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BARO-DIFFUSION IN A BINARY MIXTURE DUE TO A SPINNING CONE

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ABSTRACT

The baro-diffusion effect in a binary mixture of incompressible viscous fluids due to a steadily rotating cone has been considered. It has been found that the separations of the species in the mixture takes place because of their different molecular weights under the action of the prevailing pressure gradient.

1. *Introduction*

The problem of baro-diffusion in a binary mixture of incompressible, viscous fluids set in steady, axially symmetric motion by an infinite cone of wide opening, rotating uniformly about its axis is considered. We assume the mixture to be isothermal so that, the diffusion of the species is brought about by the concentration gradient and the pressure gradient only.

The general motivations and background to this problem area are elaborated by Sarma (1973). He discussed the problem of barodiffusion in an isothermal mixture of two incompressible viscous fluids set in steady motion by an infinite rotating disk. Wu (1959) derived the boundary layer equations for a spinning cone and showed that, with suitable transformations, the flow functions satisfy the Von-Kármán equations for an infinite rotating disk but the pressure distribution is altered. Thus, in the present configuration, we expect a different distribution of the concentrations of the species than that for the case of an infinite rotating disk.

2. *Flow at a Spining Cone*

Consider an infinite cone of opening angle 2α rotating uniformly with angular speed ω about its axis in a binary mixture of incompressible viscous fluids. We assume that one of the components in the binary mixture is present in extremely small relative amount, so that the properties of the mixture are independent of the distribution of the rarer component but the velocity $\vec{v}=(\rho_1\vec{v}_1+\rho_2\vec{v}_2)/(\rho_1+\rho_2)$ where ρ_1 and ρ_2 are the densities and, $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the rarer and more abundant components respectively.

We introduce orthogonal curvilinear coordinates (ξ, ϕ, ς) which are appropriate to the present cofiguration and are defined by the relations $x=(\xi \sin \alpha+\varsigma \cos \alpha)$, $\cos \phi$, $y=(\xi \sin \alpha+\varsigma \cos \alpha) \sin \phi$, $z=\xi \cos \alpha-\varsigma \sin \alpha$. Here (x, y, z) are rectangular certesian coordinates with origin at the vertex and z-axis along the axis of the cone. The boundary layer equations in (ξ, ϕ, ς) system are

$$\left.\begin{aligned} &\frac{1}{\xi}\frac{\partial}{\partial\xi}(u\xi)+\frac{\partial w}{\partial\varsigma}=0,\\ &u\frac{\partial u}{\partial\xi}+w\frac{\partial u}{\partial\varsigma}-\frac{v^2}{\xi}=\upsilon\frac{\partial^2 u}{\partial\varsigma^2},\\ &u\frac{\partial v}{\partial\xi}+w\frac{\partial v}{\partial\varsigma}+\frac{uv}{\xi}=\upsilon\frac{\partial^2 v}{\partial\varsigma^2},\\ &\frac{1}{\xi}v^2\cot\alpha=\frac{1}{\rho}\frac{\partial p}{\partial\varsigma}, \end{aligned}\right\} \tag{1}$$

where u, v, w are the components of $\vec{v}$ in ξ, ϕ, ζ directions respectively and $\rho=\rho_1+\rho_2$.

The boundary conditions are

$$\left.\begin{aligned} &u=0. \quad v=\xi\omega \sin\alpha, \quad w=0 \quad \text{at } \zeta=0\\ &u\to 0, \quad v\to 0 \quad \text{as } \zeta\to\infty \end{aligned}\right\} \tag{2}$$

Following *Wu* [2], we take the velocity components as

$$\begin{aligned} &u=(\xi\omega \sin\alpha)f'(\eta),\; v=(\xi\omega\sin\alpha)\,g(\eta)\\ &w=-2(\nu\omega\sin\alpha)^{\frac{1}{2}}f(\eta),\; \zeta=(\nu/\omega\sin\alpha)^{\frac{1}{2}}\eta \end{aligned} \tag{3}$$

where a prime denotes $d/d\eta$.

When these expressions are substituted in the equations (1), the equation of continuity is identically satisfied and the remaining equations transform to

$$\left.\begin{aligned} &f'''=f'^2-2ff''-g^2\\ &g''=2(f'g-fg') \end{aligned}\right\} \tag{4}$$

$$\frac{\partial p}{\partial\eta}=\rho(\xi\omega\cos\alpha)(\nu\omega\sin\alpha)^{\frac{1}{2}}g^2. \tag{5}$$

The boundary conditions (2) become

$$f(o)=f'(o)=0,\ g(o)=1\ ;\ f'\to 0,\ g\to 0 \text{ as } \eta\to\infty. \tag{6}$$

We see that the equations (4) and (6) for f and g are the familiar Von-Kármán equations for an infinite rotating disk. Cochran (1934) has obtained numerically the accurate solution to this problem.

3. *Solution of the Problem*

Writing the species conservation equation [See Sarma (1973)] in (ξ, ϕ, ζ) system and applying the same boundary layer approximations as those to the flow problem, we obtain

$$u\frac{\partial C_1}{\partial \xi}+\omega\frac{\partial C_1}{\partial \varsigma}=D\left[\frac{\partial^2 C_1}{\partial \varsigma^2}+\frac{m_2-m_1}{m_2 p_\infty}\frac{\partial}{\partial \varsigma}\left(C_1\frac{\partial p}{\partial \varsigma}\right)\right] \tag{7}$$

where $C_1=\rho_1/\rho$ is the concentration of the rarer component and D is the diffusion coefficient ; m_1, m_2 are the molecular weights of the rarer and the more abundant components, and p_∞ is the working pressure in the medium. Substituting the expressions (3) in the equation (7), we obtain

$$S\left[\xi f'\frac{\partial C_1}{\partial \xi}-2f\frac{\partial C_1}{\partial \eta}\right]=\frac{\partial^2 C_1}{\partial \eta^2}+\frac{\xi B\cos\alpha}{(\nu/\omega\ \sin\alpha)^{1/2}}\frac{\partial}{\partial \eta}(C_1 g^2) \tag{8}$$

where $S=\nu/D$ is the Schmidt number and $B=(m_2-m_1)\rho\nu\omega/m_2 p_\infty$ is the barodiffusion number.

The boundary conditions on C_1 are obtained from the fact that the normal mass flux at the surface of the cone vanishes and that there is no diffusion at large distance away from the surface. Hence the boundary conditions on C_1 are

$$\left.\begin{array}{ll}\dfrac{\partial C_1}{\partial \eta}=\dfrac{\xi\ B\cos\alpha}{(\nu/\omega\ \sin\alpha)^{1/2}}c_1 g^2 & \text{at } \eta=0\\ C_1\to(C_1)_\infty & \text{as } \eta\to\infty\end{array}\right\} \tag{9}$$

We take the concentration C_1 as

$$C_1=(C_1)_\infty[\psi_0(\eta)+(\xi\ B\cos\alpha)(\nu/\omega\ \sin\alpha)^{-\frac{1}{2}}\psi_1(\eta)]. \tag{10}$$

Substituting this expression in (8) and (9), we find that ψ_0 and ψ_1 satisfy the equations

$$\psi_0''=-2Sf\psi_0', \tag{11}$$

$$\psi''=S(f'\psi_1-2f\psi_1')-(\psi_0 g^2)' \tag{12}$$

and the boundary conditions

$$\psi_0'(0)=0\ ;\qquad \psi_0\to 1 \text{ as } \eta\to\infty \tag{13}$$

$$\psi_1'(0)+\psi_0(0)g^2(0)=0\ ;\qquad \psi_1\to 0 \text{ as } \eta\to\infty. \tag{14}$$

The equation (11) subject to the conditions (13) gives $\psi_0(\eta)=1$. To determine ψ_1 from the equation (12) and the conditions (14), we use the method of series expansions followed by Laplace's method. This method was developed by Meksyn (1956) and has been found to give good results even for a few terms in the expansions. In applying the method, we express $\psi_1(\eta)$ in powers of η as

$$\psi_1(\eta)=\sum_{n=0}^{\infty} \frac{a_n}{n!} \eta^n \tag{15}$$

where a_n's are constants to be determined. This expansion is valid only for sufficiently small values of η. The boundary conditions (14*a*) through (6*a*) gives $a_1=-1$. Substituting (15) and similar expansions for f and g in (12) and (4), and equating the coefficients of the like powers of η, we obtain a_n's after simplifications as

$$\left.\begin{array}{l} a_2=-2M,\ a_3=Ska_0-2M^2,\ a_4=-Sa_0-4k, \\ a_5=S+4+(6S-20)kM-2SMa_0,\ \text{etc.} \end{array}\right\} \tag{16}$$

where $K=f''(0)$ and $M=g'(0)$. The accurate values of K and M are known from Cochran's solutions and are given as

$$K=.510,\ M=-.616.$$

Thus, from (16), we see that ψ_1 is determined if a_0 is known. The unknown coefficient a_0 is to be determined from the conditions (14*b*) as $\eta \to \infty$.

Integrating the equation (12) once, we get

$$\psi'_1 e^F=-1+\int_0^\eta (Sf'\psi_1-2gg')e^F d\eta \equiv X(\eta) \tag{17}$$

where

$$F(\eta)=2S\int_0^\eta f(\eta)\,d\eta. \tag{18}$$

Integrating again the equation (17), we get

$$\psi_1=a_0+\int_0^\eta X(\eta)\exp(-F(\eta))d\eta. \tag{19}$$

The coefficient a_0 is given by the equation

$$a_0+\int_0^\infty X(\eta)\exp\{-F(\eta)\}\,d\eta=0. \tag{20}$$

Since $X(\eta)\to -1$ as $\eta\to\infty$, the integral can be evaluated asymptotically by Laplace's method.

Putting $F=\tau$, transforming the equation (20) to τ-variable and integrating in gamma functions, we get

$$3a_0+\sum_{n=1}^{\infty} h_n\Gamma(n/3)=0 \tag{21}$$

where the coefficients h_n are found to be

$$h_1=-(3/ks)^{\frac{1}{3}},\ h_2=-(2M+\tfrac{1}{3}k)(3/ks)^{\frac{2}{3}},$$
$$h_3=-(3/ks)\Big(\frac{3}{16k^2}+\frac{13M}{10k}+M^2+\tfrac{1}{2}ks\,a_0\Big),\ \text{etc.} \tag{22}$$

The series (21) determines a_0 for different values of S. Taking the first six terms of the series (21), we determined for $S=1$, 1.5 and 2.0. The values of a_0 are as

S	1.0	1.5	2.0
a_0	.0613	.0758	.1044

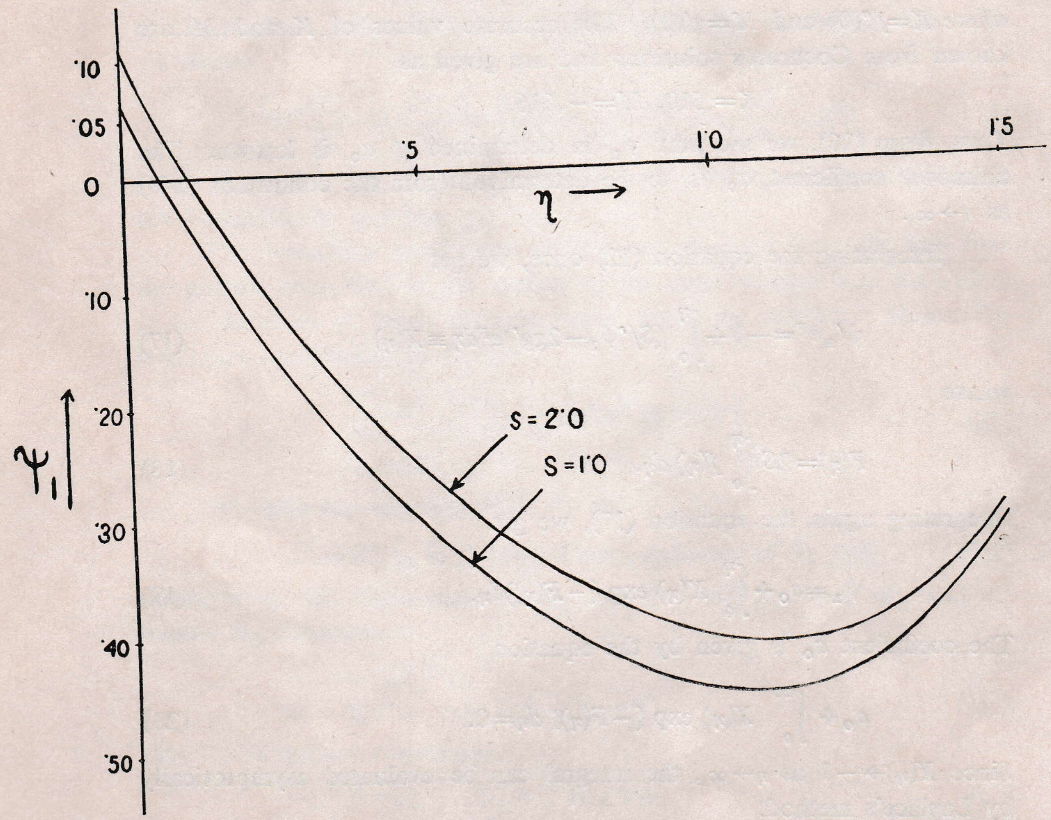

Figure 1 : Variation of $\psi_1(\eta)$ against η for $S=1.0$ and $S=20$.

The variation $\psi_1(\eta)$ near the surface of the cone is shown in figure 1 for $S=1.0$, and 2.0.

4. *Discussion*

From the expression (10), we observe that if the product $\xi\beta \cos \alpha$ is zero, then $C_1=(C_1)_\infty$ everywhere and hence there is no separation of the species in this case. We assume that $m_1<m_2$, that is, the rarer component is lighter than the other. Also, let the mixture be outside the cone so that $\alpha>90°$. Then the product $\xi B \cos \alpha$ is negative away from the vertex of the cone.

The figure 1 shows that ψ_1 is positive near the surface of the cone but becomes negative away from it. Thus, the rarer component moves away from the surface to another layer. Similar distribution of the concentrations of the species is predicted by Sarma (1973) for the case of a rotating disk. The separation of the species near the surface of the cone is enhanced by the increase in the Schimdt number S.

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