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# APPLICATION OF I-FUNCTION IN FRACTIONAL DIFFERENTIAL OPERATOR INVOLVING GENERALIZED CLASS OF POLYNOMIAL

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## 1. Introduction

The I-function of one variable is defined by Saxena [7] and further studied by Vaishya, Jain and Verma [10], Sharma and Shrivastava [8] with certain properties, Series, Summation, Integration etc. of this function we will represent and define here the I-function of one variable as follows:

$$I_R[Z] = I^{M,N}_{P_i,Q_i;R}\left[Z\middle|\begin{matrix}\ldots,\ldots\\ \ldots,\ldots\end{matrix}\right]$$

$$= I^{M.N}_{P_iQ_i:R}\left[Z\middle|\begin{matrix}[(a_j.\alpha_j)]_{I.N}\,[(a_{j_i}.\alpha_{j_i})]_{N+I.P_i}\\ [(b_j.\beta_j)]_{I.M}\,[(b_{j_i}.\beta_{j_i})]_{M+I.Q_i}\end{matrix}\right] = \frac{1}{2\pi w^{1}}\int 0(s)z^{s}ds \qquad \ldots (1.1)$$

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where $w^1 = \sqrt{-1}$ and

$$0(s) = \frac{\prod_{j=1}^{M}\Gamma(b_j - \beta_j s)\prod_{j=N+1}^{N}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{R}\left[\prod_{j=M+1}^{Q_i}\Gamma(1-b_{i_j}-\beta_{j_i} s)\prod_{j=N+1}^{P_i}\Gamma(a_{j_i}-\alpha_{j_i} s)\right]} \quad \ldots (1.2)$$

The integral (1.1) converge absolutely, if

$$|\arg z| < \frac{\beta_i \pi}{2}, \quad (\beta_i > 0), \quad (A_i = 0)$$

where $$A_i = \sum_{N+1}^{P_i}\alpha_{j_i} - \sum_{j+1}^{Q_i}\beta_{j_i}, \quad \ldots (1.3)$$

and $$B_i = \sum_{j=1}^{N}\alpha_j - \sum_{j=N+1}^{P_i}\alpha_{j_i} + \sum_{j=1}^{M}\beta_j - \sum_{j=M+1}^{\alpha_i}\beta_{j_i} \quad \ldots (1.4)$$

for further details and asymptotic expansion of the above function (1.1) refer to [8] and [9].

## 2. Preliminaries

$$D_x^{\alpha}(ax+b)^{\mu-1} = \frac{a^{\alpha}\Gamma u}{\Gamma\mu-\alpha}.(ax+b)^{\mu-\alpha-1} \quad \ldots (2.1)$$

$$D_{k,\alpha,x}(ax+b)^{\mu-1} = \frac{\Gamma(1+\mu)a^{\alpha}(ax+b)^{\mu+k}}{\Gamma(\mu-\alpha+1)}, \quad \text{where } \alpha \neq \mu \quad \ldots (2.2)$$

$$D_{k,\alpha,x}^{n}(ax+b)^{\mu} = [a]^{\frac{n\mu+n(n-1)K}{2}}\prod_{r=0}^{n-1}\frac{\Gamma(1+\mu+rK)}{\Gamma(1+\mu+rK-\alpha)}(ax+b)^{\mu+nk}\alpha \neq \mu+i$$

$$\ldots (2.3)$$

when $k = \alpha$, equation (2.3) becomes,

$$D^{n}_{\alpha,\alpha,x}(ax+b)^{\mu} = [a]^{\frac{n\mu+n(n-1)K}{2}} \frac{\Gamma(1+\mu+(n-1)\alpha)}{\Gamma(1+\mu-\alpha)}(ax+b)^{\mu+nk} \qquad \ldots (2.4)$$

taking $a = 1$, $b = 0$ in (2.1) to (2.4) we get the results due to Oldham and Spanier [5, Pg. 49]

**3. The General Class of Polynomials**

Srivastava [9] introduced and studied the general class of polynomial. Which is defined in the following manner.

$$S_N^M[x] = \sum_{K=0}^{[N/M]} \frac{(-N)_{MK}}{\lfloor K} A_{N,K}(x^K), \quad (N = 0,1,2\ldots) \qquad \ldots (3.1)$$

where $M$ is an arbitary positive integer and the coefficient $A_{N,K}$ $(N,K \geq 0)$ are arbitary constant real or complex.

**4. Required Results**

The following two theorems given by Goyal, Saxena [2] and Ronghe [6] are required to establish main theorems.

**Theorem Ist**

$$D^{m}_{K,\lambda,-\mu,t}\left\{t^{\lambda-1} S_N^M[wt^{\rho}] f[(ax+b)t]\right\} = \sum_{h=0}^{[N/M]} \frac{(-N)_{Mh}}{\lfloor h} w^h A_{N,h}\, t^{\lambda+\delta h+Km-1} \times$$

$$\times \prod_{p=0}^{m-1} \frac{\Gamma(\lambda+\rho h+\rho K)}{\Gamma(\mu+\delta h+\rho K)} \sum_{n=0}^{\infty} \frac{[-(ax+b)]^n}{\lfloor n} D_x^n\{f(ax+b)\}$$

$$_{m+1}F_m\left(\begin{matrix} -n, \lambda+\rho h, \lambda+\rho h+K, \ldots \lambda+\rho h+(m-1)K \\ \\ \mu+\rho h, \mu+\rho h+K, \ldots \mu+\rho h+(m-1)K \end{matrix} ; t\right) \qquad \ldots (4.1)$$

**Theorem II^nd^**

$$D^{m}_{K,\lambda,-\mu,t}\, t^{A} S_{N}^{M}[wt^{\rho}]f[(ax+b)t] = \sum_{h=0}^{[N/M]} \frac{(-N)_{Mn}}{\lfloor n} w^{h} A_{N,h}\, t^{\lambda+\rho h+Km-1} \prod_{p=0}^{m-1} \frac{\Gamma(\lambda+\rho h+pK)}{\Gamma(\mu+\rho h+pK)}$$

$$\sum_{n=0}^{\infty} \prod_{p=0}^{m-1} \left\{\frac{(1-\mu-\rho h-pK)_{n}}{(1-\lambda-\rho h-pK)_{n}}\right\} \frac{(-t)^{n}}{\lfloor n}.$$

$${}_{m+1}F_{m}\left(\begin{matrix}-n, \lambda+\rho h, \lambda+\rho h-n+K, \ldots \lambda+\rho h-n+(m-1)K \\ \\ \mu+\rho h-n, \mu+\rho h-n+K, \ldots \mu+\rho h-n+(m-1)K\end{matrix} ;t\right) \times$$

$$D_{x}^{n}\left\{(ax+b)^{n} f(ax+b)\right\}, \qquad \ldots (4.2)$$

Result (4.1) and (4.2) are valid if the series involved in required results and absolutely convergent.

## 5. Main Theorems

In this section we will derive two Multiplication theorems with the help of (4.1) and (4.2),

**Theorem 1**

$$(ax+b)^{-b\alpha_i} \sum_{\ell=0}^{[N/M]} \frac{(-N)_{M\ell}}{\lfloor \ell} A_{N,\ell} w^{\ell} t^{\lambda+\rho\ell+b\alpha_i+mK-1} I^{M,N+m}_{P_i+m,Q_i+m;R}$$

$$\left[ [(ax+bt]^{\beta Q_i} \left| \begin{matrix} (a_j,\alpha_j)_n, (1-\lambda+\delta\ell+b\alpha_i-pK, \beta\alpha_i)_{p=0,1,\ldots m-1}, (a_{j_i},\alpha_{j_i})_{N+1,P_i} \\ (b_j,\beta_j)_{1,M}, (1-\mu+\delta\ell+b\alpha_i-pK, \beta\alpha_i)_{p=0,\ldots m-1}, (b_{j_i},\beta_{j_i})_{M+1,Q_i} \end{matrix} \right. \right]$$

$$\sum_{h=0}^{[N/M]} \frac{(-N)_{Mn}}{\lfloor n} A_{N,h} w^{h} t^{\lambda+\delta h+Km-1} \prod_{p=0}^{m-1} \frac{\Gamma(\lambda+\delta h+pK)}{\Gamma(\mu+\delta h+pK)}$$

$$ {}_{m+1}F_m\left(\begin{matrix} -n, \lambda+\delta h, \lambda+\delta h+K, \ldots \lambda+\delta h+(m-1)K \\ ;t \\ \mu+\delta h, \mu+\delta h-n+K, \ldots \mu+\delta h+(m-1)K \end{matrix}\right) .a^n (ax+b)^{b\alpha_i} $$

$$ \times I^{M,N+m}_{P_i,Q_i;R}\left[(ax+b)^{\beta Q_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_{j_i}.\alpha_{j_i})_{N+1,P_i} \\ (b_j.\beta_j)_{1,M}(b\alpha_i+n,\beta\alpha_i)(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right] \quad \ldots (5.1) $$

**Theorem 2**

$$ (ax+b)^{-a_i}\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor \ell}A_{N,\ell}w^{\ell}t^{\lambda+\delta\ell-a_i+mK}I^{M,N+m}_{P_i+m,Q_i+m;R} $$

$$ \left[[(ax+b)t]\left|\begin{matrix}(a_j.\alpha_j)_{1,N}\,(a_1-\lambda+\delta\ell-PK,\alpha_i)_{p=0,1,\ldots m-1},(a_{j_i}.\alpha_{j_i})_{N+1,P_i} \\ (b_j.\beta_j)_{1,M}\,(a_1-\mu+\delta\ell-PK,\alpha_i)_{p=0,1,\ldots m-1},(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right] $$

$$ = a^n(ax+b)^{-a}\sum_{h=0}^{[N/M]}\frac{(-N)_{Mh}}{\lfloor h}A_{N,h}w^h t^{\lambda+\delta h+Km-1}\prod_{p=0}^{m-1}\frac{\Gamma(\lambda+\delta h+PK)}{\Gamma(\mu+\delta h+PK)} $$

$$ \prod_{p=0}^{m-1}\frac{\Gamma(1-\mu-\delta h-PK)_n}{\Gamma(1-\lambda-\delta h-PK)_n}\frac{(-t)^n}{\lfloor n}. $$

$$ {}_{m+1}F_m\left(\begin{matrix} -n, \lambda+\delta h-n, \lambda+\delta h-n+K, \ldots \lambda+\delta h-n+(m-1)K \\ ;t \\ \mu+\delta h-n, \mu+\delta h-n+K, \ldots \mu+\delta h-n+(m-1)K \end{matrix}\right) $$

$$ I^{M,N+m}_{P_i,Q_i;R}\left[(ax+b)^{\alpha_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_i-n;\alpha_i)_{N+1},(a_{j_i},\alpha_{j_i})_{N+1,P_i} \\ (b_j.\beta_j)_{1,M}(b_{j_i},\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right] \quad \ldots (5.2) $$

Provided Re $(-a_1 + max\ a_1\ (a_j/\alpha_j)) > 0$, $|\arg z| < \dfrac{\beta_i\pi}{2}$,

**Proof of the Multiplication Theorem I[st]**

For proof of (4.1) we take,

$$f(ax+b)=(ax+b)^{-bQ_i}\, I^{M,N}_{P_i,Q_i;R}\left[(ax+b)^{\beta Q_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_{j_i}.\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]$$

so the R.H.S. of (4.1) becomes,

$$\sum_{h=0}^{[N/M]}\frac{(-N)_{Mn}}{\lfloor h}w^h A_{N,h}t^{\lambda+\delta h+Km-1}\prod_{p=0}^{m-1}\frac{\Gamma(\lambda+\delta h+PK)}{\Gamma(\mu+\delta h+PK)}\sum_{n=0}^{\infty}\frac{(-1)^n}{\lfloor n}$$

$${}_{m+1}F_m\left(\begin{matrix}-n,\lambda+\delta h,\lambda+\delta h+K,\ldots\lambda+\delta h+(m-1)K\\ \\ \mu+\delta h,\mu+\delta h+K,\ldots\mu+\delta h+(m-1)K\end{matrix};t\right)$$

$$D^n_x\left\{(ax+b)^{-bQ_i}\, I^{M,N}_{P_i,Q_i;R}\left[(ax+b)^{\beta Q_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_{j_i}.\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]\right\}$$

Now we have,

$$D^n_x\left\{(ax+b)^{-bQ_i}\, I^{M,N}_{P_i,Q_i;R}\left[(ax+b)^{\beta Q_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_{j_i}.\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]\right\}$$

$$\Rightarrow D^n_x\left\{(ax+b)^{-bQ_i}\frac{1}{2\pi\omega}\int\theta(s).(ax+b)^{\beta\alpha_i s}\,ds\right\}$$

$$\Rightarrow\frac{1}{2\pi\omega}\int_L\theta(s).D^n_x(ax+b)^{-b\alpha_i s+\beta\alpha_i s-1+1}\,ds$$

$$\Rightarrow \frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j s)\prod_{j=1}^{N}\Gamma(1-a_j-\alpha_j s)}{\sum_{i=1}^{R}\left[\prod_{j=M+1}^{Q_i}\Gamma(1-b_{i_j}-\beta_{j_i}s)\prod_{j=N+1}^{P_i}\Gamma(a_{j_i}-\alpha_{j_i}s)\right]\Gamma(n-a_1+\alpha_1 s+1-n)}.ds$$

$$\Rightarrow \frac{1}{2\pi\omega}\int_L \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j s)\prod_{j=1}^{N}\Gamma(1-a_j+\alpha_j s)\Gamma[1-(a_1-n)+\alpha_1 s]a^n(ax+b)^{-a_1}(ax+b)^{-\alpha_1 s}}{\sum_{i=1}^{R}\left[\prod_{j=M+1}^{Q_i}\Gamma(1-b_{i_j}-\beta_{j_i}s)\prod_{j=N+1}^{P_i}\Gamma(a_{j_i}-\alpha_{j_i}s)\right]}.ds$$

$$\Rightarrow a^n(ax+b)^{-a_1} I^{M,N+m}_{P_i,Q_i;R}\left[(ax+b)^{\alpha_i}\left|\begin{matrix}(a_j.\alpha_j)_{2,N}(a_i-n;\alpha_i)(a_{j_i},\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i},\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]$$

therefore the R.H.S. of (4.2) becomes

$$\sum_{h=0}^{[N/M]}\frac{(-N)_{Mh}}{\lfloor h}A_{N,h}t^{\lambda+\delta h+Km-1}\prod_{p=0}^{m-1}\frac{\Gamma(\lambda+\delta h+PK)}{\Gamma(\mu+\delta h+PK)}\prod_{p=0}^{m-1}\frac{\Gamma(1-\mu-\delta h-PK)}{\Gamma(1-\lambda-\delta h-PK)}\frac{(-t)^n}{\lfloor n}$$

$${}_{m+1}F_m\left(\begin{matrix}-n,\lambda+\delta h-n,\ldots\lambda+\delta h-n+(m-1)K\\ ;t\\ \mu+\delta h-n,\ldots\mu+\delta h-n+(m-1)K\end{matrix}\right).a^n(ax+b)^{-a_1}$$

$$I^{M,N+m}_{P_i,Q_i;R}\left[(ax+b)^{\alpha}\left|\begin{matrix}(a_j.\alpha_j)_{2,N}(a_i-n;\alpha_i)(a_{j_i},\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i},\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]$$

Also the L.H.S. of (4.2) becomes

$$D^m_{K,\lambda,-\mu,t}\{t^\lambda S^M_N[wt^\delta]f[(ax+b)t]\}$$

$$\Rightarrow D^m_{K,\lambda,-\mu,t}\left\{t^\lambda\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor \ell}A_{N,\ell}w^\ell t^{\delta\ell}[(ax+b)t]^{-a}\right.$$

$$\left.I^{M,N+m}_{P_i,Q_i;R}\left[[(ax+b)]^{\mu_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_{j_i},\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i},\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]\right\}$$

$$\Rightarrow D^m_{K,\lambda,-\mu,t}\left\{t^{\lambda}\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}A_{N,\ell}w^{\ell}t^{\delta\ell}(ax+b)t^{-a_1}\frac{1}{2\pi\omega}\int_L\theta(s).(ax+b)^{\alpha_i s}t^{\alpha_i s}ds\right\}$$

$$\Rightarrow\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}w^{\ell}A_{N,\ell}(ax+b)^{-a_i+\alpha_i s}\frac{1}{2\pi\omega}\int\theta(s).D^m_{K,\lambda,-\mu,t}(t^{-\lambda+\delta\ell-a_i+\alpha_i s})ds$$

$$\Rightarrow\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}w^{\ell}A_{N,\ell}(ax+b)^{-a_i+\alpha_i s}\frac{1}{2\pi\omega}$$

$$\int\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j s)\prod_{j=1}^{N}\Gamma(1-a_j-\alpha_j s)\prod_{p=0}^{m-1}\Gamma(\lambda+\delta\ell-a_1 s+1+PK)t^{\lambda+\delta l-a_1+\alpha_i s+mK}}{\sum_{i=1}^{R}\left[\prod_{j=M+1}^{Q_i}\Gamma(1-b_{i_j}-\beta_{j_i}s)\prod_{j=N+1}^{P_i}\Gamma(a_{j_i}-\alpha_{j_i}s)\right]\prod_{p=0}^{m-1}\Gamma(\lambda+\delta\ell-a_1 s+1+PK-\lambda+\mu)}$$

$$\Rightarrow a^n(ax+b)^{-a_1}\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}A_{N,\ell}w^{\ell}t^{\lambda+\delta\ell-a_1+mK}\frac{1}{2\pi\omega}\qquad\text{(by using 2.3)}$$

$$\int 0(s).\prod_{p=0}^{m-1}\frac{[\Gamma 1-(a_1-\lambda-\delta\ell-PK)+\alpha_i s]}{[\Gamma 1-(a_1-\delta\ell-PK-\mu)+\alpha_i s]}.(ax+b)^{-\alpha_1 s}.t^{-\alpha_1 s}ds$$

$$\Rightarrow(ax+b)^{-a_1}\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}w^{\ell}A_{N,\ell}t^{\lambda+\delta\ell-a_1+mK}$$

$$I^{M,N+m}_{P_i+m,Q_i+m;R}\left[[(ax+b)t]^{\alpha_i}\left|\begin{matrix}(a_j.\alpha_j)_{1,n}(a_i-\lambda+\delta\ell-PK)_{p=0,1,\ldots m-1},(a_{j_i}.\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(a_i-\delta\ell-PK-\mu)_{p=0,\ldots m-1},(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]$$

so from L.H.S. and R.H.S. of above equation we get the desired result (5.2)

**Applications**

(i) Considering for theorem *I*, If $a = 1$, $b = 0$ in (5.1) we have,

$$\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}A_{N,\ell}w^{\ell}t^{\lambda+\delta\ell-b_{a_i}+mK-1}$$

$$I^{M,N+m}_{P_i+m,Q_i+m;R}\left[(xt)^{\beta_{Q_i}}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(1-\lambda-\delta\ell+b_{Q_i}-PK;\beta_{Q_i})_{p=0,1,\ldots m-1,}(a_{j_i}.\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(1-\mu-\delta\ell+b_{Q_i}-PK;\beta_{Q_i})_{p=0,\ldots m-1,}(b_{j_i}.\beta_{j_i})_{M+1,Q_i}\end{matrix}\right.\right]$$

$$=\sum_{h=0}^{[N/M]}\frac{(-N)_{Mn}}{\lfloor h}A_{N,h}w^h t^{\lambda+\delta h+mK-1}\prod_{p=0}^{m-1}\left[\frac{\Gamma(\lambda+\delta h+PK)}{\Gamma(\mu+\delta h+PK)}\right]\sum_{n=0}^{\infty}(-1)^n$$

$${}_{m+1}F_m\left(\begin{matrix}-n,\lambda+\delta h-n,\ldots\lambda+\delta h+(m-1)K\\ \\ \mu+\delta h,\mu+\delta h+K\ldots\mu+\delta h-n+(m-1)\end{matrix};t\right)$$

$$I^{M,N+m}_{P_i+m,Q_i+m;R}\left[(xt)^{\beta_{Q_i}}\left|\begin{matrix}(a_j.\alpha_j)_{1,N}(a_{j_i}.\alpha_{j_i})_{N+1,P_i}\\(b_j.\beta_j)_{1,M}(b_{j_i}.\beta_{j_i})_{M+1,Q_i}(b_{Q_i}.\beta_{Q_i})\end{matrix}\right.\right]$$

The condition (4.1) are satisfied.

(ii) Reconsidering for theorem I. If $a = 1$, $b = 0$, $R = 1$ in (5.1) it reduced result of Nigam [4], involving Fox's H-function [1]

$$\sum_{\ell=0}^{[N/M]}\frac{(-N)_{M\ell}}{\lfloor\ell}A_{N,\ell}w^\ell t^{\delta\ell-b_\alpha}H^{M,N+m}_{P+m,Q+m}\left[(xt)^{\beta_Q}\left|\begin{matrix}(a_j.\alpha_j)_{1,P}(1-\lambda-\delta\ell+b_Q-PK;\beta_Q)_{p=0,1,\ldots m-1}\\(b_j.\beta_j)_{1,Q}(1-\mu-\delta\ell+b_Q-PK,\beta_Q)_{p=0,\ldots m-1}\end{matrix}\right.\right]$$

$$=\sum_{h=0}^{[N/M]}\frac{(-N)_{Mn}}{\lfloor h}A_{N,h}w^h t^{\delta h}\prod_{p=0}^{m-1}\left[\frac{\Gamma(\lambda+\delta h+PK)}{\Gamma(\mu+\delta h+PK)}\right]\sum_{n=0}^{\infty}(-1)^n$$

$${}_{m+1}F_m\left(\begin{matrix}-n,\lambda+\delta h-n,\ldots\lambda+\delta h+(m-1)K\\ \\ \mu+\delta h,\mu+\delta h+K\ldots\mu+\delta h-n+(m-1)K\end{matrix};t\right).H^{M,N}_{P,Q}\left[(xt)^{\beta_Q}\left|\begin{matrix}(a_j.\alpha_j)_{1,P}\\(b_j.\beta_j)_{1,Q-1}(b_\alpha+n.\beta_Q)\end{matrix}\right.\right]$$

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