\omega_0(PX)\leq k\omega_0(X) \qquad \ldots (7)$$

Now, linking (6) and (7), we arrive at the following inequality:

$$\mu(PX) \le k\mu(X) \qquad \ldots (8)$$

The above inequality in conjunction with Theorem. 2.1 allows us to deduce that there exists a solution $x = x(t)$ of (1) in the space $BC(R_+)$. Moreover, in view of (3) and the definition of asymptotic stability, we infer that $x(t)$ is asymptotically stable on the interval R_+.

Remark 3.2: Observe that the information about the asymptotic stability of the solution $x = x(t)$ of (1) can also be deduced from the fact that the set of all solutions of (1) belongs to $\ker\mu$. Keeping in mind the description of the kernel of the measure of noncompactness μ, we obtain that every solution $x = x(t)$ of (1) is asymptotically stable.

4. Example

Consider the following functional-integral equation:

$$x(t) = \frac{t}{1+t^2}x(t) + \frac{1}{4}\operatorname{In}(1+|x(t)| + \sin t\int_0^t \frac{1}{4+t^3}\exp(-s-x^2(s))\,ds$$

$$+\cos(t\,|x(t)|)\int_0^1 \frac{\exp(-t-s)}{1+|x(s)|}ds, t \in R_+ \qquad \ldots (9)$$

The above equation takes the form of (1) with $g(t,s,x(s)) = \frac{1}{4+t^3}\exp(-s-x^2(s))$,

$h(t,s,x(s)) = \frac{\exp(-t-s)}{1+|x(s)|}, f(t,x) = \frac{t}{1+t^2}x(t)$ and $a = 1$

Let us observe that

$$|f(t,x) - f(t,y)| \le \frac{t}{1+t^2}|x-y|,$$

$$|F(t,x_1,y_1,z_1) - F(t,x_2,y_2,z_2)| \le \frac{1}{4}|x_1 - x_2| + t|y_1 - y_2| + |z_1 - z_2|$$

Now, it is easily seen that the assumptions (H2) and (H4) are satisfied

$$m(t) = \frac{t}{1+t^2},\ n_1(t) = \frac{1}{4},n_2(t) = t, n_3(t) = 1.$$

Also, we have

$$\left|g(t,s,x(s))\right| \le \frac{1}{4+t^3},$$

$$\left|h(t,s,0)\right| \le \exp(-t-s),$$

$$\left|h(t,s,x) - h(t,s,y)\right| \le \exp(-t-s)\left|x-y\right|.$$

It is clear that (9) satisfies assumptions ($H5$), ($H6$) and ($H7$) $p(t) = u(t) = \exp(-t)$ and $q(s) = v(s) = \exp(-s)$.

Taking into account the above established facts and applying Theore we infer that (9) has atleast one solution $x(t)$ which belongs to the space B and is asymptotically stable on the interval R_+.

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