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MHD OSCILLATORY CONVECTIVE FLOW PAST A VERTICAL PLATE IN SLIP - FLOW REGIME IN PRESENCE OF MAGNETIC FIELD

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Abstract

An attempt has been made to investigate the free convective oscillatory flow of a viscous incompressible and electrically conducting fluid past a vertical porous plate in slip flow regime with variable suction and periodic plate temperature in presence of a uniform transverse magnetic field. Numerical solutions to the coupled non-linear partial differential equations governing the flow and heat transfer are derived by using finite difference method. The influence of the various parameters appeared in the problem namely the Hartman number, the Grashof number, Prandtl number, Eckert number the suction parameter the frequency of oscillation, the rarefaction parameter on the velocity and temperature fields are demonstrated graphically. The skin friction coefficient and the rate of heat transfer presented in tabular form.

1. Introduction

The oscillatory free conventive flow and heat transfer problems in the presence of magnetic field have attracted the attention of many researchers due to its importance in technological applications. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology. Such flows arise due to either unsteady motion of a boundary or a boundary temperature. The unsteadiness may also be due to oscillatory free stream velocity or temperature.

Soundalgekar [1, 2, 3] studied the effects of the oscillatory free stream and free convection currents on the flow field past a vertical flat plate under a transverse magnetic field or without it when the temperature of the plate differs from the temperature of the stream, causing the flow of the free convection currents in the boundary layer. The analysis of an unsteady MHD free convection flow of a viscous imcompressible electrically conducting fluid past on infinite vertical porous plate subjected to constant suction and heat sink was made by Sahoo et.al. [4]. Anwar [5] studied the unsteady MHD free convection flow past a vertical porous plate. The transient free convection flow past an infinite plate with periodic temperature variation was investigated analytically by Das et.al. [6]. The effect of variable suction on transient free convective viscous incompressible flow past a variable plate with periodic temperature variation in a slip flow regime was investigated by Sharma and Choudhary [7]. Sharma [8] has further studied the influence of periodic temperature and concentration on unsteady free convective viscous incompressible flow and heat transfer past a vertical plate in slip flow regime. Jain and Shamra [9] have investigated the effect of viscous heating on flow past a vertical plate in slip-flow regime with periodic temperature variations. Recently Ahmed and Kalita [10] has investigated the effect of magnetic field on the oscillatory free convective flow past a vertical plate and solved the governing equations using perturbation technique.

The object of present paper is to investigate the effect of the transverse magentic field and suction parameter on the oscillatory free convective flow past a vertical plate in slip-flow regime with variable suction and periodic plate temperature. This work investigate the work of Ahmed and Kalita [10] using numerical technique.

2. Mathematical Formation:

Let us consider a flow of an oscillatory free convective flow of on electrically conducting, viscous and incompressible fluid past a vertical plate in-slip flow regime with variable suction and periodic plate temperature under the action transverse magnetic field by making the flowing assumptions:

(1) All fluid properties except the density in the buoyancy force term are constant.

(2) The magnetic dissipation term in the energy equation is negligible

(3) Induced magnetic field is negligible.

The X-axis is taken along the upward vertical plate and the Y-axis is taken perpendicular to it into the fluid region. Since the plate is of infinite length therefore all the physical quantities except the pressure p are independent of x. Under these assumptions, the physical quantities are function of y and t only.

The equations governing the flow are

Equation of Continuity:

$$\frac{\partial \bar{v}}{\partial y} = 0$$

$$\Rightarrow \bar{v} = v_0(1+\varepsilon A e^{i\overline{\omega t}}) \quad \text{... (2.1)}$$

Momentum equation:

$$\frac{\partial \bar{u}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\overline{\omega t}})\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(T - T_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\bar{u} \quad \text{... (2.2)}$$

Energy Equation:

$$c_p\left[\frac{\partial \bar{T}}{\partial \bar{t}} - v_0(1+\varepsilon A e^{i\overline{\omega t}})\frac{\partial \bar{T}}{\partial \bar{y}}\right] = \frac{k}{\rho}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \nu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \quad \text{... (2.3)}$$

The boundary conditions are

$$\bar{u} = \bar{h}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right),\ \bar{T} = \bar{T}_{\omega} + \varepsilon\left(\bar{T}_{\omega} - \bar{T}_{\infty}\right)e^{i\bar{\omega}t} \qquad \ldots (2.4)$$

at $\bar{y} = 0$ and $\bar{u} \to 0, \bar{T} \to T_{\infty}$ at $\bar{y} \to \infty$

We introduce the following non-dimensional quantities:

$$y = \frac{\bar{y}v_0}{v}, t = \frac{\bar{t}v_0^2}{4v}, \omega = \frac{4v\varpi}{v_0^2}, u = \frac{\bar{u}}{v_0}, \theta = \frac{\bar{T} - \bar{T}_{\infty}}{T_w - T_{\infty}}$$

$$G = gv\beta\frac{\bar{T} - \bar{T}_{\infty}}{v_0^3}, M = \frac{\sigma B_0^2 v}{\rho v_0^2}, h = \frac{v_0 h}{v}$$

$$E_c = \frac{v_0^2}{c_p\left(\bar{T} - \bar{T}_{\infty}\right)},\ P_r = \frac{\mu c_p}{\kappa}$$

The non-dimensional equations and boundary conditions are

$$\frac{1}{4}\frac{\partial u}{\partial t} - v_0(1 + \varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} = G\theta + \frac{\partial^2 u}{\partial y^2} - Mu \qquad \ldots (2.5)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - v_0(1 + \varepsilon A e^{i\omega t})\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots (2.6)$$

Subject to

$$\left.\begin{array}{l} y = 0;\ u = h\dfrac{\partial u}{\partial y},\ \theta = 1 + \varepsilon\, e^{i\omega t} \\ y \to \infty,\ u = 0,\ \theta = 0 \end{array}\right\} \qquad \ldots (2.7)$$

3. Solution of the Problem:

In order to solve the system (2.5) - (2.6) let us assume that

$$u = v + i\omega \qquad \ldots (3.1)$$

$$\theta = \phi + i\psi \qquad \ldots (3.2)$$

On substituting (3.1) and (3.2) in (2.4) and (2.5) and separating real and imaginary parts we get the following system of differential equations.

$$\frac{1}{4}\frac{\partial v}{\partial t} - (1+\varepsilon A\cos\omega t)\frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\varepsilon A\sin\omega t = G\phi + \frac{\partial^2 v}{\partial y^2} - Mv \qquad \ldots (3.3)$$

$$\frac{1}{4}\frac{\partial \phi}{\partial t} - (1+\varepsilon A\cos\omega t)\frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial y}\varepsilon A\sin\omega t = \frac{1}{P_r}\frac{\partial^2 \phi}{\partial y^2} + E_c\left[\left(\frac{\partial v}{\partial y}\right)^2 - \left(\frac{\partial w}{\partial y}\right)^2\right] \qquad \ldots (3.4)$$

$$\frac{1}{4}\frac{\partial \omega}{\partial t} - (1+\varepsilon A\cos\omega t)\frac{\partial v}{\partial y} - \frac{\partial \omega}{\partial y}\varepsilon A\sin\omega t = G\psi + \frac{\partial^2 w}{\partial y^2} - mw \qquad \ldots (3.5)$$

$$\frac{1}{4}\frac{\partial \psi}{\partial t} - (1+\varepsilon A\cos\omega t)\frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial y}\varepsilon A\sin\omega t = \frac{1}{\rho}\frac{\partial^2 \psi}{\partial y^2} + 2\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial y} \qquad \ldots (3.6)$$

The boundary conditions (2.6) reduces to

$$y = 0,\ v = h\frac{\partial v}{\partial y},\ w = h\frac{\partial w}{\partial y};\quad \phi = 1 + \varepsilon\cos\omega t, \psi = \varepsilon\sin\omega t \qquad \ldots (3.7)$$

$$y \to \infty;\ v = 0,\ w = 0,\ \phi = 0, \psi = 0$$

The equations (3.3) to (3.6) are coupled non-linear partial differential equations which are to solve by using boundary conditions (3.7). However, exact or approximate solutions are not possible for this set of equations and hence we solve the equations by finite difference scheme. The central difference is used for the diffusion terms and the forward difference scheme is used for the convection terms. The mesh system is divided by taking $\Delta y = 0.1$ and $\Delta t = 0.01$. An iterative technique is applied until the boundary conditions are satisfied.

The important characteristics of the problem are the skin friction and heat transfer rate at the plate.

The coefficient of skin friction at the surface is given by

$$C_f = \text{real part of } \frac{\tau}{\rho_\infty v_0^2} = \text{real part of } \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho_\infty v_0^2} = \frac{\mu\left(\frac{\partial v}{\partial y}\right)_{y=0}}{\rho_\infty v_0^2} \quad \ldots (3.8)$$

The rate of heat transfer in terms of Nusselt number at the surface is given by

$$Nu = \text{real part of } \frac{q_w \theta}{k_\infty v_0 (T_w - T_\infty)} = \text{real part of } \frac{-k\left(\frac{\partial T}{\partial y}\right)_{y=0}}{k_\infty v_0 (T_w - T_\infty)} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0} \quad \ldots (3.9)$$

4. Results and Discussion:

As a result of numerical calculations the velocity and temperature distribution for the flow are obtained from equations (3.3) to (3.7) which are displayed in figures (4.1) to (4.10) for different values of M (hydromagnetic parameter), G (Grashof number), h (rarefaction parameter) for fluid with Prandtl number $Pr = 0.7$ and Eckert number $Ec = 0.01$. Throughout our investigation we take suction parameter $A = 0.2$, frequency parameter $\omega = 1$, and amplitude oscillation $\varepsilon = 0.1$. Also the skin friction coefficient C_f and the Nusselt number Nu is calculated from equations (4.1.1) and (4.2.1) and is tabulated in table (1) and table (3) for same set of parameters.

Table (1) and (2) respectively depict that skin friction coefficient C_f and the Nusselt number Nu for the Hartman number (M = 1, 2, 3, 4, 5) for (h = 0, 0.2, 0.4). It is observed that C_f increases and Nu decreases as M increases. It is evident from table (3) that C_f decreases and Nu increases for increasing value of G.

The variations of velocity profile are shown from fig. (1) - fig. (5) for different values of combination of parameters. From these figures it is observed that velocity increases with the increase of the parameters G, A, M and Pr while velocity increases for the increase of the parameter Ep. Also there is no significant changes of velocity for the parameters A and Pr. The temperature profile are presented for the different combination of the parameters from fig. 6 - fig. 9. From these figures it is observed that temperature increases with the increases of Pr, G and A also it is observed that there is no significant in temperature for the variation of the parameters M, G and A.

For the entire investigation the values of the parameters are taken as ep = 0.10,

Pr = 0.10,M = 0.50 G = 1.00, w = 1.00, h = 0.40, A = 0.20, Ec = 0.10

Table-1: Values of skin friction coefficient C_f for different values of M, P_r=0.7, G=5 and

M	h=0	h=0.2	h=0.4
1	3.87877	-0.54133	-1.42488
2	4.237863	-0.32575	-1.23817
3	4.606745	-0.10299	-1.04463
4	4.984626	0.127031	-0.84419
5	5.371603	0.364404	-0.63673

Table-2: Values of Nusselt number N_u different values of M, P_r=0.7, G=5 and E_c=0.01

M	h=0	h=0.2	h=0.4
1	1.086324	1.095217	1.096647
2	1.083954	1.09466	1.096305
3	1.08142	1.094039	1.09592
4	1.078723	1.093357	1.09488
5	1.075853	1.092607	1.094999

Table-3: Values of skin friction coefficient C_t and Nusselt number N_u for different values of G and h = 0.4, P_r = 0.7, M = 0.5 and E_c = 0.01.

G	C_f	N_u
2	-0.76846	1.0092897
3	-1.07142	1.093893
4	-1.26650	1.095191
5	-1.51570	1.096803
6	-1.76504	1.098719
7	-2.01454	1.100941
8	-2.326421	1.103469
9	-2.51405	1.106304
10	-2.76410	1.109442

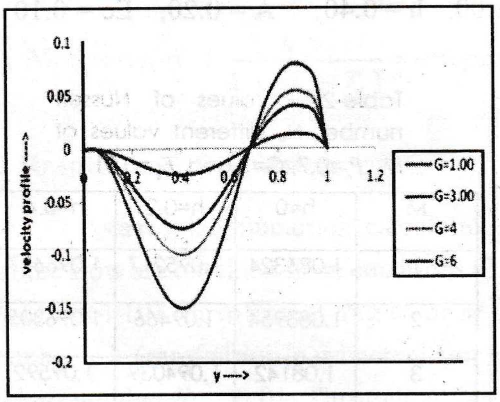

Fig: 1- Velocity profile with respect to the parameter G for epsilon=0.10, Pr=0.10, M=0.50, w=1.00, h=0.40, A=0.20,

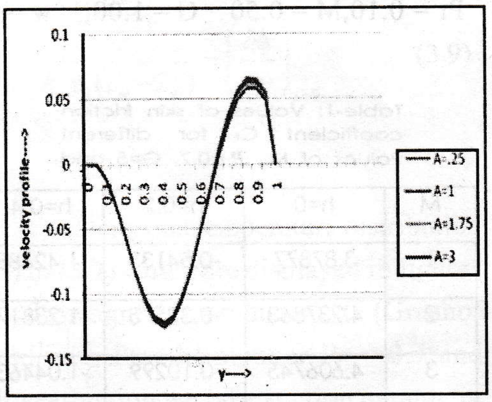

Fig: 2- Velocity profile with respect to the parameter A for epsilon=0.10, Pr=0.10, M=0.50, w=1.00, h=0.40, G=1.0,

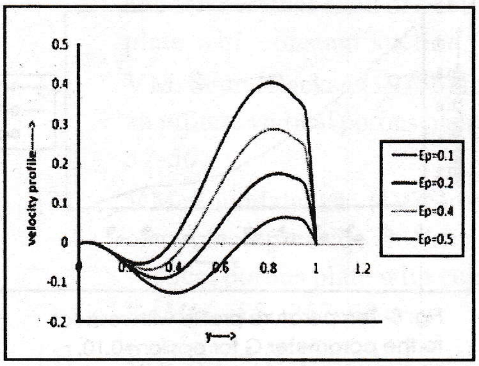

Fig: 3- Velocity profile with respect to the parameter Ep for Pr=0.10, M=0.50, w=1.00, h=0.40, G=1.0, A=0.20, Ec=0.10

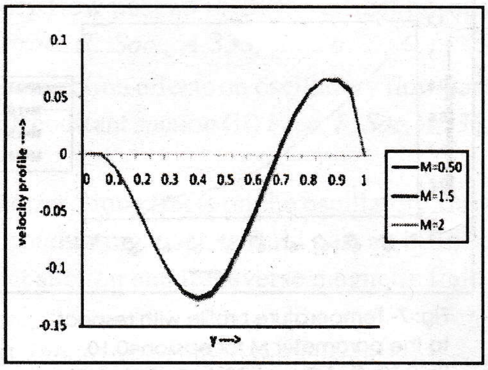

Fig: 4- Velocity profile with respect to the parameter M for epsilon=0.10, Pr=0.10, w=1.00, G=1.0, h=0.40, A=0.20,

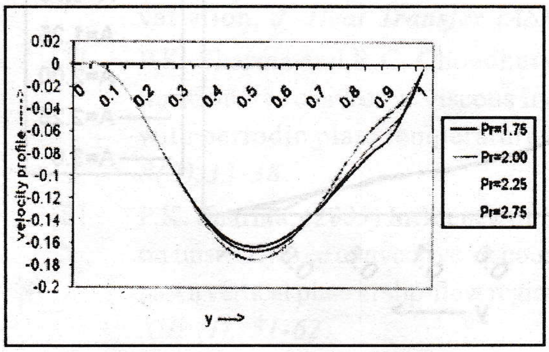

Fig: 5- Velocity profile with respect to the parameter Pr for epsilon=0.10, M=0.50, w=1.00, G=1.0, h=0.40, A=0.20,

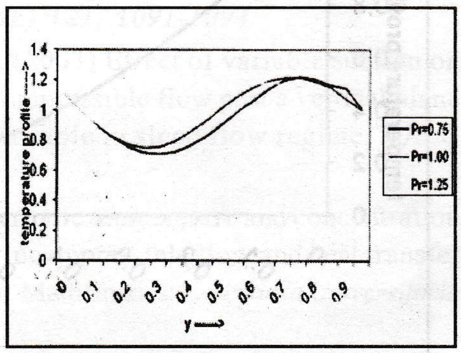

Fig: 6- Temperature profile with respect to the parameter Pr for epsilon=0.10, M=0.50, w=1.00, G=1.0, h=0.40, A=0.20,

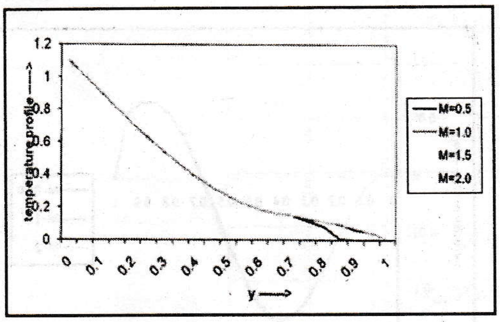

Fig: 7- Temperature profile with respect to the parameter M for epsilon=0.10, Pr=0.10, G=1.0, w=1.00, h=0.40, A=0.20,

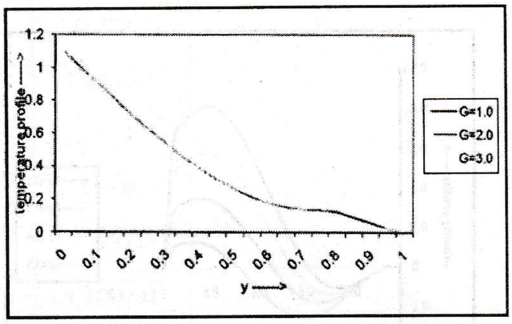

Fig: 8- Temperature profile with respect to the parameter G for epsilon=0.10, Pr=0.10, M=0.50, w=1.00, h=0.40, A=0.20,

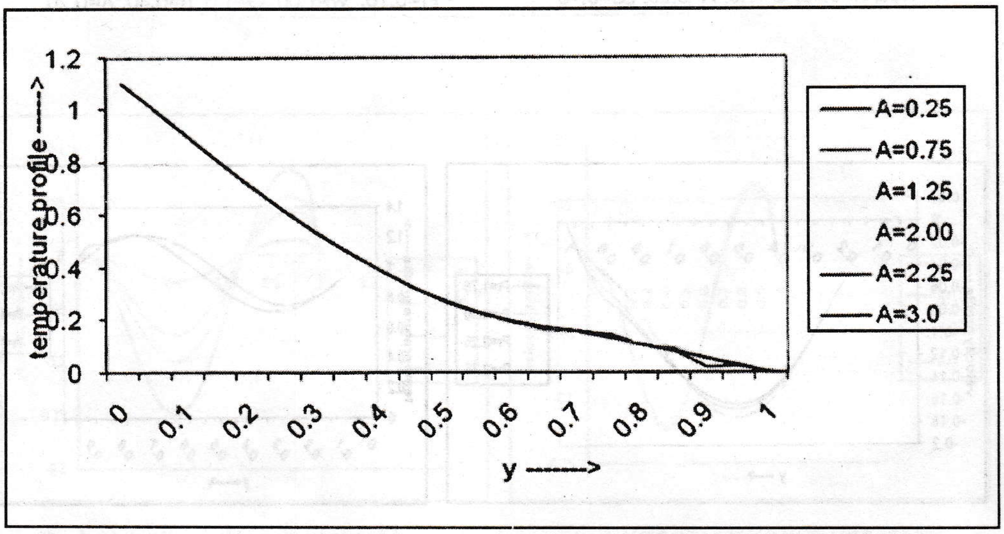

Fig: 9- Temperature profile with respect to the parameter A for epsilon=0.10, Pr=0.10, M=0.50, w=1.00, h=0.40, G=1.020, Ec=0.10

REFERENCES:

[1] V.M. Soundalgekar, (1973a) free convection effects on the mean velocity and temperature field of oscillatory flow past an infinite vertical porous plate with constant suction (i) *Proc. R. Soc., A 333, 25-26.*

[2] V.M. Soundalgekar, (1973b) free convection effects on oscillatory flow past an infinite vertical porous plate with constant suction (II) *Proc. R. Soc, A 333, 37-50.*

[3] V.M. Soundalgekar, (1975) free convection effects on the oscillatory flow of an incompressibe electrically conducting viscous fluid past an infinite vertical porous plate with constant suction and transverse magnetic field. *ZAMM, 55, 257-68.*

[4] P.K. Sahoo, N. Dutta and S. Biswal, (2003) Magnetohydrodynamic unsteady free convection past an infinite vertical plate with constant suction and heat sink, India J. Pure Appl. Math., 34(1), 145-155.

[5] K. Anwar, (1998) MHD unsteady free convection flow past an vertical porous plate, *ZAMM 78, 255-270.*

[6] U.N.N. Das, R.K. Deka and V.M. Soundalgekar (1998) 'Transient free convection flow past an infinite vertical plate with periodic temperature variation,' *J. Heat Transfer (ASME) 121, 1091-1094.*

[7] P.K. Sharma and R.C. Choudhury, (2003) Effect of variable suction on transient free convective viscous incompressible flow past a vertical plane with periodic plate temperature variable in sleep flow regime, *EJER 8(2),33-38.*

[8] P.K. Sharma, (2005) Influence of periodic temperature and concentration on unsteady free convective viscous incompressible flow and heat transfer past a vertical plate in slip-flow regime, Mathematics: *Ensenanza Universitaria XIII (1), 51-62.*

[9] S.R. Jain, and P.K. Sharma, (2006) Effect of Viscous heating on flow past a vertical plate in slip-flow regime with periodic temperature variations, *J. Rajasthan Acad Phy Sci. Vol. 5, No. 4, pp. 383-39.*

[10] N. Ahmed and D. Kalita, (2008) MHD free convective flow past a vertical plate in slip flow regime with variable suction and periodic plate temperature, J., Heat and Technology, Vol. 26, n.2.