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# ALMOST CONTRA-$\Omega^*g\alpha$-CONTINUOUS FUNCTIONS

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## Abstract

The notion of contra continuous functions was introduced by Dontchev. In this paper we apply the notion of $\Omega^*$-open sets in topological space to present and study a new class of functions called almost contra-$\Omega^*g\alpha$-continuous functions as a new generalization of contra continuity. Futhermore, we obtain basic properties and preservation theorems of almost contra-$\Omega^*g\alpha$-continuity and investigate the relationship between almost contra-$\Omega^*g\alpha$-continuity and $\Omega^*g\alpha$-regular graph.

***Key words:*** M-$\Omega^*g\alpha$-closed map, Almost contra-$\Omega^*g\alpha$-continuity, $\Omega^*g\alpha$-regular graph.

## 1. Introduction

Dontchev [3] introduced the notions of contra-continuity in topological spaces. He defined a function $f: X \to Y$ is contra continuous if the preimage of every open set of $Y$ is closed in $X$. Recently Ganster and Reilly [6] introduced a new class

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of functions called regular set connected functions (in 1999). Jafari and Noiri [7] introduced contra-pre-continuous functions. Almost contra-pre-continuous functions were introduced by Ekici [4]. J. Mercy and I. Arockiarani [12] introduced On $\Omega$*-closed sets and $\Omega p$-closed sets in topological spaces. In this paper we introduce and study a new class of functions called almost contra-$\Omega^* g\alpha$-continuous functions which generalize classes of regular set connected [6] contra continuous [3] and perfectly continuous [13] functions. Moreover, the relationship between almost contra-$\Omega^*$-$g\alpha$-continuity and $\Omega^* g\alpha$-regular graphs are also investigated.

## 2. Preliminaries

Throughout this paper, spaces $(X,\tau)$ and $(Y,\sigma)$ or (Simply $X$ and $Y$) always mean topological spaces on which no separation axioms are assumed unless explicity stated. For a subset $A$ of $(X,\tau)$, cl($A$) and int($A$) represent the closure of $A$ and interior of $A$ with respect to $\tau$ respectively.

**Definition 2.1:** A subset A of a topological space $(X,\tau)$ is said to be preopen [11] (resp. preclosed) if $A \subset \text{Int}(\text{cl}(A))$ (resp.$\text{cl}(\text{int}(A) \subset A)$).

**Definition 2.2:** A subset $A$ of a topological space $(X,\tau)$ is said to be regular open [15] (resp. regular closed) if $A = \text{int}(cl(A))$ (resp. $A = cl(\text{int}(A))$).

**Definition 2.3:** A subset $A$ of a topological space $(X,\tau)$ is said to be $\alpha$-closed [14] (resp. $\alpha$-closed) if $Cl(\text{Int}(Cl(A))) \subset A$ (resp. $A \subset \text{Int}(Cl(\text{Int}(A)))$.

**Definition 2.4:** The intersection of all $\alpha$-closed sets containing $A$ is called $\alpha$-closure of $A$ and is denoted by $\alpha$-cl($A$).

**Definition 2.5:** The $\alpha$-interior of $A$ is defined by the union of $\alpha$-open sets contained in $A$ and is denoted by $\alpha$-int($A$).

**Definition 2.6:** A subset $A$ of a topological space $(X,\tau)$ is said to be generalized $\alpha$-closed set [10] (briefly $g\alpha$-closed) if $\alpha\text{-cl}(A) \subset U$ whenever $A \subset U$ and $U$ is $\alpha$-open.

**Definition 2.7:** A function $f: (X,\tau) \to (Y,\sigma)$ is called

1. Contra-continuous [3] if $f^{-1}(V)$ is closed in $(X,\tau)$ for every open set $V$ of $(Y,\sigma)$.
2. Regular set connected [6] if $f^{-1}(V)$ is clopen in $X$ for every $V \varepsilon RO(Y)$.
3. Perfectly-continuous [13] if $f^{-1}(V)$ is both open and closed in $(X,\tau)$ for every open set $V$ of $(Y,\sigma)$.
4. Almost-continuous [16] if $f^{-1}(V)$ is open in $X$ for every regular open set $V$ of $(Y,\sigma)$.

**Definition 2.8:** A subset $A$ of a topological space $(X,\tau)$ is said to be $\pi g\alpha$-closed [1] if $\alpha$-cl$(A) \subset U$ whenever $A \subset U$ and $U$ is $\pi$-open.

**Definition 2.9:** A function $f: (X,\tau) \to (Y,\sigma)$ is called $\pi g\alpha$-continuous [2] if $f^{-1}(V)$ is $\pi g\alpha$-open in $(X,\tau)$ for every open set $V$ of $(Y,\sigma)$.

**Definition 2.10:** A function $f : (X,\tau) \to (Y,\sigma)$ is said to be almost contra-$\pi g\alpha$-continuous [8] if $f^{-1}(V) \varepsilon\ \pi G\alpha C(X,\tau)$ for every $V \varepsilon RO(Y,\sigma)$.

**Definition 2.11:** A subset $A$ of a topological space $(X,\tau)$ is said to be $\Omega^*$-closed [12] if pcl$(A) \subset$ Int$(U)$, whenever $A \subset U$ and $U$ is pre-open in $(X,\tau)$.

## 3. Almost Contra-$\Omega^*g\alpha$-Continuous Functions

**Definition 3.1:** A subset $A$ of a topological space $(X,\tau)$ is said to be

(a) $\Omega^*g\alpha$-closed if $\alpha$-cl$(A) \subset U$ whenever $A \subset U$ and $U$ is $\Omega^*$-open.

(b) $\Omega^*g\alpha$-open if $X$-$A$ is $\Omega^*g\alpha$-closed.

The family of all $\Omega^*g\alpha$-closed sets of $X$ (resp. $\Omega^*g\alpha$-open sets) are denoted by $\Omega^*G\alpha C(X,\tau)$ (resp. $\Omega^*G\alpha O(X,\tau)$).

**Definition 3.2:** A function $f: (X,\tau) \to (Y,\sigma)$ is called

1. $\Omega^*g\alpha$-continuous if $f^{-1}(V)$ is $\Omega^*g\alpha$-open in $(X,\tau)$ for every open set $V$ of $(Y,\sigma)$.
2. Almost-$\Omega^*g\alpha$-continuous if $f^{-1}(V)$ is $\Omega^*g\alpha$-open in $X$ for every regular open set $V$ of $(Y,\sigma)$.

3. Contra-$\Omega^*g\alpha$-continuous if $f^{-1}(V)$ is $\Omega^*g\alpha$-closed in $(X,\tau)$ for every open set $V$ of $(Y,\sigma)$.
4. M-$\Omega^*g\alpha$-open (resp. M-$\Omega^*g\alpha$-closed) if image of each $\Omega^*g\alpha$-open set (resp. $\Omega^*g\alpha$-closed) is $\Omega^*g\alpha$-open (resp. $\Omega^*g\alpha$-closed).

**Definition 3.3:** A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be almost contra-$\Omega^*g\alpha$-continuous if $f^{-1}(V) \varepsilon \Omega^*G\alpha C(X,\tau)$ for every $V \varepsilon RO(Y,\sigma)$.

**Theorem 3.4:** Let $(X,\tau)$ and $(Y,\sigma)$ be topological spaces. The following statements are equivalent for a function $f : X \rightarrow Y$.

1. $f$ is almost contra-$\Omega^*g\alpha$-continuous.
2. $f^{-1}(F) \varepsilon \Omega^*G\alpha O(X,\tau)$ for every $F \varepsilon RC(Y,\sigma)$.
3. for each $x \varepsilon X$ and each regular closed set $F$ in $Y$ containing $f(x)$, there exists a $\Omega^*g\alpha$-open set $U$ in $X$ containing $x$ such that $f(U) \subset F$.
4. for each $x \varepsilon X$ and each regular open set $V$ in $Y$ not containing $f(x)$, there exists a $\Omega^*g\alpha$-closed set $K$ in $X$ not containing $x$ such that $f^{-1}(V) \subset K$.
5. $f^{-1}(\text{int}(\text{cl}(G)) \varepsilon \Omega^*G\alpha C(X,\tau)$ for every open subset $G$ of $Y$.
6. $f^{-1}(\text{cl}(\text{int}(F)) \varepsilon \Omega^*G\alpha O(X,\tau)$ for every closed subset $F$ of $Y$.

**Proof:** (1)⇒(2) : Let $F \varepsilon RC(Y)$. Then $Y\text{-}F \varepsilon RO(Y,\sigma)$. By (1), $f^{-1}(Y\text{-}F) = X\text{-}f^{-1}(F) \varepsilon \Omega^*G\alpha C(X,\tau)$. This implies $f^{-1}(F) \varepsilon \Omega^*G\alpha O(X,\tau)$.

(2)⇒(1) : Let $V \varepsilon RO(Y,\sigma)$. Then $Y\text{-}V \varepsilon RC(Y,\sigma)$. By (2) $f^{-1}(Y\text{-}V) = X\text{-}f^{-1}(V) \varepsilon \Omega^*G\alpha O(X,\tau)$. This implies $f^{-1}(V) \varepsilon \Omega^*G\alpha C(X,\tau)$.

(2)⇒(3) : Let $F$ be any regular closed set in $Y$ containing $f(x)$. By(2), $f^{-1}(F) \varepsilon \Omega^*G\alpha O(X,\tau)$ and $x \varepsilon f^{-1}(F)$. Take $U = f^{-1}(F)$. Then $f(U) \subset F$.

(3)⇒(2) : Let $F \varepsilon RC(Y,\sigma)$ and $x \varepsilon f^{-1}(F)$. From (3), there exists a $\Omega^*g\alpha$-open set $Ux$ in $X$ containing $x$ such that $Ux \subset f^{-1}(F)$. We have $f^{-1}(F) = \bigcup_{x \varepsilon f^{-1}(F)} Ux$. Thus, $f^{-1}(F)$ is $\Omega^*g\alpha$-open.

(3)⇒(4) : Let $V$ be a regular open set in $Y$ not containing $f(x)$. Then $Y\text{-}V$ is a regular closed set containing $f(x)$. By (3) there exists a $\Omega^*g\alpha$-open set $U$ in $X$ containing $x$ such that $f(U) \subset Y\text{-}V$. Hence $U \subset f^{-1}(Y\text{-}V) \subset X\text{-}f^{-1}(V)$ and then $f^{-1}(V) \subset X\text{-}U$. Take $K = X\text{-}U$. We obtain a $\Omega^*g\alpha$-closed set $K$ in $X$ not containing $x$.

(4)⇒(3) : Let $F$ be regular closed set in $Y$ containing $f(x)$. Then $Y\text{-}F$ is a regular open set in $Y$ not containing $f(x)$. By (4) there exist a $\Omega^*g\alpha$-closed set $K$ in $X$ not containing $x$ such that $f^{-1}(Y\text{-}F) \subset K$. This implies $X\text{-}f^{-1}(F) \subset K \Rightarrow X\text{-}K \subset f^{-1}(F) \Rightarrow f(X\text{-}K) \subset F$. Take $U = X\text{-}K$. Then $U$ is a $\Omega^*g\alpha$-open set in $X$ containing $x$ such that $f(U) \subset F$.

(1)⇒(5) : Let $G$ be an open subset of $Y$. Since int(cl($G$)) is regular open, then by (1) $f^{-1}(\text{int}(\text{cl}(G)))\ \varepsilon\ \Omega^*G\alpha C(X,\tau)$.

(5)⇒(1) : Let $V\ \varepsilon\ RO(Y,\sigma)$. Then $V$ is open in $Y$. By (5) $f^{-1}$ (int(cl($V$))) $\varepsilon$ $\Omega^*G\alpha C(X,\tau) \Rightarrow f^{-1}(V)\ \varepsilon\ \Omega^*g\alpha$-closed in $(X,\tau)$.

(2)⇒(6) : The proof is obvious from the definitions.

**Remark 3.5:** The following diagram holds.

| | | | | |
|---|---|---|---|---|
| Perfectly continuous | ⇒ | Contra continuous | ⇒ | Contra-$\Omega^*g\alpha$-continuous |
| ⇓ | | | | ⇓ |
| Regular set connected | | ⇒ | | Almost Contra $\Omega^*g\alpha$-continuous |

None of the implications is reversible for almost Contra $\Omega^*g\alpha$-continuity as shown by the following examples.

**Example 3.6:** Let $X = \{a,b,c\}$, $\tau = \{\Phi, X, \{a\}\}$ and $\sigma = \{\Phi, X, \{b\}, \{c\}, \{b,c\}\}$. Then the identity function $f: (X,\tau) \rightarrow (X,\sigma)$ is almost contra-$\Omega^*g\alpha$-continuous but not regular set connected.

**Example 3.7:** Let $X = \{a,b,c,d\}$, $\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,d\}, \{a,c,d\}\}$ and $\sigma = \{X, \Phi, \{a\}, \{a,b\}, \{a,c,d\}\}$. Then the identity function $f : (X,\tau) \rightarrow (X,\sigma)$ is almost contra-$\Omega^*g\alpha$-continuous but not contra-$\Omega^*g\alpha$-continuous.

**Example 3.8:** Let $X = \{a,b,c\}$, $\tau = \{X, \Phi, \{a,b\}\}$ and $\sigma = \{X, \Phi, \{a\}, \{a,b\}\}$. Then the identity function $f : (X,\tau) \rightarrow (X,\sigma)$ is contra-$\Omega^*g\alpha$-continuous but not contra-continuous.

**Theorem 3.9 :** Suppose that $\Omega^*g\alpha$-closed sets are closed under finite intersection. If $f : X \rightarrow Y$ is almost contra-$\Omega^*g\alpha$-continuous function and $A$ is $\Omega^*g\alpha$-open subset of $X$, Then the restriction $f/A$: $A \rightarrow Y$ is almost contra-$\Omega^*g\alpha$-continuous.

**Proof:** Let $F \ \varepsilon \ RC(Y)$. Since $f$ is almost contra-$\Omega^*g\alpha$-continuous then $f^{-1}(V) \ \varepsilon \ \Omega^*G\alpha O(X,\tau)$. Since $A$ is $\Omega^*g\alpha$-open in $X$ if follow that $(f/A)^{-1}(F) = A \cap f^{-1}(F)) \ \varepsilon \ \Omega^*G\alpha O(A,\tau)$. Therefore, $f/A$ is almost contra-$\Omega^*g\alpha$-continuous function.

**Remark 3.10:** Every restriction of an almost contra-$\Omega^*g\alpha$-continuous function is not necessarily almost contra-$\Omega^*g\alpha$-continuous.

**Example 3.11:** Let $X = \{a,b,c,d\}$, $\tau = \{\Phi, X,\{a\},\{d\},\{a,d\},\{c,d\},\{a,c,d\}\}$ and $\sigma = \{\Phi, X,\{b\},\{c\},\{b,c\}\}$. Then the identity function $f : (X,\tau) \rightarrow (X,\sigma)$ is almost contra-$\Omega^*g\alpha$-continuous but if $A = \{a,b,c\}$, where $A$ is not $\Omega^*g\alpha$-open in $(X,\tau)$ and $\tau_A = \{\Phi,\{a,b,c\},\{a\},\{c\},\{a,c\}\}$ is the relative topology on $A$ induced by $\tau$, then $f/A$: $(A,\tau_A) \rightarrow (X,\sigma)$ is not almost contra-$\Omega^*g\alpha$-continuous. Note that $\{a,b,d\}$ is regular closed in $(X,\tau)$ but that $(f/A)^{-1}\{a,b,d\} = A \cap \{a,b,d\} = \{a,b,c\} \cap \{a,b,d\} = \{a,b\}$ is not $\Omega^*g\alpha$-open in $(A,\tau_A)$.

**Definition 3.12:** A cover $\Sigma = \{U\alpha : \alpha \ \varepsilon \ I\}$ of subsets of $X$ is called a $\Omega^*g\alpha$-cover if $U\alpha$ is $\Omega^*g\alpha$-open for each $\alpha \ \varepsilon \ I$.

**Theorem 3.13:** Suppose that $\Omega^*G\alpha O(X,\tau)$ sets are closed under finite intersection. Let $f : X \rightarrow Y$ be a function and $\Sigma = \{U\alpha : \alpha \ \varepsilon \ I\}$ be a $\Omega^*g\alpha$-cover of $X$. If for each $\alpha \ \varepsilon \ I$, $f/U\alpha$ is almost contra-$\Omega^*g\alpha$-continuous, then $f : X \rightarrow Y$ is almost contra-$\Omega^*g\alpha$-continuous.

**Proof:** Let $V \varepsilon RC(Y)$. Since $f/U\alpha$ is almost contra-$\Omega^*g\alpha$-continuous function, $(f/U\alpha)^{-1}(V) \varepsilon \Omega^*G\alpha O(U\alpha)$. Since $U\alpha \varepsilon \Omega^*G\alpha O(X)$, by the result if $U$ is $\Omega^*g\alpha$-open in $X$ and $V$ is $\Omega^*g\alpha$-open in $X$, it follows $(f/U\alpha)^{-1}(V)) \varepsilon \Omega^*G\alpha O(X)$ for each $\alpha \varepsilon I$. Then $f^{-1}(V) = \cup (f/U\alpha)^{-1}(V) \varepsilon \Omega^*G\alpha O(X)$. This gives $f$ is almost contra-$\Omega^*g\alpha$-continuous $\alpha \varepsilon I$ function.

**Theorem 3.14:** Let $f : X \to Y$ and let $g : X \to X \times Y$ be the graph function of $f$ defined by $g(x) = (x, f(x))$ for every $x \varepsilon \in X$. If $g$ is almost contra-$\Omega^*g\alpha$-continuous then $f$ is almost contra-$\Omega^*g\alpha$-continuous.

**Proof:** Let $V \varepsilon RC(Y)$, then $X \times V = X \times \text{cl}(\text{int}(V) = \text{cl}(\text{int}(X) \times \text{cl}(\text{int}(V) = \text{cl}(\text{int}(X \times V))$. Therefore $X \times V \varepsilon RC(X \times Y)$. Since $g$ is almost contra-$\Omega^*g\alpha$-continuous, $g^{-1}(X \times V) \varepsilon \Omega^*g\alpha$-open in $X$. This implies $f^{-1}(V) = g^{-1}(X \times V) \varepsilon \Omega^*g\alpha$-open in $X$. Thus, $f$ is almost contra-$\Omega^*g\alpha$-continuous.

**Theorem 3.15:** Let $f : X \to Y$ and $g : Y \to Z$ be function. Then, the following properties hold:

1) If $f$ is almost contra-$\Omega^*g\alpha$-continuous and $g$ is regular set connected, then $gof : X \to Z$ is almost contra-$\Omega^*g\alpha$-continuous and almost $\Omega^*g\alpha$-continuous.
2) If $f$ is almost contra-$\Omega^*g\alpha$-continuous and $g$ is perfectly continuous then $gof : X \to Z$ is $\Omega^*g\alpha$-continuous and contra-$\Omega^*g\alpha$-continuous.
3) If $f$ is almost contra-$\Omega^*g\alpha$-continuous and $g$ is regular set-connected then $gof : X \to Z$ is almost contra-$\Omega^*g\alpha$-continuous almost $\Omega^*g\alpha$-continuous.

**Proof:** Let $V \varepsilon RO(Z)$ Since $g$ is regular set connected $g^{-1}(V)$ is clopen in $Y$. Since $f$ is almost contra-$\Omega^*g\alpha$-continuous, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $\Omega^*g\alpha$-open and $\Omega^*g\alpha$-closed. Therefore $gof$ is almost contra-$\Omega^*g\alpha$-continuous and almost $\Omega^*g\alpha$-continuous. (2) and (3) can be obtained similarly.

**Theorem 3.16:** If $f : X \to Y$ is a surjective $M$-$\Omega^*g\alpha$-open and $g : X \to Z$ is a function such that $gof: X \to Z$ is almost contra-$\Omega^*g\alpha$-continuous, then $g$ is almost contra-$\Omega^*g\alpha$-continuous.

**Proof:** Let $V$ be any regular closed set in $Z$. Since gof is almost contra-Ω*$g\alpha$-continuous, $(gof)^{-1}(V))$ ε Ω*$g\alpha$-open in $(X,\tau)$. Since $f$ is surjective, $M$-Ω*$g\alpha$-open map, $f((gof)^{-1}(V)) = f(f^{-1}(g^{-1}(V)) = g^{-1}(V)$ is Ω*$g\alpha$-open. Therefore $g$ is almost contra-Ω*$g\alpha$-continuous.

**Theorem 3.17:** If $f : X \to Y$ is a surjective $M$-Ω*$g\alpha$-closed map and $g : X \to Z$ is a function such that $gof : X \to Z$ is almost contra-Ω*$g\alpha$-continuous, then $g$ is almost contra-Ω*$g\alpha$-continuous.

**Proof:** Similarly as the previous theorem.

**Theorem 3.18:** If a function $f : X \to Y$ is almost contra-Ω*$g\alpha$-continuous and almost continuous then $f$ is regular set-connected.

**Proof:** Let $V$ ε $RO(Y)$. Since $f$ is almost contra-Ω*$g\alpha$-continuous and almost continuous $f^{-1}(V)$ is Ω*$g\alpha$-closed and open. Hence $f^{-1}(V)$ is clopen. Hence $f$ is regular set-connected.

**Definition 3.19:** A filter base Λ is said to be Ω*$g\alpha$-convergent (resp. rc-convergent) to a point $x$ in $X$ if for any $U$εΩ*$g\alpha$-open in $X$ containing $x$ (resp. $U$ε$RC(X)$) there exist a $B$ ε Λ Such that $B \subset U$.

**Theorem 3.20:** If a function $f : X \to Y$ is almost contra-Ω*$g\alpha$-continuous, then for each point $x$ε$X$ and each filter base Λ in $X$ Ω*$g\alpha$-converging to $x$, the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.

**Proof:** Let $x$ ε $X$ and Λ be any filter base in $X$ Ω*$g\alpha$-converging to $x$. Since $f$ is almost contra-Ω*$g\alpha$-continuous then for any $V$ε$RC(Y)$ containing $f(x)$ there exist $U$ εΩ*$g\alpha$-open in $X$ containing $x$ such that $f(U) \subset V$. Since Λ is Ω*$g\alpha$-converging to $x$, there exist a $B$ ε Λ such that $B \subset U$. This means that $f(B) \subset V$ and therefore the filter base $f(\Lambda)$ is rc-convergent to $f(x)$.

Note that a function $f : X \to Y$ is almost contra-Ω*$g\alpha$-continuous at $x$ if each regular closed set $F$ in $Y$ containing $f(x)$, there exist Ω*$g\alpha$-open set $U$ in $X$ containing $x$ such that $f(U) \subset F$.

**Theorem 3.21:** Let $f: X \rightarrow Y$ be a function and $x \varepsilon X$. If there exist $U \varepsilon \Omega^*g\alpha$-open in $X$ such that $x \varepsilon U$ and the restriction of $f$ to $U$ is almost contra-$\Omega^*g\alpha$-continuous at $x$ then $f$ is almost contra-$\Omega^*g\alpha$-continuous at $x$.

**Proof:** Suppose that $F \varepsilon RC(Y)$ containing $f(x)$. Since $f/U$ is almost contra-$\Omega^*g\alpha$-continuous at $x$, there exists $V \varepsilon \Omega^*g\alpha$-open set $U$ in $X$ containing $x$ such that $f(V) = (f/U)(V) \subset F$. Since $U \varepsilon \Omega^*g\alpha$-open in $X$ containing $x$ it follows that $V \varepsilon \Omega^*g\alpha$-open in $X$ containing $x$. This shows clear that $f$ is almost contra-$\Omega^*g\alpha$-continuous at $x$.

## 4. The Preservation Theorems

In this section, we investigate the relationships among almost contra-$\Omega^*g\alpha$-continuous functions, separation axioms, connectedness and compactness.

**Definition 4.1:** A space $X$ is said to be weakly Hausdorff [19] if each element of $X$ is an intersection of regular closed sets.

**Definition 4.2:** A space $X$ is said to be $\Omega^*g\alpha$-To if for each pair of distinct points in $X$ there exists a $\Omega^*g\alpha$-open set of $X$ containing one point but not the other.

**Definition 4.3:** A space $X$ is said to be $\Omega^*g\alpha$-$T_1$ if for each pair of distinct points $x$ and $y$ in $X$ there exists a $\Omega^*g\alpha$-open sets $U$ and $V$ containing $x$ and $y$ respectively such that $y \notin U$ and $x \notin V$.

**Definition 4.4 :** A space $X$ is said to be $\Omega^*g\alpha$-Hausdorff if for each pair of distinct points $x$ and $y$ in $X$ there exists $U \varepsilon \Omega^*g\alpha$-open in $(X, x)$ and $V \varepsilon \Omega^*g\alpha$-open in $(Y, y)$ such that $U \cap V = \phi$.

**Theorem 4.5:** If $f: X \rightarrow Y$ is an almost contra-$\Omega^*g\alpha$-continuous injection and $Y$ is weakly Hausdorff then $X$ is $\Omega^*g\alpha$-$T_1$.

**Proof:** Suppose that $Y$ is weakly Hausdorff. For any distinct points $x$ and $y$ in $X$ there exist $V$, $W \varepsilon RC(Y)$ such that $f(x) \varepsilon V$, $f(y) \varepsilon W$, $f(x) \notin W$, $f(y) \notin V$. Since $f$ is almost $\Omega^*g\alpha$-continuous, $f^{-1}(V)$ and $f^{1}(W)$ are $\Omega^*g\alpha$-open subsets of $X$ such that

$x \in f^{-1}(V)$ and $y \in f^{-1}(W)$, $y \notin f^{-1}(V)$, $x \notin f^{-1}(W)$, This shows that $X$ is $\Omega^*g\alpha$-$T_1$.

**Definition 4.6:** A topological space $X$ is called $\Omega^*g\alpha$-ultra connected if every two non-void $\Omega^*g\alpha$-closed subsets of $X$ intersect.

**Definition 4.7:** A topological space $X$ is called hyper connected [20] if every open set is dense.

**Theorem 4.8:** If $X$ is $\Omega^*g\alpha$-ultra connected and $f: X \to Y$ is almost contra-$\Omega^*g\alpha$-continuous and surjective, then $Y$ is hyper connected.

**Proof:** Assume that $Y$ is hyper connected. Then there exist an open set $V$ such that $V$ is not dense in $Y$. Then there exist disjoint non-empty regular open subsets $B_1$ and $B_2$ in $Y$ namely $B_1$ = int cl($V$) and $B_2$ = $Y$-cl($V$). Since $f$ is almost contra-$\Omega^*g\alpha$-continuous and surjective, $A_1 = f^{-1}(B_1)$ and $A_2 = f^{-1}(B_2)$ are disjoint non-empty $\Omega^*g\alpha$-closed subsets of $X$ which is a contradiction to the fact that $X$ is $\Omega^*g\alpha$-ultra connected. Hence $Y$ is hyper connected.

**Definition 4.9:** A space $X$ is called $\Omega^*g\alpha$-connected provided that $X$ is not the union of two disjoint non-empty $\Omega^*g\alpha$-open sets.

**Theorem 4.10:** If $f: X \to Y$ is almost contra-$\Omega^*g\alpha$-continuous surjection and $X$ is $\Omega^*g\alpha$-connected then $Y$ is connected.

**Proof:** Suppose that $Y$ is not connected. Then there exist non-empty disjoint open sets $V_1$ and $V_2$ such that $Y = V_1 \cup V_2$. Therefore $V_1$ and $V_2$ are clopen in $Y$. Since $f$ is almost contra-$\Omega^*g\alpha$-continuous, $f^{-1}(V_1)$ and $f^{-1}(V_2)$ are disjoint and $X = f^{-1}(V_1) \cup f^{-1}(V_2)$ which is a contradiction to the fact that $X$ is $\Omega^*g\alpha$-connected. Hence $Y$ is connected.

**Definition 4.11:** A space $X$ is said to be

a) $\Omega^*g\alpha$-closed if every $\Omega^*g\alpha$-closed cover of $X$ has a finite subcover.

b) Countable $\Omega^*g\alpha$-closed if every countable cover of $X$ by $\Omega^*g\alpha$-closed sets has a finite subcover.

c) $\Omega^*g\alpha$-Lindelof if every cover of $X$ by $\Omega^*g\alpha$-closed sets has a countable cover.

d) Nearly compact if every regular open cover of $X$ has a finite subcover. [17]

e) Nearly countably compact if every countably cover of $X$ by regular open sets has a finite subcover. [5, 18]

f) Nearly Lindelof [4] if every cover of X by regular open sets has a countable subcover.

**Theorem 4.12:** Let $f : X \to Y$ be an almost contra-$\Omega$-continuous surjection. Then the following statements hold.

a) If $X$ is $\Omega^*g\alpha$-closed then $Y$ is nearly compact.

b) If $X$ is $\Omega^*g\alpha$-lindelof then $Y$ is nearly lindelof.

c) If $X$ is countably-$\Omega^*g\alpha$-closed, then $Y$ is nearly countably compact.

**Proof:** Let $\{V\alpha : \alpha \in I\}$ be any regular open cover of $Y$. Since $f$ is almost contra-$\Omega^*g\alpha$-continuous, then $\{f^{-1}(V\alpha) : \alpha \in I\}$ is a $\Omega^*g\alpha$-closed cover of $X$. Since $X$ is $\Omega^*g\alpha$-closed there exist a finite $Io$ of $I$ such that $X = \cup \{f^{-1}(V\alpha) : \alpha \in Io\}$. Thus we have $Y = \cup \{V\alpha : \alpha \in Io\}$ and $Y$ is nearly compact.

Proof of b) and c) are analogue to a).

**Definition 4.13 :** *A* space $X$ is said to be

(a) Mildly $\Omega^*g\alpha$-compact if every $\Omega^*g\alpha$-clopen cover of X has a finite subcover.

(b) Mildly countably-$\Omega^*g\alpha$-compact if every $\Omega^*g\alpha$-clopen countable cover of $X$ has a countable subcover.

(c) Mildly $\Omega^*g\alpha$-Lindelof if every $\Omega^*g\alpha$-clopen cover of $X$ has a countable subcover.

**Theorem 4.14:** If $f : X \to Y$ is an almost contra-$\Omega^*g\alpha$-continuous and almost contra-$\Omega^*g\alpha$-continuous surjection. Then

(a) If $X$ is mildly $\Omega^*g\alpha$-compact then $Y$ is nearly compact.

(b) If $X$ is mildly countably-$\Omega^*g\alpha$-compact then $Y$ is nearly countably compact.

(c) If $X$ is mildly $\Omega^*g\alpha$-lindelof then $Y$ is nearly Lindelof.

**Proof:** (a) $V \varepsilon RO(Y)$. Then since f is almost contra-Ω*gα-continuous almost Ω*gα-continuous, $f^{-1}(V)$ is clopen. Let $\{V\alpha : \alpha \varepsilon I\}$ be any regular open cover of $Y$. Then $\{f^{-1}(V\alpha) : \alpha \varepsilon I\}$ is a clopen cover of $X$. Since $X$ is mildly Ω*gα-compact, there exist a finite subset Io of I such that $X = \cup \{f^{-1}(V\alpha) : \alpha \varepsilon I\}$ Hence $Y$ is nearly compact.

Proof of (b) and (c) are similar to (a).

## 5. Ω*ga-Regular Graphs

In this we define Ω*gα-regular graphs and investigate the relationships between Ω*gα-regular graphs and almost contra-Ω*gα-continuous functions.

**Definition 5.1:** For a function $f : X \rightarrow Y$ the subset $\{(x, f(x)/x \in X\} \subset X \times Y$ is called the graph of $f$ and is denoted by $G(f)$ [4].

**Definition 5.2:** A graph $G(f)$ of a function $f : X \rightarrow Y$ is said to be Ω*gα-regular if for each $(x,y) \varepsilon X \times Y\text{-}G(f)$, there exist a Ω*gα-closed set $U$ in $X$ containing $x$ and $V \varepsilon RO(Y)$ containing $y$ such that $(U \times V) \cap G(f) = \Phi$.

**Lemma 5.3:** The following properties are equivalent for a graph G(f) of a function

1. $G(f)$ is Ω*gα-regular.
2. for each point $(x,y) \varepsilon X \times Y\text{-}G(f)$ there exist a Ω*gα-closed set $U$ in $X$ containing $x$ and $V \varepsilon RO(Y)$ containing y such that $f(U) \cap V = \Phi$.

**Proof :** It follows from definition and the fact that for any subsets $U \subset X$, $V \subset Y$ $(U \times V) \cap G(f) = \Phi$ iff $f(U) \cap V = \Phi$.

**Theorem 5.4:** If $f : X \rightarrow Y$ is almost contra-Ω*gα-continuous and $Y$ is $T_2$, then $G(f)$ is Ω*gα-regular graph in $X \times Y$.

**Proof:** Let $(x,y) \varepsilon X \times Y\text{-}G(f)$. It follows that $f(x) \neq y$. Since $Y$ is $T_2$, there exist open sets $V$ and $W$ containing $f(x)$ and $y$ respectively such that $V \cap W = \Phi$. We have $\text{int}(\text{cl}(V)) \cap \text{int}(\text{cl}(W)) = \Phi$. Since $f$ is almost contra-Ω*gα-continuous, $f^{-1}(\text{int}(\text{cl}(V)))$

is $\Omega^*g\alpha$-closed in $X$ containing $x$. Take $U = f^{-1}(\text{int}(\text{cl}(V)))$. Then $f(U) \subset \text{int}(\text{cl}(V))$ Therefore $f(U) \cap \text{int}(\text{cl}(W)) = \Phi$. Hence $G(f)$ is $\Omega^*g\alpha$-regular.

**Theorem 5.5:** Let $f : X \rightarrow Y$ have $\Omega^*g\alpha$-regular graph $G(f)$. If $f$ is injective, then $X$ is $\Omega^*g\alpha$-$T_1$.

**Proof:** Let $x$ and $y$ be any two distinct points of $X$. Then we have $(x, f(y)) \varepsilon X \times Y$-$G(f)$. By definition of $\Omega^*g\alpha$-regular graph, there exist a $\Omega^*g\alpha$-closed set $U$ of $X$ and $V \varepsilon RO(Y)$ such that $(x, f(y)) \varepsilon U \times V$ and $U \cap f^{-1}(V) = \Phi$. Therefore we have $Y \notin U$. Thus $y \varepsilon X$-$U$. $x \notin X$-$U$. $X$-$U \varepsilon \Omega^*g\alpha$-open in $(X,\tau)$ implies $X$ is $\Omega^*g\alpha$-$T_1$.

**Theorem 5.6:** Let $f : X \rightarrow Y$ have $\Omega^*g\alpha$-regular graph $G(f)$ If $f$ is surjective, then $Y$ is weakly $T_2$.

**Proof:** Let $y_1$ and $y_2$ be any two distinct points of $Y$. Since $f$ is surjective $f(x) = y_1$ for some $x \varepsilon X$ and $(x, y_2) \varepsilon X \times Y$-$G(f)$. By lemma 5.3, there exist a $\Omega^*g\alpha$-closed set $U$ of $X$ and $F \varepsilon RO(Y)$ such that $(x, y_2) \varepsilon U \times F$ and $f(U) \cap F = \Phi$. Hence $y_1 \notin F$. Then $y_2 \notin Y$-$F \varepsilon RC(Y)$ and $y_1 \varepsilon Y$-$F$. This implies that $Y$ is weakly $T_2$.

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