Mathematical Forum
Vol.XXI, 2008-2009

MODELLING A SOLUTION CONCEPT TO COOPERATIVE GAMES WITH FUZZY COALITIONS THROUGH NEGOTIATION VIA MEDIATOR

Surajit Borkotokey
Department of Mathematics, Dibrugarh University,
Dibrugarh, India-786004
surajitbor@yahoo.com

Abstract

We have considered the problem of distribution of payoffs among the players in an n-person cooperative game with fuzzy or partial participation through mediated negotiation. We have provided a framework in which each of the participating players provides the membership value of satitsisfaction upon receiving a proposal offered by the mediator. The corresponding convergence theorem is established.

AMS Subject Classiffcations [2000] 91A12, 91A99,03E72

Key words : Fuzzy coalitions, Belief, core solution, negotiation process.

1. Introduction

A cooperative game with side payments can be completely characterized by a real valued set function v, called the characteristic function over the set of all possible coalitions from the players' set. This characteristic function assigns to each coalition, a nonnegative real number called the worth of that coalition. A solution is a distribution of payoffs to the players which is a vector in $\mathbf{R}^n$.

In crisp games, the members are forming coalitions with full participation, however this idea is not very interesting while dealing with practical situations. There are numerous instances (see [8, 9]), where players would prefer only partial participations in the coalitions. To model such situations Aubin [5], Tijs [8], Tsurumi [7] et. al. extended the notion of crisp games into fuzzy games where players are participating partially. Thus in their games, a characteristic function assigns to each fuzzy coalition a real number. Consequently, the corresponding solution concepts were also modified.

Here, we consider a class of fuzzy games which are recently studied by Azrieli and Lehrer [10]. This class seems to be more general than the other existing classes and includes the class of crisp games as a subclass.

We take $N = \{1, 2, 3, \ldots, n\}$ as a finite set (the set of players). For every non-negative vector $Q = (q_1, q_2, \ldots, q_n) \in \mathbf{R}^n$, let $F(Q)$ be the box given by

$$F(Q) = \{c \in \mathbf{R}^n : 0 \leq c \leq Q\}$$

The point Q is interpreted as the 'grand coalition' in fuzzy sense. A fuzzy game is a pair (Q, v) such that

(i) $Q \in \mathbf{R}^n$ and $Q \geq 0$.

(ii) $v : F(Q) \to \mathbf{R}$ is bounded and satisfies $v(0) = 0$

where $0 \in F(Q)$ is the zero vector signifying 0-membership to all the players. [see [10] for more details.]

A core solution to the game (Q, v) is a payoff distribution $x \in \mathbf{R}^n$ such that

(i) $xc \geq v(c)$

(ii) $xQ = v(Q)$

Following the dynamic process of negotiation and bargaining, Lai and Lin [1] suggests a fuzzy set of satisfaction for each player on the basis of which they would decide whether or not to accept a proposal. They propose that a player would offer a proposal which has to be accecpted by all the other players based on his/her satisfaction level. The proposals are so made that finally an agreement among the players would be reached that benefits all agents with a high degree of satisfaction. Furthermore, Lehrer [4] uses Blackwell Approachability Principle in obtaining a solution vector through an allocation process while Yager [2] shows that a solution can be achieved through some mediation steps. Tohme et al [3] proposes a process of coalition formation by taking help from a belief function. No literature till date is found on the Dynamic Process of Fuzzy Coalition Formation and Allocation according to their rate of participation.

In this paper, we have considered the problem of distribution of payoffs among the players in an n-person cooperative game with fuzzy or partial participation through mediated negotiation.

We have provided a framework in which each of the participating players provides the membership value of satitsisfaction upon receiving a proposal offered by the mediator. The objective of our study is to provide a systematic treatment of satisfaction level as a basis for negotiation among rational agents, who can participate in various coalitions with varied rate of participations simultaneously. The mediator has her own preference of distributions and would offer alternative proposals to the players judging on their reactions to the previous offers. A stopping rule is developed and the process of updating the belief of the mediator by use of a suitable probability measure towards the possible reactions of the players upon different offers is proposed. Furthermore, a similarity relation is defined to measure the similarity between the satisfaction levels of the mediator and the individual players over a single proposal. The negotiation strategy is so designed that the similarity value thus defined would be increased at each stage of the negotiation process and finally in the limiting case it converges to 1 where the game will have a solution. The solution thus obtained is found to be a core solution. What we have also kept in mind that in the negotiation process, each of the players has a single motive: maximizing its individual payoff, which is well represented by the monotonic increasing functions characterising the fuzzy sets of their satisfactions. The mediator, on the other hand wants to restrict them to go beyond a controllable limit. However, negotiation appeals a player to accommodate the desires and views of all the other players. This suggests that an appropriate negotiation process should reward those players who are more open in forming coalitions. Our model shows that the negotiation process thus defined speeds up for cooperating players. We provide an example to show the usefulness of our proposed model.

2. Our Model

A set of $n+1$ players comprising of a set of n self interested players each having a vague idea of forming coalitions with the others and a mediator with her own ineterests represented by a fuzzy set of satisfaction is considered. All the players save the mediator would offer their participations in various coalitions among them. If Q represents the vector whose i^{th} coordinate signifies the total resource of the ith player then we call Q as the grand coaltion and any $c \sqsubseteq Q$ is a possible fuzzy coalition. A coalition structure $\{c^j\}_{j=1}^m$ is a set of all coalitions formed by the players by offering their resources. After finalizing the participation rates in a coalition c^j (say) and calculating its worth, its time for the mediator to offer a proposal to the players about their respective payoffs (an n tuple of real numbers). Upon receipt of an offer at the i^{th} stage, the players would react by announcing their level of satisfaction. The mediator, unaware of the actual fuzzy sets of satisfaction of the players instead updates her belief (fuzzy sets of the satisfactions of the players as she believes them to be) from the hitherto information of the players. The mediator proposes the next offer so that the similarity relation between her own satisfaction degree and the satisfaction of the players as she believes is more close than the previous. When all the players are satisfied, the negotiation stops and the corresponding proposal would be the required solution with respect to the coalition.

Furthermore let $S\_M_i(.)$ represent the fuzzy set of satisfaction of the mediator in offering a proposal to the i^{th} player. We denote and define Similarity between two vectors of fuzzy sets μ and ν as follows:

Definition 2.1. *For $z \in \mathbf{R}$, given the universal set X, and vectors $\mu = <\mu_i>_{i=1}^{n}$ and $\nu = <\nu_i>_{i=1}^{n}$, with each μ_i and ν_i a fuzzy subset of X, the similarity between μ and ν is given by*

$$Sim(\mu(z), \nu(z)) = 1 - \frac{1}{n}\sqrt{\sum_{i=1}^{n}(\mu_i(z) - \nu_i(z))^2},$$

Definition 2.2. *Possibility that a given offer is accepted by all the players is given by:*

$$Po(A|z) = Sim(\mu^t(z), S\_M(z))$$

μ^t is the belief function of the mediator at the t^{th} stage.

The Belief Function is defined as follows:

The initial belief function $\mu^0(.)$ is determined by the mediator. At $(t+1)^{th}$ stage, the i^{th} component of the belief function μ^{t+1} is defined as :

$$\mu_i^{t+1}(z) = Po(A\,|\,z) S\_P_i(x_t) \vee \mu_i^t(z),\ z \neq x_1, \ldots x_t$$
$$= S\_P_i(x_k);\ z = x_k,\ k = 1, 2, \ldots, t.$$

for all $z \in C^{t+1}$.

Where, x_t is the offer made in the t^{th} stage and $S\_P_i(.)$ is the fuzzy set of satisfaction of the i^{th} player.

Let, $C^{t+1} = \{z \,|\, (\mu_i^{t+1}(z) \geq \alpha_i) \wedge (S\_M_i(z) \geq \beta_i), \forall\, i = 1, 2, \ldots, n\}$

α : Acceptable Threshold computed in the begining as $\alpha_i = S\_P_i(X(i, c^j))$.

β : Presentable Threshold. If negotiation is logged down, it may be modified.

The mediator offers the initial payoff distribution : $X(i;c^j)$. Let $x^t = X(i;c^j)$ when $t = 1$.The players announce their satisfaction memberships $S\_P_i(x^t)$ at every stage t. The Mediator updates her belief function $\mu^t(.)$ to form $\mu^{t+1}(.)$ based on these information.

Let $D^{t+1} = \{z \mid Sim(S\_M(z), \mu^{t+1}(z)) \geq Sim(S\_M(x_t), S\_P(x_t))\}$. The mediator chooses the next offer $x_{t+1} \in \text{argmin}_{z \in D^{t+1}} \{\| S\_M(z) - \mu^{t+1}(z) \|\}$.

2.1 Protocol

Step 1 : Each Player i will announce her degree of satisfaction upon receiving the offer made by the mediator, based on her own satisfaction represented by a fuzzy set $SP_i(.)$.

Step 2 : A Stopping Rule is tested : "'Whether all the players are satisfied?'"

1. If the condition is met GOTO step 3.
2. If the condition is not met GOTO step 4.

Step 3: The Solution is obtained and the process terminates.

Step 4: Every player's degree of satisfaction at the tth stage is used by the mediator to update her belief for the $t+1^{\text{th}}$ stage. She will then choose from a possible set of alternatives, the best distribution under some rationality condition.

Step 5: GOTO step 1.

2.2 Main Theorem

Before stating and proving the main theorem, we would like to make the following assumptions:

1. **Assumption :** Players' domains, objects are all finite so that during the course of negotiation, the mediator at a given satisfaction value can only submit finite offer. Searching process does not repeat infinitely.
2. **Assumption :** The characteristic function representing the fuzzy set of satisfaction of all the players are continuous.
3. **Assumption :** The mediator wants the players to get satisfied with the offer, however her interest is not to give payoffs at her company's loss. Thus the set $D^t \neq \theta$.
4. **Assumption :** Each decrease of tolerance if exists is minimized so that no possible offer exists between two presentable thresholds β_t and β_t+1.

The following theorem states that there exists a solution to the game in the sense that if all the players agree to a consensus then the negotiation process stops there.

Theorem 2.3. *The negotiation will terminate and converge on a solution through the protocol and strategies used by the players and the mediator.*

Proof. At each stage, D_t gets reduced so that the similarity between the satisfaction levels of the players as believed by the mediator and the mediator herself get closer to 1. Thus the negotiation process will converge on a solution say $x^*.x_t \rightarrow x^*$ as $t \rightarrow \infty$. It remains to show $\mu_i^t(x^*) - S\_P_i(x^*)$ as $t \rightarrow \infty$ for all i. If $\mu_{t+1}^i(x_{t+1}) = \mu_t^i(x_t)$, The mediator goes to the next tolerance level β_i, $\forall i$. As $t \rightarrow \infty$, $x_t \rightarrow x^*$, $S\_P_i(x_t) \rightarrow S\_P_i(x^*)$. $\forall i$. (All players are satisfied at x^*). We observe that $Po(x^*|x_t) \rightarrow 1$, as $t \rightarrow \infty$.

Theorem 2.4. *At each stage of the negotiation process, the set of possible solutions to the game is getting smaller. This reduces, at each stage, the labour of seeking alternative solutions. Thus the negotiation process converges rapidly.*

Theorem 2.5. *When there is a solution, as a result of the agreement among the players,no further improvement of the distribution gives a better solution with increased satisfaction to all the players including the mediator simultaneously. A Pareto optimality condition.*

Example 2.6. Let $v(1) = v(2) = v(3) = 100$, $v(1;2) = v(2;3) = v(1;3) = 300$, $v(1;2;3) = 600$. Let $c = (0.5, 0.3, 0.2)$ and using Choquet Integral Type Fuzzy game, we have : $v(c) = 420$. $v(0\|x_1) = 100 \times 0.5$, $v(0\|x_2) = 100 \times 0.3$, $v(0\|x_3) = 100 \times 0.2$.

First Offer (210, 126, 84)

Second Offer (305, 212, 242) if similarity of satisfactions is more than 0.756.

The process will continue depending upon the requirement of the similarity, but not indefinitely.

REFERENCES

[1] Lai, K .R and Lin, M .W. (2004) : Modeling Agent Negotiation via Fuzzy Constraints in E-Business, Computational Intelligence, Vol. 20, Number 4.

[2] Yager, R . (2007) : Multiagent negotiation using linguistically expressed mediation rules,Group Decision and Negotiation,16,1-23.

[3] Tohme, F. and Sandholm,T., Coalition Formation Process with belief revision among bounded self interested agents, (Source : Internet, Open access Journal).

[4] E. Lehrer, (2002): Allocation processes in cooperative games, Int J Game Theory, 31, 651-654.

[5] J. P. Aubin, (1982) Mathematical Methods of Game and Economic Theory' (rev. ed.)., North-Holland, Ams-terdam.

[6] M. Mares, M. Vlach, (2006) : Fuzzy coalitional structures, Mathware and Soft Comput XIII,1, 59-70.

[7] M. Tsurumi, T. Tanino, M. Inuiguchi, (2001) : Theory and methodology{A Shapley function on a class of cooperative fuzzy games, European Journal of Operation Research, 129 , 596-618.

[8] R. Branzei, D. Dimitrov, S. Tijs, (2004) : Models in Cooperative Game Theory: Crisp, Fuzzy and Multichoice Games, Lecture Notes in Economics and Mathematical Systems, Springer , 556, Berlin.

[9] S. Borkotokey, (2008) : Cooperative Games with fuzzy coalitions and fuzzy characteristic functions, Fuzzy Sets and Systems, Elsevier, 159, 138-151.

[10] Y. Azrieli, E. Lehrer, (2007) : On some families of cooperative fuzzy games, International Journal of Game Theory,36, 1-15.