Mathematical Forum Vol. IV, No. 1, March 1981, pp. 23-31

FLOW OF INCOMPRESSIBLE SECOND ORDER FLUID THROUGH CONTRACTING AND EXPANDING PIPE

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(Received on 1.9.1981).

Abstract

Unsteady flow of an incompressible second order fluid through a semi-infinite circular pipe whose radius varies with time and whose one end is closed has been investigated. A perturbation scheme in small parameter α which represents the rate of contraction or expansion of the pipe has been used in solving the problem. It is seen that the absolute values of shearing stress and frictional drag decrease or increase on account of viscoelasticity when the pipe contracts or expands.

Introduction :

The main part of the cardiovascular pump is a valved vessel. When the blood in the left ventricle is being forced by systole into the aorta, the mitral valve is closed while the atrioventricular valve is open. At this stage the left ventricle forms a vessel with one end closed.

A vein of medium size also has a valve system when it contracts the valve at the upstream end is closed and that at the downstream end is open. In such a vessel or tube with one end closed the contents are ejected into the adjoining section mainly by the simple contraction of the tube diameter, even though the peristalsis, if any, may help to transport the contents. By co-operating with motions of valve a vein with a valve system can act as a local pumping station powered by the action of muscles.

The blood vessel closed at one end by a valve, or the thin bronchial tube may be modelled by a semi-infinite pipe with one end closed by an idealized membrane and the blood may be represented by some suitable rheological model. Uchida and Aoki (1) have discussed this flow problem by taking blood to be a Newtonian fluid. Since, blood exhibits some elastic properties, we consider the problem by taking blood to be an incompressible second order fluid whose constitutive equation is given by Coleman and Noll (2) as

$$\tau_{ij} = -p\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)i}{}^{\alpha} A_{(1)\alpha j} \tag{1}$$

Here p is an indeterminate pressure which differs, in general, from the mean pressure, and $A_{(1)ij}$ and $A_{(2)ij}$ are Rivlin-Ericksen tensors given by

$$A_{(1)ij} = v_{i,j} + v_{j,i}.$$
$$A_{(2)ij} = a_{i,j} + a_{j,i} + 2v^m{}_{,i} v_{m,j} \tag{2}$$

μ_1, μ_2, μ_3, are material constants and have been determined experimentally by Markovitz and Brown (3) and others for the solutions of polyisobutylene in cetane of various concentrations. The constant μ_2 is negative.

We assume that the one end of semi-infinite pipe is closed with an idealized complaint membrane which prevents only the axial motion but leaves the radial motion completely unchecked. The radius of the pipe is assumed to be a function of time t. Introducing a non-dimensional parameter $\alpha(t) = \frac{a\rho}{\mu_1}\frac{da}{dt}$, where a is the radius of the pipe and ρ is the density of the fluid, we have obtained the solution for the unsteady flow produced by a single contraction or a single expansion of the pipe by expanding the flow functions as power series in α. The pressure and friction at the wall have been calculated for different values of the elasticoviscous parameters.

Formulation of the Problem :

Unsteady flow of an incompressible second order fluid through a semi-infinite circular pipe whose radius varies with time is considered. The pipe is closed at $z=0$ by an elastic membrane which prevents axial motion but is fully complaint to radial motion produced by the wall. This situation may easily be achieved by making the flow symmetrical about the plane $z=0$.

Referring to cylindrical polar co-ordinates (r, θ, z) and denoting the velocity components by u, o and w in the directions of r, θ and z respectively, axisymmetric unsteady flow is governed by the following equations of continuity and motion.

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{3}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}\right)=\frac{\partial \tau_{rr}}{\partial r}+\frac{\partial \tau_{rz}}{\partial z}+\frac{\tau_{rr}-\tau_{\theta\theta}}{r} \tag{4}$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=\frac{\partial \tau_{rz}}{\partial r}+\frac{\partial \tau_{zz}}{\partial z}+\frac{\tau_{rz}}{r} \tag{5}$$

where τ_{rr}, $\tau_{\theta\theta}$, τ_{zz}, τ_{rz}, are the physical components of the stress given by (1) and (2). The wall of the pipe moves only in the radial direction and its radius is assumed to be a function of time only. The radial velocity of fluid at the wall denoted by u_w is equal to the wall velocity $da/dt=\dot{a}$. The boundary condition are therefore

$$u=\dot{a},\quad w=0 \text{ at } r=a(t)$$

$$u=0,\quad \frac{\partial \omega}{\partial r}=0 \text{ at } r=0$$

$$w=0 \text{ at } z=0. \tag{6}$$

we make the following transformations

$$\mu=-\frac{\nu_1}{a}\frac{F}{\eta},\qquad \omega=\frac{\nu_1 z}{a^2}\frac{F'}{\eta},\qquad \eta=\frac{r}{a}$$

$$p(r,z)=\frac{\rho\nu_1^2}{a^2}\left[p_1(\eta)+\frac{z^2}{a^2}p_2(\eta)\right] \tag{7}$$

where a prime denotes $d/d\eta$.

The equation of continuity is identically satisfied by the forms of u and w in (7). Using the transformations (7), the equations of motion (4) and (5) become

$$\begin{aligned}
p_1' = \Bigg[\Bigg\{ & \frac{a^2}{v_1}\frac{\dot{F}}{\eta}-\alpha\eta\left(\frac{F}{\eta}\right)'-\alpha\frac{F}{\eta}-\frac{F}{\eta}\left(\frac{F}{\eta}\right)'-\left(\frac{F}{\eta}\right)''+\frac{1}{\eta}\left(\frac{F}{\eta}\right)'+\frac{1}{\eta^2}\frac{F}{\eta}\Bigg\}+\beta\Bigg\{-2\alpha\left(\frac{\dot{F}}{\eta}\right)'' \\
& +2\alpha\eta\left(\frac{F}{\eta}\right)'''+6\alpha\left(\frac{F}{\eta}\right)''+\alpha\left(\frac{\dot{F}'}{\eta}\right)'-\alpha\eta\left(\frac{F'}{\eta}\right)''-3\alpha\left(\frac{F'}{\eta}\right)'-2\alpha\frac{1}{\eta}\left(\frac{\dot{F}}{\eta}\right)' \\
& +2\alpha\left(\frac{F}{\eta}\right)''+2\alpha\frac{1}{\eta}\left(\frac{F}{\eta}\right)'+2\alpha\frac{1}{\eta^2}\frac{\dot{F}}{\eta}-2\alpha\frac{1}{\eta^2}\frac{F}{\eta}+\frac{F}{\eta}\left(\frac{F}{\eta}\right)'''+\frac{1}{\eta}\frac{F}{\eta}\left(\frac{F}{\eta}\right)'' \\
& +9\left(\frac{F}{\eta}\right)'\left(\frac{F}{\eta}\right)''+3\frac{1}{\eta}\left(\frac{F}{\eta}\right)'\left(\frac{F}{\eta}\right)'+\frac{1}{\eta^2}\frac{F}{\eta}\left(\frac{F}{\eta}\right)'+4\frac{F'}{\eta}\left(\frac{F'}{\eta}\right)'\Bigg\}+\gamma\Bigg\{8\left(\frac{F}{\eta}\right)'\left(\frac{F}{\eta}\right)'' \\
& +2\frac{1}{\eta}\frac{F}{\eta}\left(\frac{F'}{\eta}\right)'+4\frac{1}{\eta}\left(\frac{F}{\eta}\right)'\left(\frac{F}{\eta}\right)'-4\frac{1}{\eta^3}\left(\frac{F}{\eta}\right)^3\Bigg\}\Bigg]
\end{aligned} \tag{8}$$

$$p_2' = (2\beta+\gamma)\left\{2\left(\frac{F'}{\eta}\right)'\left(\frac{F'}{\eta}\right)'' + \frac{1}{\eta}\left(\frac{F'}{\eta}\right)'\left(\frac{F'}{\eta}\right)'\right\} \tag{9}$$

$$\begin{aligned}
&\Bigg[-\frac{a^2}{\nu_1}\left(\frac{F'}{\eta}\right)' + 3\alpha\left(\frac{F'}{\eta}\right)' + \alpha\eta\left(\frac{F'}{\eta}\right)'' + \left(\frac{F}{\eta}\right)'\left(\frac{F'}{\eta}\right)' + \frac{F}{\eta}\left(\frac{F'}{\eta}\right)'' \\
&-2\left(\frac{F'}{\eta}\right)'\frac{F'}{\eta} + \left(\frac{F'}{\eta}\right)''' + \frac{1}{\eta}\left(\frac{F'}{\eta}\right)'' - \frac{1}{\eta^2}\left(\frac{F'}{\eta}\right)'\Bigg] + \beta\Bigg[\alpha\left(\frac{\dot{F}'}{\eta}\right)''' - \alpha\frac{1}{\eta^2}\left(\frac{\dot{F}'}{\eta}\right)' \\
&+\alpha\frac{1}{\eta}\left(\frac{\dot{F}'}{\eta}\right)'' - 6\alpha\left(\frac{F'}{\eta}\right)''' - \alpha\eta\left(\frac{F'}{\eta}\right)'''' + 3\alpha\frac{1}{\eta^2}\left(\frac{F'}{\eta}\right)' - 3\alpha\frac{1}{\eta}\left(\frac{F'}{\eta}\right)'' \\
&-\left(\frac{F}{\eta}\right)'''\left(\frac{F'}{\eta}\right)' - 3\left(\frac{F}{\eta}\right)''\left(\frac{F'}{\eta}\right)'' - 3\left(\frac{F}{\eta}\right)'\left(\frac{F'}{\eta}\right)''' - \left(\frac{F}{\eta}\right)\left(\frac{F'}{\eta}\right)'''' \\
&+4\left(\frac{F'}{\eta}\right)'''\frac{F'}{\eta} + 4\left(\frac{F'}{\eta}\right)''\left(\frac{F'}{\eta}\right)' + \frac{1}{\eta^2}\left(\frac{F}{\eta}\right)'\left(\frac{F'}{\eta}\right)' - \frac{1}{\eta}\left(\frac{F}{\eta}\right)''\left(\frac{F'}{\eta}\right)' \\
&-2\frac{1}{\eta}\left(\frac{F}{\eta}\right)'\left(\frac{F'}{\eta}\right)'' + \frac{1}{\eta^2}\frac{F}{\eta}\left(\frac{F'}{\eta}\right)'' - \frac{1}{\eta}\frac{F}{\eta}\left(\frac{F'}{\eta}\right)''' - 4\frac{1}{\eta^2}\left(\frac{F'}{\eta}\right)'\frac{F'}{\eta} \\
&+4\frac{1}{\eta}\left(\frac{F'}{\eta}\right)''\frac{F'}{\eta}\Bigg] + \gamma\Bigg[2\frac{1}{\eta^2}\left(\frac{F}{\eta}\right)'\left(\frac{F'}{\eta}\right)' - 2\frac{1}{\eta}\left(\frac{F}{\eta}\right)''\left(\frac{F'}{\eta}\right)' - 4\frac{1}{\eta}\left(\frac{F}{\eta}\right)'\left(\frac{F'}{\eta}\right)'' \\
&+2\frac{1}{\eta^2}\frac{F}{\eta}\left(\frac{F'}{\eta}\right)'' - 2\frac{1}{\eta}\left(\frac{F}{\eta}\right)\left(\frac{F'}{\eta}\right)''' - 8\left(\frac{F'}{\eta}\right)'\left(\frac{F'}{\eta}\right)'' - 2\frac{1}{\eta}\left(\frac{F'}{\eta}\right)'\left(\frac{F'}{\eta}\right)'\Bigg] = 0.
\end{aligned} \tag{10}$$

where $\alpha(t) = \dfrac{a\hat{a}}{\nu_1}, \quad \beta = \dfrac{\nu_2}{a^2}, \quad \gamma = \dfrac{\nu_3}{a^2}.$ (11)

The relation (9) has been used in obtaining the equation (10). The boundary conditions (6) become

$$\frac{F}{\eta} = 0, \quad \left(\frac{F'}{\eta}\right)' = 0 \text{ at } \eta = 0$$

$$\frac{F}{\eta} = -\alpha, \quad \frac{F'}{\eta} = 0 \text{ at } \eta = 1. \tag{12}$$

Solutions of the Equation :

In order to analyse the fundamental properties of the present flow, a full solution similar in both space and time which preserves the non-linear characteristies of the problem is studied here.

Such a similar solution can be obtained by assuming $\dot{F} = 0$ and $\alpha =$ constant in (10), which means that the function F is a function only of η

containing α as a constant parameter. The value of α is taken as its initial value

$$\frac{a\dot{a}}{\nu_1}=\alpha=\frac{a_0\dot{a}_0}{\nu_1} \tag{13}$$

where, a_0 and $\dot{a}_0=\left(\frac{da}{dt}\right)_0$ are the initial values of the radius and of its expansion rate, respectively. Contracting and expanding pipes thus have $\alpha<0$ and $\alpha>0$ respectively. The parameter $|\alpha|$ is the Reynolds number which represents the dynamical scale of the present motion.

Here, we obtain a series solution for small values of α which is seen to be a parameter representing wall Reynolds number we put

$$F(\eta)=\sum_{n=1}^{\infty}\alpha^n F_n(\eta). \tag{14}$$

The boundary conditions on F_n are

$$\frac{F_1}{\eta}=0,\ \left(\frac{F'_1}{\eta}\right)'=0 \text{ at } \eta=0$$

$$F_1=-1,\ F'_1=0 \text{ at } \eta=1. \tag{15}$$

$$\left.\begin{array}{l}\dfrac{F_n}{\eta}=0,\ \left(\dfrac{F'_n}{\eta}\right)'=0 \text{ at } \eta=0\\ F_n=0,\ F'_n=0 \text{ at } \eta=1\end{array}\right\} \text{ for } n\geqslant 2. \tag{16}$$

Substituting expressions (14) into (10) with $\frac{\partial^2 F}{\partial\eta\partial t}=0$ and equating the co-efficients of different powers of α to zero, we get a set of equations for F_1, F_2, F_3, etc. and K_1, K_2, K_3. etc.

The integrations of these equations under boundary conditions (15) and (16) gives

$$F_1=-2\eta^2+\eta^4. \tag{17}$$

$$\begin{aligned}F_2=&-\frac{5}{18}\eta^2+\frac{7}{12}\eta^4-\frac{1}{3}\eta^6+\frac{1}{36}\eta^8+\beta\left(-\frac{1}{4}\eta^2+\frac{1}{2}\eta^4-\frac{1}{4}\eta^6\right)\\&+\gamma(4\eta^2-8\eta^4+4\eta^6).\end{aligned} \tag{18}$$

$$\begin{aligned}F_3=&-\frac{1057}{10800}\eta^2+\frac{271}{1080}\eta^4-\frac{47}{216}\eta^6+\frac{2}{27}\eta^8-\frac{7}{720}\eta^{10}+\frac{1}{5400}\eta^{12}\\&+\beta\left[-\frac{2389}{720}\eta^2+\frac{10639}{1440}\eta^4-\frac{39}{8}\eta^6+\frac{247}{288}\eta^8-\frac{19}{360}\eta^{10}\right]\end{aligned}$$

$$+\gamma\left[\frac{211}{80}\eta^2-\frac{167}{20}\eta^4+9\eta^6-\frac{7}{2}\eta^8+\frac{17}{80}\eta^{10}\right]$$

$$+\beta^2\left[-\frac{269}{96}\eta^2+\frac{191}{32}\eta^4-\frac{113}{32}\eta^6+\frac{35}{96}\eta^8\right]$$

$$+\gamma^2\left[-\frac{16}{3}\eta^2+72\eta^4-80\eta^6+\frac{88}{3}\eta^8\right]$$

$$+\beta\gamma\left[\frac{134}{3}\eta^2-\frac{224}{3}\eta^4+60\eta^6-\frac{23}{3}\eta^8\right] \tag{19}$$

etc. and values of the k_n may also be determined

$$k_1=16$$

$$k_2=-\frac{44}{3}+8\beta-128\gamma.$$

$$k_3=-\frac{208}{135}+\frac{10189}{9}\beta-\frac{268}{5}\gamma+\frac{191}{2}\beta^2-\frac{3584}{3}\beta\gamma+1152\gamma^2. \tag{20}$$

Let $k=\alpha k_1+\alpha^2 k_2+\alpha^3 k_3+\ldots\ldots$

$$=16\alpha-\left(\frac{44}{3}-8\beta+128\gamma\right)\alpha^2-\left(\frac{208}{135}-\frac{10189}{90}\beta+\frac{268}{5}\gamma\right.$$

$$\left.-\frac{191}{2}\beta^2+\frac{3584}{3}\beta\gamma-1152\gamma^2\right)\alpha^3+\ldots. \tag{21}$$

The pressure can be calculated by substituting the values of F_1, F_2, F_3 etc. from (17) to (19) into (8) and (9). Now, denoting the pressure on the axis by p_c (z, t), we get the pressure distribution at a fixed z as

$$\frac{p-p_c}{\rho\nu_1{}^2/a^2}=\alpha\left[\left\{-4\eta^2+\alpha\left\{-\frac{7}{3}\eta^2+3\eta^4-\frac{13}{18}\eta^6+\beta\left(-16\frac{1}{\eta^2}-\frac{273}{2}\eta^2+\frac{105}{2}\eta^4\right)\right.\right.\right.$$

$$\left.\left.+\gamma\left(-70\eta^2+24\eta^4\right)\right\}+\ldots\ldots\right\}+\frac{z^2\alpha}{a^2}\left\{\left(172\eta^2\beta+86\eta^2\gamma\right)\right.$$

$$+\alpha\left\{\beta\left(224\eta^2-320\eta^4+\frac{448}{9}\eta^6\right)+\gamma\left(112\eta^2-160\eta^4+\frac{224}{9}\eta^6\right)\right.$$

$$+\beta^2(48\eta^2-240\eta^4)+\gamma^2(-1536\eta^2+1920\eta^4)$$

$$\left.\left.+\beta\gamma(-3048\eta^2+3720\eta^4)\right\}+\ldots\ldots\right\}\Big]. \tag{22}$$

Taking $\eta=1$, the above relation yields the pressure difference between the wall and the central axis as

$$\frac{p_w-p_c}{\rho\nu_1{}^2/a^2}=\alpha\left[\left\{-4-\alpha\left(\frac{1}{18}+68\beta+46\gamma\right)+\ldots\ldots\right\}+\frac{z^2\alpha}{a^2}\left\{172\beta+86\gamma\right.\right.$$

$$\left.\left.+\alpha\left(-\frac{416}{9}\beta-\frac{208}{9}\gamma-192\beta^2+384\gamma^2+672\beta\gamma\right)+\ldots\ldots\right\}\right]. \tag{23}$$

The pressure at an arbitrary point is calculated as

$$
\begin{aligned}
\frac{p-p_{cl}}{\rho\nu_1^2/a^2} &= \alpha\Big[\Big\{-4\eta^2+\alpha\Big\{-\frac{7}{3}\eta^2+3\eta^4-\frac{13}{14}\eta^6+\beta\Big(-16\frac{1}{\eta^2}-\frac{273}{2}\eta^2+\frac{105}{2}\eta^4\Big) \\
&\quad +\gamma\Big(-70\eta^2+24\eta^4\Big)\Big\}+\ldots\ldots\Big\}+\frac{z^2\alpha}{a^2}\Big\{172\eta^2\beta+86\eta^2\gamma \\
&\quad +\alpha\Big\{\beta\Big(224\eta^2-320\eta^4+\frac{448}{9}\eta^6\Big)+\gamma\Big(112\eta^2-160\eta^4+\frac{224}{9}\eta^6\Big) \\
&\quad +\beta^2(48\eta^2-240\eta^4)+\gamma^2(-1536\eta^2+1920\eta^4) \\
&\quad +\beta\gamma\Big(-3048\eta^2\Big[+3720\eta^4\Big)\Big\}+\ldots\ldots\Big\}\Big]+\frac{z^2-l^2}{a^2}\frac{k}{2}.
\end{aligned}
\tag{24}
$$

when p_{cl} is the reference pressure at a fixed point $z=l$, $r=0$. Taking $\eta=1$ the above relation yields the pressure along the axis as

$$
\begin{aligned}
\frac{P_\omega-P_{cl}}{\rho\upsilon_1^2/a^2} &= \alpha\Big[\Big\{-4-\alpha\Big(\frac{1}{18}+100\beta+46\gamma)+\ldots\ldots\Big\}+\frac{z^2\alpha}{a^2}\Big\{172\beta+86\gamma \\
&\quad +\alpha\Big(-\frac{416}{9}\beta-\frac{208}{9}\gamma-192\beta^2+384\gamma^2+672\beta\gamma\Big)+\ldots\ldots\Big\} \\
&\quad +\frac{k}{2}\frac{z^2-l^2}{a^2}.
\end{aligned}
\tag{25}
$$

The necessary power for the external forces driving the present motion can be calculated as

$$
P=-2\pi a\dot{a}\int_0^l p_\omega\,dz. \tag{26}
$$

Integrating (26) the non-dimensional form of p is given by

$$
\begin{aligned}
\frac{P}{\rho\upsilon_1^3} &= 2\pi l\alpha\Big[-\frac{p_{cl}}{\rho\nu_1^2}+\frac{\alpha}{a^2}\Big\{4+\alpha\Big(\frac{1}{18}+100\beta+46\gamma\Big)+\ldots\ldots\Big\} \\
&\quad +\frac{l^2\alpha}{a^4}\Big\{\frac{16}{3}-\alpha\Big(\frac{44}{9}+\frac{164}{3}\beta+\frac{214}{3}\gamma\Big)+\ldots\ldots\Big\}\Big].
\end{aligned}
\tag{27}
$$

The expression for shearing stress at the wall can be given as

$$
\tau_\omega=(\tau_{rz})_{\eta=1}=\frac{\rho\nu_1^2}{a^2}=z\tau_1 \tag{28}
$$

where,

$$
\begin{aligned}
\tau_1 &= \alpha\Big[-\alpha(2+37\beta-16\gamma)-\alpha^2\Big(\frac{56}{135}+\frac{777}{60}\beta-\frac{11}{5}\gamma \\
&\quad +\frac{71}{2}\beta^2-\frac{1052}{3}\beta\gamma\Big)+\ldots\ldots\Big].
\end{aligned}
\tag{29}
$$

The values T_1 with respect to $\alpha=-.2, -.1, .1, .2$ at $\eta=1$ for $(\alpha, \beta)=(0, 0)$, $(-.05, .2)$, $(-.05, .3)$, $(-.1, .3)$ are calculated and given in Table 1.

TABLE 1

Values of T_1 for different values of (α, β)

α	$\beta=0$ $\gamma=0$	$\alpha=-\cdot05$ $\gamma=\cdot2$	$\beta=-\cdot05$ $\gamma=\cdot3$	$3=-\cdot1$ $\gamma=\cdot3$
—.2	—1.6766816	—1.4546188	—1.3783524	—1.2653232
—.1	—0.8195852	—0.76657735	—0.75504405	—0.7256654
.1	0.7795852	0.82757735	0.84204404	0.8556654
.2	1.5166816	1.6986188	1.7503524	1.785332

And the non-dimensional form of the frictional drag Df acting on the part of the pipe between $z=0$ and $z=l$ is

$$\frac{Df}{\rho v_1{}^2}=\frac{2\pi a}{\rho v_1{}^2}\int_0^l (\tau_{rz})_{\eta=1}\, dz$$

$$=\frac{\rho v_1{}^2 l^2}{a}\pi\tau_1. \tag{30}$$

Discussion :

The expression for the pressure at the wall of the pipe is given by (25). This shows that the viscoelastic parameters affect the pressure distribution. The necessary power for the external forces driving the motion in the pipe is given by (27). When $\beta=\gamma=0$, the expressions (25) and (27) give the corresponding results for the case of a Newtonian fluid.

The shearing stress T_W at the wall of the pipe and the frictional drag D acting on the pipe between $z=0$ to $z=1$ are given by (23) and (30) respectively in terms of T_1. The values of T_1 for different values of the parameters α, β, γ are given in table 1. The table 1 shows that for $\alpha<0$ (contracting pipe) $|\tau_1|$ decreases when $|\beta|$ and/or γ increases while for $\alpha>0$ (expanding pipe) $|\tau_1|$ increases when either or both viscoelastic parameters increase. Thus the effect of viscoelasticity on the shearing stress at the wall and the frictional drag is to decrease them when pipe contracts and to increase them when the pipe expands.

ACKNOWLEDGEMENT

Sincere thanks to Dr. M. S. Saroa for suggesting this problem and for his generous help throughout the investigation.

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