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SKIN FRICTION AND MASS TRANSFER IN POROUS MEDIUM USING BRINKMAN-FORCHHEIMER MODEL

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Abstract

The flow of a steady incompressible viscous fluid through porous medium confined between two horizontal walls is presented by using Brinkman-Forchheimer model. The governing equation of motion is solved analytically by perturbation method. The solution obtained for the velocity profile is presented graphically for various values of the permeability parameter (σ) and the Ergun parameter (E_r). It is found that σ and E_r affects the flow significantly. The skin-friction coefficient and average velocity are tabulated and both of them increase with the decrease in the values of the parameters σ and E_r.

Key words : Viscous fluid, porous medium, Brinkman-Forchheimer model, skin-friction coefficient.

1. Introduction

A study of flow of viscous fluids through porous medium is of utmost importance because of its applications in a large number of fields of natural sciences as well as several branches of technology. These include geophysics, soil mechanics, rheology, metal casting, ceramic engineering, and the technology of paper, textiles, and insulating materials. The mathematical theory of flow of a fluid through porous medium was initiated by Darcy [1]. For the steady flow it was postulated that the viscous forces were large in comparison to inertia forces so that the latter could be neglected and that led him to conclude that the viscous forces were in equilibrium with the external forces due to pressure differences and body forces. Deviations from Darcy's law occur when the Reynolds number based on the mean pore diameter exceeds a value in the range 1-10. Inertial forces then become comparable to viscous forces. Several empirical formulations have been developed to account for such effects. More recent formulations, such as those developed by Brinkman [2], Ergun [3], and others, account for the thermal properties of both fluid and porous material Sharma [4], Sharma and Barman [5], Srivastava and Sharma [6], Shrivastava and Srivastava [7], Padmavathi et. al. [8], Shrivastava and Saxena [9], and many others [10-21] applied Brinkanam model to study problems under various geometries.

The results obtained by various workers discussed above may be extended by taking into account the thermal properties of both fluid and porous material, to gain generality, by employing Brinkman-Forchheimer model. In this paper we have studies the flow of an incompressible viscous fluid due to a constant pressure gradient in a porous medium bounded by two parallel impervious walls. The equations governing the motion are solved analytically by using perturbation technique. The velocity profile is represented graphically to exhibit the effects of permeability parameter and Ergun parameter. The average velocity and the skin-friction coefficient have also been tabulated for various values of the parameters.

2. Formulation of the problem:

We consider here two-dimensional steady flow of a viscous incompressible fluid flowing through a porous medium bounded by two parallel impervious walls (see Fig. 1) using Cartesian coordinate system. The x-axis is taken in the middle of the channel parallel to the walls and the y-axis perpendicular to the walls of the channel i.e. both the plates lie at $y = h$ and $y = -h$. The walls of the channel at $y = h$ and $y = -h$ are taken to be at rest. The flow is due to the constant pressure gradient present in the medium.

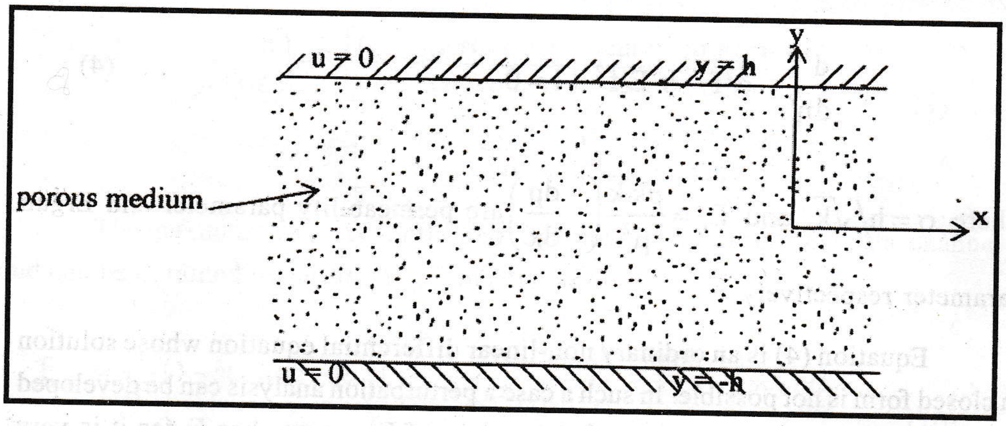

Fig. 1. Geometry of the problem

We assume the velocity components in the form $[u(y), 0, 0]$. For this form of velocity components the equation of continuity is satisfied identically and the equation of motion under the above assumption becomes

$$\frac{d^2u}{dy^2} - \frac{u}{k} - \frac{\rho k'}{\mu k} u^2 = \frac{1}{\mu}\frac{dp}{dx} \qquad \ldots (1)$$

where k, k', μ, ρ, p are permeability coefficient, inertial coefficient, coefficient of viscosity, density of the fluid and the pressure in the fluid respectively.

The boundary condition of the problem are

$$u = 0 \text{ at } y = \pm h. \qquad \ldots (2)$$

Assuming $y = h\eta$ in the equation (1) we get

$$\frac{d^2u}{d\eta^2} - \frac{uh^2}{k} - \frac{h^2\rho k'}{\mu k} u^2 = \frac{h^2}{\mu}\frac{dp}{dx} \qquad \ldots (3)$$

Substituting the non-dimensional form of velocity as $u = -\frac{h^2}{\mu}\frac{dp}{dx} f(\eta)$ in equation (3), we get

$$\frac{d^2 f}{d\eta^2} - \sigma^2 f - \sigma^4 E_r f^2 + 1 = 0 \quad \ldots (4)$$

where $\sigma = h/\sqrt{k}$ and $E_r = \frac{\rho k' k}{\mu^2}\left(-\frac{dp}{dx}\right)$ are permeability parameter and Ergun parameter respectively.

Equation (4) is an ordinary non-linear differential equation whose solution in closed form is not possible. In such a case a perturbation analysis can be developed for the non-dimensional velocity $f(\eta)$ in terms of Ergun number E_r (as it is very small) and hence

$$f = f_0 + E_r f_1 + \ldots \quad \ldots (5)$$

Substituting (5) in (4) and comparing various powers of E_r (neglecting E_r^2 and its higher powers), we get

$$\frac{d^2 f_0}{d\eta^2} - \sigma^2 f_0 + 1 = 0 \quad \ldots (6)$$

and $$\frac{d^2 f_1}{d\eta^2} - \sigma^2 f_1 - \sigma^4 f_0^2 = 0 \quad \ldots (7)$$

The boundary conditions (2) for the problem become

$$f_0 = f_1 = 0 \quad \text{at} \quad \eta = \pm 1 \quad \ldots (8)$$

3. Solution of the problem:

Equations (6)-(7) are solved under the boundary condition (8) and the solution is obtained as

$$f=\frac{1}{\sigma^2}\left(1-\frac{\cosh\sigma\eta}{\cosh\sigma}\right)+E_r\left[\left(\frac{1}{\sigma^2}-\frac{\cosh 2\sigma-3}{6\sigma^2\cosh^2\sigma}+\frac{\tanh\sigma}{\sigma}\right)\frac{\cosh\sigma\eta}{\cosh\sigma}-\frac{1}{\sigma^2}+\frac{\cosh 2\sigma\eta-3}{6\sigma^2\cosh^2\sigma}-\frac{\eta\sinh\sigma\eta}{\sigma\cosh\sigma}\right]. \quad \ldots(9)$$

The maximum velocity for the above geometry is at the center of the channel and can be obtained by putting $\eta=0$ in (9) as

$$f_{max.}=\frac{1}{\sigma^2}(1-\mathrm{sech}\,\sigma)+E_r\left[\left(\frac{1}{\sigma^2}-\frac{\cosh 2\sigma-3}{6\sigma^2\cosh^2\sigma}+\frac{\tanh\sigma}{\sigma}\right)\mathrm{sech}\,\sigma-\frac{1}{\sigma^2}-\frac{1}{3\sigma^2}\mathrm{sech}^2\sigma\right]. \quad \ldots(10)$$

The average velocity is obtained by integrating

$$f_{average}=\frac{1}{2}\int_{-1}^{1}f(\eta)d\eta \quad \text{yielding}$$

$$f_{average}=\frac{1}{2\sigma^2}\left(1-\frac{\tanh\sigma}{\sigma}\right)+\frac{E_r}{2}\left[\left(\frac{2}{\sigma^2}-\frac{\cosh 2\sigma-3}{6\sigma^2\cosh^2\sigma}+\frac{\tanh\sigma}{\sigma}\right)\frac{\tanh\sigma}{\sigma}-\frac{2}{\sigma^2}+\frac{\sinh 2\sigma-6\sigma}{12\sigma^3\cosh^2\sigma}\right]. \quad \ldots(11)$$

The expression for the skin-friction coefficient can be obtained as

$$c_f=\frac{2}{R_e}\times\frac{f'(1)}{(f_{average})^2}, \quad \ldots(12)$$

where average velocity is given by (11) and the $f'(1)$ is given as follows

$$f'(1) = -\frac{\tanh\sigma}{\sigma} + E_r\left[\left(\frac{2}{3\sigma} - \frac{\cosh 2\sigma - 3}{6\sigma\cosh^2\sigma} + \tanh\sigma\right)\tanh\sigma - 1\right] \quad \ldots (13)$$

4. Results and discussion:

The graph of velocity profile f(η) is plotted in Fig.2 for different combinations of the values of the parameters σ and E_r viz. σ = 0, E_r = 0; σ = 1, E_r = 0; σ = 2, E_r = 0; σ = 2, E_r = 0.2; σ = 2, E_r = 0.4. against the width of the channel from -1 to 1. From the graph it can be concluded that the flow is maximum if we neglect the combined effect of the parameters σ and E_r. The flow decreases as we increases the values of the parameter σ keeping the parameter Er = 0 fixed. This concludes that the effect of the parameterσ is to decrease the flow in the porous media. The graph also reveals that the increases in the values of the parameter E_r such as 0, 0.2, 0.4 keeping the parameter σ = 2 fixed decreases the flow. Therefore, this can be concluded from the graph that the combine effect of the parameters σ and E_r is to decrease the flow in the porous medium.

Finally, the effects of the parameters σ and E_r on the skin-friction coefficient cf and the average velocity are shown in the Table 1. The average velocity explains the rate of mass transfer as the rate of mass transfer is the product of average velocity and a constant (the product of density and the cross section of the channel). The discussion regarding the behaviour of the parameters on skin-friction coefficient and the rate of mass transfer are self evident from the table and hence are not discussed for brevity.

5. Conclusions

Thus we conclude that inclusion of Forchheimer [22] term in the equation of motion results in (i) decrease of magnitude of the velocity in the channel (ii) reduction in the magnitude of the average velocity in the channel consequently reduces the rate of mass transfer in the channel and (iii) reduction in the magnitude of the coefficient of skin friction coefficient.

Table. 1. Average velocity and the skin friction coefficient.

E_r \ σ	Average Velocity 1	2	3	E_r \ σ	Skin-friction Coefficient at $R_e = 8$ 1	2	3
0	0.119203	0.064749	0.037129	0	-13.3995	-28.7437	-60.15174
0.1	0.118180	0.062254	0.034881	0.1	-13.5483	-30.3665	-66.03868
0.2	0.117156	0.059759	0.032634	0.2	-13.7003	-32.1657	-73.03255
0.3	0.116132	0.057264	0.030387	0.3	-13.8555	-34.17	-81.44953

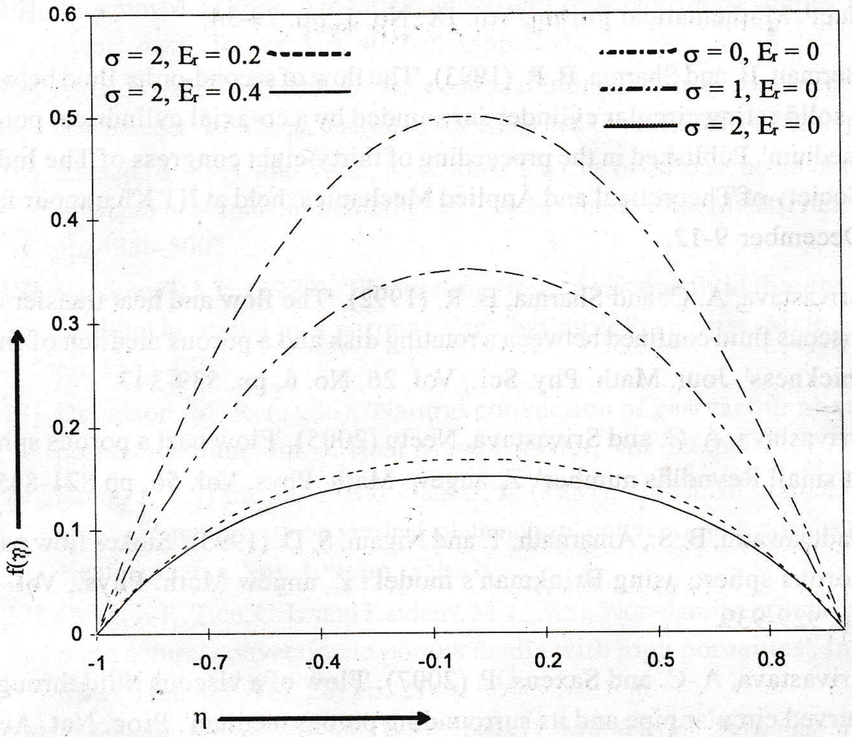

Fig. 2. The velocity profile against the breadth of the channel for various Values of σ and E_r.

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