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DECAY LAW OF TEMPERATURE FLUCTUATIONS IN DUSTY FLUID MHD TURBULENCE BEFORE THE FINAL PERIOD

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Abstract

Using Deissler's method the decay of temperature fluctuations in dusty fluid MHD turbulence before the final period is studied and considered correlations between fluctuating quantities at two and three-points. In this study two and three point correlation equations in presence of dust particle is obtained and the set of equation is made to determinate by neglecting the quadruple correlations in comparison to the second and third-order correlations. The correlation equations are converted to the spectral form by taking their Fourier transforms. Finally integrating the energy spectrum over all wave numbers, the solution is obtained and this solution gives the decay law of temperature fluctuations in MHD dusty fluid turbulence before the final period.

Keywords : Deissler's method, MHD dusty fluid turbulence, temperature fluctuation

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1. Introduction

Deissler [1,2] developed a theory for homogeneous tubulence, which was valid for times before the final period. Using Deissler's theory Loeffler and Deissler [3] studied the decay of temperature fluctuations in homogeneous turbulence before the final period. Sarker and Rahman [5] studied the decay of temperature fluctuations in MHD turbulence before the final period. Islam and Sarker [7] studied the first order reactant in MHD turbulence before the final period of decay for the case of multi-point and multi-time. Kumar and Patel [15] also studied on fist-order reactant in homogeneous turbulence before the final period of decay for the case of multipoint and multi-time. Sarker and Islam [4] studied the decay of MHD turbulence before the final period for the case of multi-point and multi-time. Sarker and Kishore [6] had done further work along this same line for the case of multi-point and single time. In their approach they considered two and three-point correlations after neglecting higher order correlation terms compared to the second-and third-order correlation terms. Sarker and Islam [8] studied the decay of dusty fluid turbulence before the final period.

Later on Azad and Sarker [9], Sarker and Azad [10], Azad and Sarker [11], Sarker and Azad [12,13] also studied the decay of MHD turbulence, Homogeneous turbulence for the case of multi-point and multi-time in presence of dust particle in a rotaing system.

Following the above theories, we have studied the decay of temperature fluctuation in dusty fluid MHD turbulence before the final period. Here two-and three-point correlation equations have been considered after neglecting fourth-order correlation terms in comparison to the second-and third-order correlation terms. Finally, the energy decay law of temperature fluctuations in dusty fluid MHD turbulence before the final period is obtained.

Basic Equations:

The equation of motion and continuity for viscous, incompressible MHD dusty fluid turbulent flow are given by

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_k}(u_i u_k - h_i k_k) = -\frac{\partial w}{\partial x_i} + v\frac{\partial^2 u_i}{\partial x_k \partial x_k} + f(u_i - v_i) \quad \ldots (1)$$

$$\frac{\partial h_i}{\partial t} + \frac{\partial}{\partial x_k}(h_i u_k - u_i h_k) = \frac{v}{p_M}\frac{\partial^2 h_i}{\partial x_k \partial x_k} \quad \ldots (2)$$

$$\frac{\partial v_i}{\partial t} + v_k \frac{\partial v_i}{\partial x_k} = -\frac{k}{m_s}(v_i - u_i) \quad \ldots (3)$$

with

$$\frac{\partial u_i}{\partial x_i} = \frac{\partial v_i}{\partial x_i} = \frac{\partial h_i}{\partial x_i} = 0 \quad \ldots (4)$$

and the equation of energy for an incompressible fluid with constant properties and for neglligible frictional heating

$$\frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \left(\frac{v}{p_r}\right)\frac{\partial^2 T}{\partial x_i \partial x_i} \quad \ldots (5)$$

The subscripts can take on the value 1, 2 or 3.

Here, u_i, turbulent velocity component; h_i, magnetic field fluctuation compnent, v_i, dust velocity component

$W(\hat{x},t) = \frac{p}{\rho} + \frac{1}{2}\langle h^2 \rangle$, total MHD pressure, $p(\hat{x},t) =$ hydrodymanic pressure, ρ = fluid density,

$p_M = \frac{v}{\lambda}$, magnetic prandtl number, $p_r = \frac{v}{\gamma}$, prandtl number, v = kinematic viscotiy,

$\gamma = \dfrac{K}{\rho c_p}$, thermal diffusivity, $\lambda = (4\pi\mu\sigma)^{-1}$, magnetic diffusivity, c_p = heat capacity at constant pressure, $f = \dfrac{kN}{\rho}$, dimension of frequency; N, constant number density of dust particle,

$m_s = \dfrac{4}{3}\pi R_s^3 \rho_s$, mass of single spherical dust particle of radius R_s, ρ_s = constant density of the material in dust particle, x_k = Space co-ordinate, the subscripts can take on the values 1, 2 or 3.

Two-point Correlation and Spectral Equations:

The induction equation of a magnetic field at the point p is

$$\frac{\partial h_i}{\partial t} + u_k \frac{\partial h_i}{\partial x_k} - h_k \frac{\partial u_i}{\partial x_k} = \left(\frac{\nu}{p_M}\right) \frac{\partial^2 h_i}{\partial x_k \partial x_k} \qquad \ldots (6)$$

and the energy equation at the point p' is

$$\frac{\partial T_j'}{\partial t} + u_k' \frac{\partial T_j'}{\partial x_k'} = \left(\frac{\nu}{p_r}\right) \frac{\partial^2 T_j'}{\partial x_k' \partial x_k'} \qquad \ldots (7)$$

Multiplying equation (6) by T_j' and (7) by h_i, adding and taking ensemble average, we get

$$\frac{\partial \langle h_i T_j' \rangle}{\partial t} + u_k \frac{\partial \langle h_i T_j' \rangle}{\partial x_k} + u_k' \frac{\partial \langle h_i T_j' \rangle}{\partial x_k^1} - h_k \frac{\partial \langle u_i T_j' \rangle}{\partial x_k'} = \nu \left[\frac{1}{p_M} \frac{\partial^2 \langle h_i T_j' \rangle}{\partial x_k \partial x_k} + \frac{1}{p_r} \frac{\partial^2 \langle h_i T_j' \rangle}{\partial x_k' \partial x_k'} \right] \qquad \ldots (8)$$

Angular bracket $\langle \ldots\ldots\ldots \rangle$ is used to denote an ensemble averae and the continuity equation is $\dfrac{\partial u_k}{\partial x_k} = \dfrac{\partial u_k'}{\partial x_k'} = 0$. ... (9)

Substituting equation (9) into equation (8) yields

$$\frac{\partial\langle h_i T_j'\rangle}{\partial t}+\frac{\partial\langle u_k h_i T_j'\rangle}{\partial x_k}+\frac{\partial\langle u_k' h_i T_j'\rangle}{\partial x_k'}-\frac{\partial\langle u_i h_k T_j'\rangle}{\partial x_k}=v\left[\frac{1}{p_M}\frac{\partial^2\langle h_i T_j'\rangle}{\partial x_k dx_k}+\frac{1}{p_r}\frac{\partial^2\langle h_i T_j'\rangle}{\partial x_k'\partial x_k'}\right] \quad \ldots (10)$$

Using the transformations

$$\frac{\partial}{\partial r_k}=-\frac{\partial}{\partial x_k}=\frac{\partial}{\partial x_k'}$$

and the Chandrasekhar relation [14].

$$\langle u_k h_i T_j'\rangle=-\langle u_k' h_i T_j'\rangle.$$

Equation (10) become

$$\frac{\partial}{\partial t}\langle h_i T_j'\rangle+2\frac{\partial}{\partial r_k}\langle u_k' h_i T_j'\rangle+\frac{\partial\langle u_i h_k T_j'\rangle}{\partial r_k}=v\left[\frac{\partial^2\langle h_i T_j'\rangle}{\partial r_k\partial r_k}\left(\frac{1}{p_M}+\frac{1}{p_r}\right)\right] \quad \ldots (11)$$

Now we write equation (10) in spectral form in order to reduce it to an ordinary differential equation by use of the following three-dimensional Fourier transforms.

$$\langle h_i T_j'(\hat{r})\rangle=\int_{-\infty}^{\infty}\langle \psi,\tau_j'(\hat{K})\rangle\exp[i(\hat{K},\hat{r})]d\hat{K} \quad \ldots (12)$$

$$\langle u_i h_k T_j'(r)\rangle=\int_{-\infty}^{\infty}\langle \phi_i\psi_k\tau_j'(\hat{K})\rangle\exp[i(\hat{K},\hat{r})]d\hat{K} \quad \ldots (13)$$

$$\langle u_k' h_i T_j'(r)\rangle=\langle u_k h_i T_j'(-\hat{r})\rangle=\int_{-\infty}^{\infty}\langle \phi_k\psi_i\tau_j'(-\hat{K})\rangle\exp[i(\hat{K},\hat{r})]d\hat{K} \quad \ldots (14)$$

Equation (14) is obtained by interchanging the subscripts i and j and then the points p and p'.

Substitution of equations (12) to (14) into equation (11) leads to the Spectral equation

$$\frac{\partial\langle\psi_i,\tau_j\rangle}{\partial t}+iK_k\left[2\langle\phi_k\psi_i\tau'_j(-\hat{K})\rangle+\langle\phi_i\psi_k\tau'_j(\hat{K})\rangle\right]=-v\left[\left(\frac{1}{p_M}+\frac{1}{p_r}\right)K^2\langle\psi_i\tau'_j(\hat{K})\rangle\right] \quad \text{. . . (15)}$$

The tensor equation (15) becomes a scalar equation by contraction of the indices i and j

$$\frac{\partial\langle\psi_i,\tau'_i(\hat{K})\rangle}{\partial t}+iK_k\left[2\langle\phi_k\psi_i\tau'(-\hat{K})\rangle+\langle\phi_i\psi_k\tau'_i(\hat{K})\rangle\right]=-v\left[\left(\frac{1}{p_M}+\frac{1}{p_r}\right)K^2\langle\psi_i\tau'_i(\hat{K})\rangle\right] \quad \text{. . . (16)}$$

Three-point Correlation and Spectral Equations:

Similar procedure can be used to find the three points correlation equation. For this purpose we take the momentum equation of MHD turbulence at the point P, the induction equation at the point P' and the energy equation at P'' as

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial k_k}-h_k\frac{\partial h_i}{\partial x_k}=-\frac{\partial w}{\partial x_i}+v\frac{\partial^2 u_i}{\partial x_k\partial x_k}+f(u_i-v_i) \quad \text{. . . (17)}$$

$$\frac{\partial h'_i}{\partial t}+u'_k\frac{\partial h'_i}{\partial x_k}-h'_k\frac{\partial u'_i}{\partial x'_k}=\left(\frac{v}{p_M}\right)\frac{\partial^2 h'_i}{\partial x'_k\partial x'_k} \quad \text{. . . (18)}$$

and

$$\frac{\partial T''_j}{\partial t}+u''_k\frac{\partial T''_j}{\partial x''_k}=\left(\frac{v}{p_r}\right)\frac{\partial^2 T''_j}{\partial x''_k\,\partial x''_k} \quad \text{. . . (19)}$$

where $W(\hat{x},t)=\frac{P}{\rho}+\frac{1}{2}\langle h^2\rangle$, total MHD pressure inclusive of potential and centrifugal force $P(\hat{x},t)$, hydrodynamic pressure; $f=\frac{kN}{\rho}$, dimension frequency; N, constant number density of dust particle.

Multiplying equation (17) by $h_i'T_j''$, (18) by u_iT_j'' and (19) by u_ih_i' adding and taking ensemble average, one obtains

$$\frac{\partial\langle u_ih_i'T_j''\rangle}{\partial t}+\frac{\partial\langle u_iu_kh_i'T_j\rangle}{\partial x_k}-\frac{\partial\langle h_ih_kh_i'T_j''\rangle}{\partial x_k''}+\frac{\partial\langle u_iu_k'h_i'T_j''\rangle}{\partial x_k'}-\frac{\partial\langle u_iu_i'h_k'T_j''\rangle}{\partial x_k'}+\frac{\partial\langle u_ih'u_k''T_i''\rangle}{\partial x_k''}$$

$$=-\frac{\partial\langle wh_i'T_i''\rangle}{\partial x_i}+v\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k\partial x_k}+v\left[\frac{1}{p_M}\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k'\partial x_k'}+\frac{1}{p_r}\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial x_k''\partial x_k''}\right]+f\left(\langle u,h_i''T_j''\rangle-\langle v_ih_i''T_j''\rangle\right) \quad \ldots (20)$$

Using the transformations

$$\frac{\partial}{\partial x_k}=-\left(\frac{\partial}{\partial r_k}+\frac{\partial}{\partial r_k'}\right),\frac{\partial}{\partial x_k'}=\frac{\partial}{\partial r_k},\frac{\partial}{\partial x_k''}=\frac{\partial}{\partial r_k'} \quad \ldots (21)$$

into equations (21)

$$\frac{\partial\langle u_ih_i'T_j''\rangle}{\partial t}-v\left[\left(1+\frac{1}{p_M}\right)\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial r_k\partial r_k}+\left(1+\frac{1}{p_r}\right)\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial r_k'\partial r_k'}+2\frac{\partial^2\langle u_ih_i'T_j''\rangle}{\partial r_k\partial r_k'}\right]$$

$$=\frac{\partial\langle u_iu_kh_i'T_j''\rangle}{\partial r_k}+\frac{\partial\langle u_iu_kh_i'T_j''\rangle}{\partial r_k'}-\frac{\partial\langle h_ih_kh_i'T_j''\rangle}{\partial r_k}-\frac{\partial\langle h_ih_kh_i'T_j''\rangle}{\partial r_k'}-\frac{\partial\langle u_iu_k'h_i'T_j''\rangle}{\partial r_k}$$

$$+\frac{\partial\langle u_iu_i'h_k'T_j''\rangle}{\partial r_k}-\frac{\partial\langle u_iu_k''h_i'T_j''\rangle}{\partial r_k'}+\frac{\partial\langle wh_i'T_j''\rangle}{\partial r_i}+\frac{\partial\langle wh_i'T_j''\rangle}{\partial r_i'}+f\left(\langle u_ih_i'T_j''\rangle-\langle v_ih_i'T_j''\rangle\right)$$

$$\ldots (22)$$

In order to write the equation to spectral form, we can define the following six dimensional Fourier transform:

$$\left\langle u_i h_i'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_i \beta_i'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (23)$$

$$\left\langle u_i u_k h_i'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_i \varphi_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (24)$$

$$\left\langle h_i h_k h_i'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \beta_i \beta_k \beta_i'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (25)$$

$$\left\langle u_i u_k' h_i'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_i \phi_k(\hat{k}) \beta_i'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (26)$$

$$\left\langle u_i u_i'(\hat{r}) h_k'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \phi_i \phi_i'(\hat{k}) \beta_i'(\hat{k})\theta_k'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (27)$$

$$\left\langle w h_i'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \gamma \beta_i'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (28)$$

$$\left\langle v_i h_i'(\hat{r}) T_j''(\hat{r}')\right\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle \mu_i \beta_i'(\hat{k})\theta_j''(\hat{k}')\right\rangle \exp\left[\hat{i}(\hat{k}.\hat{r}+\hat{k}'.\hat{r}')\right] d\hat{k}\,d\hat{k}' \quad \ldots (29)$$

Interchanging the points p' and p'' along with the indices i and j result in the relations $\left\langle u_i u_k'' h_i' T_j''\right\rangle = \left\langle u_i u_k' h_i' T_j''\right\rangle$.

By use of this facts the equation (22) may be transformed as

$$\frac{\partial\left\langle \phi_i \beta_i' \theta_j''\right\rangle}{\partial t} + v\left[\left(1+\frac{1}{p_M}\right)k^2 + \left(1+\frac{1}{p_r}\right)k'^2 + 2k_k k_k' - \frac{f}{v}\right]\left\langle \phi_i \beta_i' \theta_j''\right\rangle$$

$$= i(k_k+k_k')\left\langle \phi_i \phi_k \beta_i' \theta_j''\right\rangle - i(k_k+k_k')\left\langle \beta_i \beta_k \beta_i' \theta_j''\right\rangle - i(k_k+k_k')\left\langle \phi_i \phi_k' \beta_i' \theta_j''\right\rangle + ik_k\left\langle \phi_i \phi_i' \beta_k' \theta_j''\right\rangle$$

$$+ i(k_i+k_i')\left\langle \gamma \beta_i' \theta_j''\right\rangle - f\left\langle \mu_i \beta_i' \theta_j''\right\rangle \quad \ldots (30)$$

The tensor equation (29) can be converted to scalar equation by contraction of the indices i and j

$$\frac{\langle \partial \langle \phi_i \beta_i' \theta_i'' \rangle}{\partial t} + v\left[\left(1+\frac{1}{p_M}\right)k^2 + \left(1+\frac{1}{p_r}\right)k'^2 + 2k_k k_k' - \frac{f}{v}\right]\langle \phi_i \beta_i' \theta_i'' \rangle$$

$$= i(k_k + k_k')\langle \phi_i \phi_k \beta_i' \theta_i'' \rangle - i(k_k + k_k')\langle \beta_i \beta_k \beta_i' \theta_i'' \rangle - i(k_k + k_k')$$

$$\times \langle \phi_i \phi_k \beta_i' \theta_i'' \rangle + ik_k \langle \phi_i \phi_i' \beta_k' \theta_i'' \rangle + i(k_i + k_i')\langle \gamma_i \beta_i' \theta_i'' \rangle - f\langle \mu_i \beta_i' \theta_i'' \rangle \qquad \text{. . . (31)}$$

If derivative with respect to x_i is taken of the momentum equation (17) for the point p, the equation multiplied through by $h_i T_j''$ and time average taken, the resulting equation

$$-\frac{\partial^2 \langle w h_i' T_j'' \rangle}{\partial x_i \partial x_i} = \frac{\partial^2}{\partial x_i \partial x_k}\left(\langle u_i u_k h_j' T_j'' \rangle - \langle h_i h_k h_i' T_j'' \rangle\right) \qquad \text{. . . (32)}$$

Writing this equation in terms of the independent variables $\hat{r}$ and $\hat{r}'$

$$-\left[\frac{\partial^2}{\partial r_i \partial r_i} + 2\frac{\partial^2}{\partial r_i \partial r_i'} + \frac{\partial^2}{\partial r_i' \partial r_i'}\right]\langle w h_i' T_j'' \rangle = \left[\frac{\partial^2}{\partial r_i \partial r_k} + \frac{\partial^2}{\partial r_i' \partial r_k} + \frac{\partial^2}{\partial r_i \partial r_k'} + \frac{\partial^2}{\partial r_i' \partial r_k'}\right]$$

$$\times \left(\langle u_i u_k h_i' T_j'' \rangle - \langle h_i h_k h_i' T_j'' \rangle\right) \qquad \text{. . . (33)}$$

Now taking the Fourier transforms of equation (33)

$$-\langle \gamma \beta_i' \theta_j'' \rangle = \frac{(k_i k_k + k_i' k_k + k_i k_k' + k_i' k_k')\left(\langle \phi_i \phi_k \beta_i' \theta_j'' \rangle - \langle \beta_i \beta_k \beta_i' \theta_j'' \rangle\right)}{k_i k_i + 2k_i' k_i + k_i' k_i'} \qquad \text{. . . (34)}$$

Equation (34) can be used to eliminate $\langle \gamma \beta_i' \theta_j'' \rangle$ from equation (30).

Solution for Times Before the Final Period:

It is known equation for final period of decay is obtained by considering the two-point correlations after neglecting the 3rd order correlation terms. To study the decay for times before the final period, the three point correlations are considered and the quadruple correlation terms are neglected because the quadruple correlation terms decays faster than the lower-order correlation terms. Equation (34) shows that term $\langle \gamma\beta_i'\theta_j'' \rangle$ associated with the pressure fluctuations should also be neglected. Thus neglecting all the terms on the right hand side of equation (31),

$$\frac{\partial\langle \phi_i\beta_i'\theta_j'' \rangle}{\partial t}+\nu\left[\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2+2k_kk_k'-\frac{fS}{\nu}\right]\langle \phi_i\beta_i'\theta_i'' \rangle=0\,, \qquad \ldots (35)$$

where $\langle \mu_i\beta_i'\theta_i'' \rangle = R\langle \phi_i\beta_i'\theta_i'' \rangle$ and $1\text{-}R = S$, here R and S are arbitrary constant.

Integrating the equation (35) between t_0 and t with inner multiplication by k_k and gives

$$k_k\langle \phi_i\beta_i'\theta_i'' \rangle = k_k\left[\phi_i\beta_i'\theta''\right]_0 \exp\left[-\nu\left\{\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2+2kk'\cos\theta-\frac{fS}{\nu}\right\}(t-t_0)\right]$$
$$\ldots (36)$$

where θ is the angle k and k' and $\langle \phi_i\beta_i'\theta_i'' \rangle_0$ is the value of $\langle \phi_i\beta_i'\theta_i'' \rangle$ at $t = t_0$.

Now by letting $r' = 0$ in equation (23) and comparing with equations (13) and (14), we get

$$\left\langle \phi_i\psi_k\tau_i'(\hat{k}) \right\rangle = \int_{-\infty}^{\infty}\langle \phi_i\beta_i'\theta_i'' \rangle d\hat{k}', \qquad \ldots (37)$$

$$\left\langle \phi_i\psi_i\tau_i'(-\hat{k}) \right\rangle = \int_{-\infty}^{\infty}\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')d\hat{k}' \qquad \ldots (38)$$

Substituting equation (36) - (38) in equation (16), we get

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t}+v\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\langle\psi_i\tau_i'(\hat{k})\rangle=-\int_{-\infty}^{\infty}ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle+2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0$$

$$\times\exp\left[-v(t-t_0)\left\{\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2+2kk'\cos\theta-\frac{fS}{v}\right\}\right]d\hat{k}' \qquad \ldots (39)$$

Now, $d\hat{k}'$ can be expressed in terms of k' and θ as $-2\pi k'^2 d(\cos\theta)dk'$ (cf. Deissler [1]).

Hence $d\hat{k}'=-2\pi k'^2 d(\cos\theta)dk'$ . . . (40)

Putting equation (40) in equation (39) yields

$$\frac{\partial\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t}+v\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2\langle\psi_i\tau_i'(\hat{k})\rangle=-\int_{-\infty}^{\infty}2\pi ik_k\left[\langle\phi_i\beta_i'\theta_i''\rangle+2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0 k'^2$$

$$\times\left[\int_{-1}^{1}\exp\left\{-v(t-t_0)\left[\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2+2kk'\cos\theta-\frac{fS}{v}\right]\right\}d(\cos\theta)dk'\right] \qquad \ldots (41)$$

In order to find the solution completely and following Loeffler and Deissler [72] we assume that

$$ik_k\left[\langle\phi_i\beta_i'(\hat{k})\theta_i''(\hat{k}')\rangle+2\langle\phi_k\beta_i'(-\hat{k})\theta_i''(-\hat{k}')\rangle\right]_0=\frac{\beta_0}{(2\pi)^2}\int_0^{\infty}(k^2k'^4-k^4k'^2) \qquad \ldots (42)$$

where β_0 is a constant depending on the initial conditions. Substituting equation (42) into equation (41) and completing the integration with respect to cosθ, one obtains

$$\frac{\partial 2\pi\langle\psi_i\tau_i'(\hat{k})\rangle}{\partial t}+v\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2 2\pi\langle\psi_i\tau_i'(\hat{k})\rangle=-\frac{\beta_0}{2v(t-t_0)}\int_0^{\infty}(k^3k'^5-k^5k'^3)$$

$$[\exp\left\{-\nu(t-t_0)\left[\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2-2kk'-\frac{fS}{\nu}\right]\right\}$$

$$+-\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty (k^3k'^5-k^5k'^3)\exp\left\{-\nu(t-t_0)\left[\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2+2kk'-\frac{fS}{\nu}\right]dk'\right.$$

$$\ldots (43)$$

Multiplying both sides of equation (43) by k^2, we get

$$\frac{\partial Q}{\partial t}+\nu\left(\frac{1}{p_M}+\frac{1}{p_r}\right)k^2Q=F, \qquad \ldots (44)$$

where $Q=2\pi k^2\left\langle \psi_i\tau'(\hat{k})\right\rangle$. $\qquad \ldots (45)$

Q is the Magnetic energy Spectrum function.

And

$$F=-\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty (k^3k'^5-k^5k'^3)\exp\left\{-\nu(t-t_0)\left[\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2-2kk'-\frac{fS}{\nu}\right]\right\}$$

$$+\frac{\beta_0}{2\nu(t-t_0)}\int_0^\infty (k^3k'^5-k^5k'^3)\exp\left\{-\nu(t-t_0)\left[\left(1+\frac{1}{p_M}\right)k^2+\left(1+\frac{1}{p_r}\right)k'^2+2kk'-\frac{fS}{\nu}\right]\right\}d\hat{k}'$$

$$\ldots (46)$$

Integrating equation (46) with respect to k', we have

$$F=-\frac{\beta_0\sqrt{\pi}\,p_r^{5/2}}{2\nu^{3/2}(t-t_0)(1+p_r)^{5/2}}\exp\left[\left\{\frac{fS}{\nu}\right\}(t-t_0)\right]\exp\left[-\nu(t-t_0)\left(1+\frac{1}{p_M}-\frac{p_r}{1+p_r}\right)k^2\right]$$

$$\left[\frac{15p_rk^4}{4\nu^2(t-t_0)^2(1+p_r)}+\left\{\frac{5p_r^2}{(1+p_r)^2}-\frac{3}{2}\right\}\frac{k^6}{\nu(t-t_0)}+\left\{\frac{p_r^3}{(1+p_r)^3}-\frac{p_r}{(1+p_r)}\right\}k^8\right]$$

$$\ldots (47)$$

The series of equation (47) contains only even powers of k and start with k^4 and the equation represents the transfer function arising owing to consideration of magnetic field at three points at a time.

It is interesting to note that if we integrate equation (47) over all wave numbers, we find that

$$\int_0^\infty F dk = 0 \qquad \ldots (48)$$

which indicates that the expression for F satisfies the condition of continuity and homogeneity.

The linear equation (44) can be solved to give

$$Q = \exp\left[-\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_0)\right]\int F\exp\left[\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_0)\right]dt$$

$$+J(k)\exp\left[-\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_0)\right], \qquad \ldots (49)$$

where $J(K)=\frac{N_0 k^2}{\pi}$ is constant of integration. Substitution the values of F from equation (47) in to equation (49) and integrating with respect to t, we get

$$Q(\hat{k},t)=\frac{N_0 k^2}{\pi}\exp\left[-\nu k^2\left(\frac{1}{p_M}+\frac{1}{p_r}\right)(t-t_0)\right]+\frac{\beta_0\sqrt{\pi}\,p_r^{3/2}}{2\nu^{3/2}(1+p_r)^{7/2}}$$

$$\times\exp[fS(t-t_0)]\exp\left[-\nu k^2(t-t_0)\left\{\frac{1+p_r+p_M}{p_M(1+p_r)}\right\}\right]$$

$$\left[\frac{3p_r k^4}{2\nu^2(t-t_0)^{5/2}}+\frac{p_r(7p_r-6)k^6}{3\nu(1+p_r)(t-t_0)^{3/2}}-\frac{4(3p_r^2-2p_r+3)k^8}{3(1+p_r)^2(t-t_0)^{1/2}}+\frac{8\sqrt{\nu}(3p_r^2-2p_r+3)k^9}{3(1+p_r)^{5/2}\sqrt{p_r}}N(\omega)\right]$$

$$\ldots (50)$$

where $N(\omega)=e^{-\omega^2}\int_0^{\omega} e^{x^2}\,dx, \omega = k\sqrt{\dfrac{\lambda(t-t_0)}{p_r(1+p_r)}}$.

The function has been calculated numerically and tabulated in [4].

By setting $\hat{r}=0, j=i, d\hat{K}=-2\pi k^2 d(\cos\theta)dk$ and $Q=2\pi k^2\langle \psi_i \tau_i'(\hat{K})\rangle$ in equation (12), we get the expression for temperature energy decay as

$$\frac{\langle T^2\rangle}{2}=\frac{T_i T_i'}{2}=\int_0^{\infty} Q(\hat{k})d\,\hat{k} \qquad \ldots (51)$$

Substituting equation (50) into (51) and after integration, we get

$$\frac{\langle T^2\rangle}{2}=\frac{N_0 P_r^{3/2} p_M^{3/2}(t-t_0)^{-3/2}}{4\sqrt{\pi}\, v^{3/2}(p_r+p_M)^{3/2}}+\exp[fS]\frac{\beta_0 \pi p_r^{7/2} p_M^{5/2}(t-t_0)^{-5}}{2v^6(1+p_r)(1+p_r+p_M)^{5/2}}\times$$

$$\left\{\frac{9}{16}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}-\frac{35p_M^2(3p_M^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6.p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5\ldots(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\right\}$$

or

$$\frac{\langle T^2\rangle}{2}=\frac{N_0 P_r^{3/2} p_M^{3/2}(t-t_0)^{-3/2}}{4\sqrt{\pi}\, v^{3/2}(p_r+p_M)^{3/2}}+\beta_0 z v^{-6}(t-t_0)^{-5}\times\exp[fS] \qquad \ldots (52)$$

where

$$Z=\frac{\pi p_r^{7/2} p_M^{5/2}}{2(1+p_r)(1+p_r+p_M)^{5/2}}$$

$$\left\{\frac{9}{16}+\frac{5p_M(7p_r-6)}{16(1+p_r+p_M)}-\frac{35p_M^2(3p_r^2-2p_r+3)}{8p_r(1+p_r+p_M)^2}+\frac{8p_M^3(3p_r^2-2p_r+3)}{3.2^6.p_r^2(1+p_r+p_M)^3}\sum_{n=0}^{\infty}\frac{1.3.5\ldots(2n+9)}{n!(2n+1)2^{2n}(1+p_r)^n}\right\}$$

Thus the energy decay law for temperature field fluctuation of dusty fluid MHD turbulence before the final period may be written as

$$\langle T^2\rangle = X(t-t_0)^{-3/2}+\exp[fS]Y(t-t_0)^{-5}, \qquad \ldots (53)$$

where

$$X = \frac{N_0 P_r^{3/2} p_M^{3/2}}{2\sqrt{\pi} v^{3/2}(p_r + p_M)} \qquad \text{and} \qquad Y = 2\beta_0 Z v^{-6}.$$

$\langle T^2 \rangle$ is the total "energy" (the mean square of the temeperature fluctuations) t is the time, x and t_0 are constants determined by the initial conditions. The constant Y depends on both initial conditions and the fluid Prandtl number.

Results and Discussion:

In equation (53) we obtained the decay law of temperature fluctuations in MHD turbulence before the final period in presence of dust particle considering three-point correlation equation after neglecting quadruple correlation terms. If the fluid is clean then $f = 0$, the equation (53) becomes.

$$\langle T^2 \rangle = X(t - t_0)^{-3/2} + Y(t - t_0)^{-5} \qquad \ldots (54)$$

which was obtained earlier by Sarker and Rahman [5]

In the absence of a magnetic field, magnetic Prandtl number coincides with the Prandtl number (i.e. $p_r = p_M$) and the fluid is clean the equation (52) becomes

$$\frac{\langle T^2 \rangle}{2} = \frac{N_0 p_r^{3/2}}{8\sqrt{2\pi} v^{3/2}(t - t_0)^{3/2}} + \frac{\beta_0 Z}{v^6 (t - t_0)^5} \qquad \ldots (55)$$

which was obtained earlier by Loeffler and Deissler [3].

Here we conclude that due to the effect of dust particles in the flow field, the turbulent energy decays more rapidly than the energy for clean fluid.

The 1st term of the right hand side of equation (53) corresponds to the thermal energy for two-point correlation and second term represents thermal energy for three-point correlation. For large times the last term in the equation (53) becomes negligible, leaving the 3/2 power decay law for the final period. If higher order correlations are considered in the analysis, it appears that more terms of higher power of time would be added to the equation (53).