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APPROXIMATE ESTIMATION OF SECOND ORDER WAVE FORCE ON SQUARE CAISSONS

M. RAHMAN

Department of Applied Mathematics
Technical University of Nova Scotia
Halifax, Nova Scotia
Canada, B3J 2x4

Abstract

A second order solution of wave diffraction is derived and used to evaluate the forces exerted by waves on large vertical square-section caissons. The recent nonlinear theory of Rahman and Heaps (10) applicable to large circular cylinders has been extended to predict the wave forces on large square caissons. The theoretical predictions are compared with the available experimental data. The comparison shows excellent agreement.

Introduction

In recent years, considerable attention has been paid in the research of estimating wave loads on various shapes of monolithic offshore structures; however, very little knowledge is available concerning wave loadings on large vertical square-section caissons resting on the ocean bottom and extending through the water surface. Numerical methods for the solution of wave loads on large bodies including square caissons have been described by many authors including Hogben and Standing (1975) and Garrison and Chow (1972). Hogben and Standing (1975) presented some results of wave loads measured on a model of a square caisson, and compared their measured values with the approximate solution based on the diffraction theory of MacCamy and Fuchs (1954).

In this paper we have assumed that the wave forces on a square caisson or cylinder occur in a similar manner to the wave forces on a circular cylinder. That is, if the size of the square cylinder is small compared to the incident wave length, the cylinder does not deform the incident wave, and the wave forces can be calculated using the Morrison equation, i.e. the wave force consists of the sum of the inertial and viscous drag forces. But if the size of the square cylinder is large compared to the wave length, it causes reflection and diffraction of the incident waves and the viscous drag forces may be neglected in comparison with the inertial forces.

It has been found experimentally that the linear diffraction theory of MacCamy and Fuchs (1954) for small amplitude waves is inadequate for the prediction of wave forces on a vertical circular cylinder under extreme wave conditions. In real situations, the character of ocean waves is generally a nonlinear function of the disturbance.

The nonlinear theory of wave diffraction has been attempted by many previous researchers including Chakrabarti (1975), Yamaguchi and Tsuchiya (1974), and Raman and Venkatanarasaiah (1976). Molin (1979) formulated a nonlinear diffraction theory using the Haskind reciprocal relations as outlined by Lighthill (1979). However, all these theories appear to be incorrect one way or the other.

Hunt and Williams (1982) provided a nonlinear theory to calculate wave loads on a circular cylinder using perturbation technique. They have derived a second order solution for the diffraction of a nonlinear, progressive wave in shallow water, incident on a vertical, surface-piercing, circular cylinder. Recently, a very elegant second-order theory of wave diffraction based on a direct perturbation technique utilizing complex variable analysis has been developed by Rahman and Heaps (1983). The theory has been applied to predict the wave forces on large circular cylinders and theoretical predictions have been compared with many available experimental data. The comparison shows very good correlation between the prediction and the experimental data.

This paper is mainly concerned with the estimation of wave forces on large vertical square caissons. The theory of Rahman and Heaps (1983) for the circular cylinders has been extended to predict the wave forces on large square caissons. The theoretical predictions are compared with the experimental data collected by the National Research Council of Canada. The comparison shows excellent agreement.

Mathematical Formulation

Consider a rigid vertical surface piercing square caisson of side length b acted on by a regular surface wave of height H that progresses in the direction of the positive x-axis. In the absence of the wave the water depth is h and in the presence of the wave the free surface height is η above the mean surface level. We introduce a rectangular co-ordinate system with the origin at the still water surface, x in the direction of propagation of waves, y perpendicular to this in the plane of the still water level, and z vertically upwards. The incident wave train has amplitude $H/2$, wave-length L, wave number $k=2\pi/L$, and wave frequency $\sigma=2\pi/$

T, where T is the period. Assuming irrotational incompressible flow theory, the fluid velocity is given by $q=\triangle\emptyset$, in which the velocity potential must satisfy Laplace's equation. The boundary conditions are same as those of Rahman and Heaps (1983). Following their work, we use Taylor's expansion about $z=0$ together with perturbation technique for expansion of $\emptyset$ and η in powers of ϵ [$=0\,(kH/2)$], and equate like powers of ϵ. Considering upto second order term, we get conditions for ϕ_1 and ϕ_2. Bernoulli's equation gives pressure $P(x, y, z, t)$ which, using the perturbation expansion of ϕ upto second order term, yields

$$P=-\rho gz-\varepsilon\rho\frac{\partial\phi_1}{\partial t}-\varepsilon^2\rho\left\{\frac{\partial\phi_2}{\partial t}+\frac{1}{2}(V\phi_1)^2\right\}+0(\varepsilon^3). \tag{1}$$

The force evaluation on a square caisson has been described in the following section :

Force Calculations on Square Caissons :

It is assumed that the wave force on a large rigid square-section cylinder or caisson be expressed as an inertial force if the coefficient of mass used includes the effects of wave reflection and diffraction. Thus the force on the offshore structures based on the work of Morison *et. al.* (1950) may be expressed after neglecting the drag force as (justified if the dimension of the structure is large relative to wave length)

$$F=\rho V\, C_{Ms}\frac{du}{dt} \tag{2}$$

where p is the fluid density, V is the volumetric displacement of the body, $\frac{du}{dt}$ is fluid acceleration along the horizontal direction, and C_M is the coefficient of mass of the body.

Then the horizontal force per unit length on the caisson in the x-direction is expressed as

$$f_{xs}=\rho\, b^2\, C_{Ms}\frac{du}{dt} \tag{3}$$

where C_{Ms} is the coefficient of mass for square-section caisson, and b is the side length of the caisson.

Similarly, if the square caisson is replaced by a circular cylinder, then the horizontal force per unit length on the circular cylinder with exactly the same conditions as for the square cylinder, in the x-direction is given by

$$f_x=\rho\pi\,\frac{D_e^2}{4}C_M\frac{du}{dt} \tag{4}$$

where D_e is the diameter of the cylinder and C_M is the coefficient of mass of the circular cylinder.

For design purposes, we may approximate the effects of wave scattering by the square caisson and its associated wave loading by that of a circular cylinder of "equivalent diameter". Now if the inertial forces exerted on both the structures are the same, then by equating these forces, we have

$$\rho b^2 C_{Ms}\frac{du}{dt}=\rho\pi(D_e^2/4)C_M\frac{du}{dt} \qquad (5)$$

This concept, then, enables us to find the diameter of an equivalent circular cylinder

$$D_e=2b(C_{Ms}/\pi C_M)^{\frac{1}{2}} \qquad (6)$$

We know that the characteristics of the scattered wave fields are fundamentally different in each case ; however, in this paper, we are mainly concerned with the approximate estimation of second order wave forces on the square caissons for engineering design purposes. It has been reported in the offshore engineering literatures that $C_{Ms}=2.19$ and $C_M=2$ approximately, and therefore, as a first approximation we have used in this study, $C_{Ms}\approx C_M$ such that equation (19) can be reduced to a very simple expression

$$D_e=2b/(\pi)^{\frac{1}{2}} \qquad (7)$$

Thus with the knowledge of this equivalent diameter concept, we get the expression for the horizontal force exerted on the square caisson same as (28)—(30) of Rahman and Heaps (1983).

Resultant Horizontal Forces on Square Caissons

We are now in a position to rewrite the theoretical expressions to predict the wave forces on square caisson. The first order horizontal force on an equivalent diameter circular cylinder with diameter D_e can be written as (53) of Rahman and Heaps (1983). Now replacing D_e and a_c by the diameter of the square caisson *i.e.* $D_e=(2b/\sqrt{\pi})$ and $a_e=(b/\sqrt{\pi})$, we can write the non-dimensional expression for εF_{x_1} as :

$$\frac{\varepsilon F_{x_1}}{\rho g b^3}=\frac{1}{2}C_M\frac{\left(\frac{H}{L}\right)}{\left(\frac{b}{L}\right)}\tanh kh\cos(\sigma t-\alpha_1) \qquad (8)$$

and

$$C_M = \frac{4}{(kb)^2 \left| H_1^{(1)'}\left(\frac{kb}{\sqrt{\pi}}\right) \right|}. \tag{9}$$

where C_M is the coefficient of added mass due to linear theory.

The second order forces may be expressed as the sum of steady and oscillating parts :

$$F_{x_2} = F^{SS}_{x_2} + F^{OS}_{x_2} \tag{10}$$

in which

$$F^{SS}_{x_2} = -\frac{2\rho g a_e}{\pi k_2}\left(\frac{1}{ka_e}\right)^3 \sum_{l=0}^{\infty} \left\{\left[1 - \frac{l(1+l)}{k^2 a_e^2}\right]\left(1 + \frac{2kh}{\sinh 2kh}\right) E_l\right\} \tag{11}$$

$$F^{OS}_{x_2} = \left\{\frac{\rho g \tanh kh}{k} e^{-2i\sigma t} \int_{k_2=0}^{\infty} G(k_2)\, dk_2 + C.C\right\}$$

$$-\frac{2\rho g a_e}{\pi k^2}\left(\frac{1}{ka_e}\right)^2 \sum_{l=0}^{\infty} (-1)^l \left\{\left(3 - \frac{2kh}{\sinh 2kh}\right) + \left(1 + \frac{2kh}{\sinh 2kh}\right)\frac{l(1+l)}{k^2 a_e^2}\right\}$$

$$\times (C_l \cos 2\sigma t - S_l \sin 2\sigma t) \tag{12}$$

where

$$E_l = \frac{J'_l\, Y'_{l+1} - J'_{l+1}\, Y'_l}{(J'^2_l + Y'^2_l)(J'^2_{l+1} + Y'^2_{l+1})} \tag{13}$$

$$C_l = \frac{Y'_l J'_{l+1} + Y'_{l+1} J'_l}{(J'^2_l + Y'^2_l)(J'^2_{l+1} + Y'^2_{l+1})} \tag{14}$$

$$S_l = \frac{Y'_1\, Y'_{l+1} - J'_l\, J'_{l+1}}{(J'^2_l + Y'^2_l)(J'^2_{l+1} + Y'^2_{l+1})} \tag{15}$$

the Bessel functions argument being ka_e, and

$$G(k_2 a) = \frac{\int_{ka}^{\infty} A_1(k_3 r)\, B_1(kr)\,(kr)\, d(kr)}{(k_2 a_e)\left[(k_2 a_e) - \frac{4ka_e \tanh kh}{\tanh k_2 h}\right] H_1^{(1)\prime}(k_2 a_e)} \tag{16}$$

in which

$$B_1(kr) = 8 \sum_{m=0}^{\infty} (-1)^{m+1} \left[A_{m,m+1} + \frac{2 \coth kh}{k^2 r^2} m(m+1) A_m A_{m+1}\right] \tag{17}$$

$$A_{m,n} = (3 \tanh kh - \coth kh)\, A_m A_n + 2 A_m A_n \coth kh \tag{18}$$

and

$$A_m(kr) = J_m(kr) - \frac{J'_m(ka_e)}{H_m^{(1)\prime}(ka_e)} H_m^{(1)\prime}(kr) \tag{19}$$

$$A_1(k_2r)=J_1(k_2r)-\frac{J'_1(k_2a_e)}{H_1^{(1)}(k_2a_e)}H_1^{(1)'}(k_2r). \tag{20}$$

Now replacing D_e and a_e by the dimension of the square caisson, we obtain the following expression for the second order component of the total horizontal force as

$$\frac{\varepsilon F_{x_2}}{\rho g b^3}=\left\{\frac{(H/L)^2}{2(b/L)}e^{-2i\sigma t}\tanh kh\int_0^\infty G(k_2)\,dk_2+c.c.\right\}$$

$$-\frac{(H/L)^2}{2\pi^{3/2}\left(\frac{b}{L}\right)^2}\sum_{l=0}^{\infty}(-1)^l\left\{\left(3-\frac{2kh}{\sin 2kh}\right)\right.$$

$$\left.+\frac{l(l+1)}{4\pi\left(\frac{b}{L}\right)^2}\left(1+\frac{2kh}{\sinh 2kh}\right)\right\}\{C_l\cos 2\sigma t-S_l\sin 2\sigma t\}$$

$$-\frac{(H/L)^2}{8\pi^{3/2}\left(\frac{b}{L}\right)^4}\sum_{l=0}^{\infty}\left\{\left[1-\frac{l(l+1)}{4\pi\left(\frac{b}{L}\right)^2}\right]\left(1+\frac{2kh}{\sinh 2kh}\right)E_l\right\}. \tag{21}$$

The total horizontal force upto second order term on a square caisson in nondimensional form may be expressed as

$$F=\frac{\varepsilon F_{x_1}}{\rho g b^3}+\frac{\varepsilon^2 F_{x_2}}{\rho g b^3}. \tag{22}$$

Discussion of Results

This paper describes an application to the nonlinear theory of wave diffraction developed by Rahman and Heaps (1983). The wave force estimation on a large square caisson has been presented and the results are compared with available experimental data. It has been reported by many previous researchers that large monolithic structures subjected to the extreme ocean waves pose many difficulties in estimating the wave loads by linear wave diffraction theory developed by MacCamy and Fuchs (1954). Especially Mogridge and Jamieson (1976) reported that the linear wave diffraction theory can be used with an accuracy of 6% for the parameters $\frac{h}{L}$ and $\frac{b}{L}$ greater than 0.09. However, for $\frac{h}{L}$ and $\frac{b}{L}$ less than 0.09, the percentage differences are greater than 6% and very large errors occur particularly for large wave steepnesses.

In this study the variation of wave forces with wave steepness in non-dimensional form has been plotted in Figures 1-4. Linear and non-linear forces in nondimensional forms have been compared with the experimental

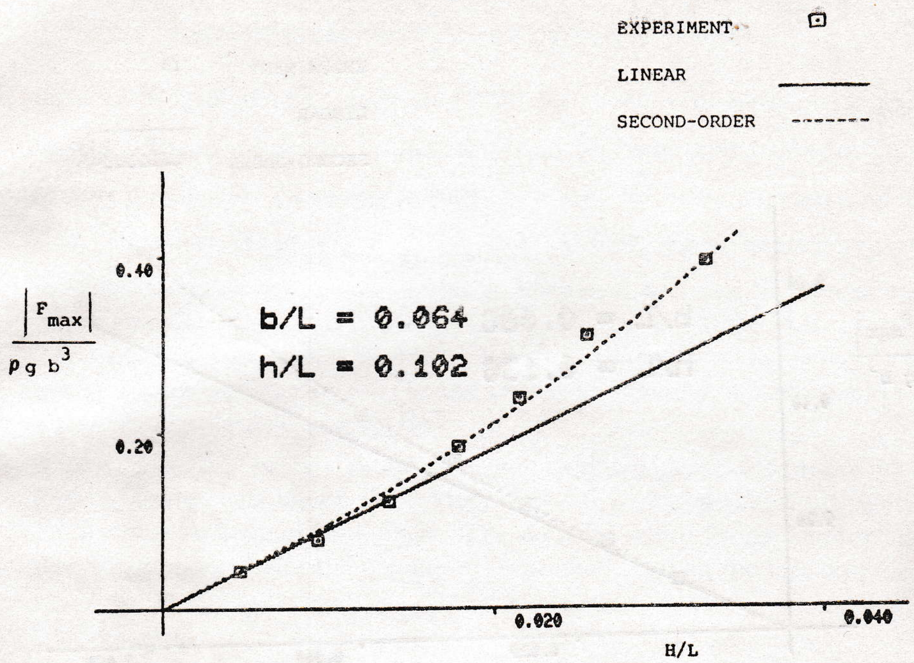

Fig. 1
Comparison of linear and second order forces with experiment.

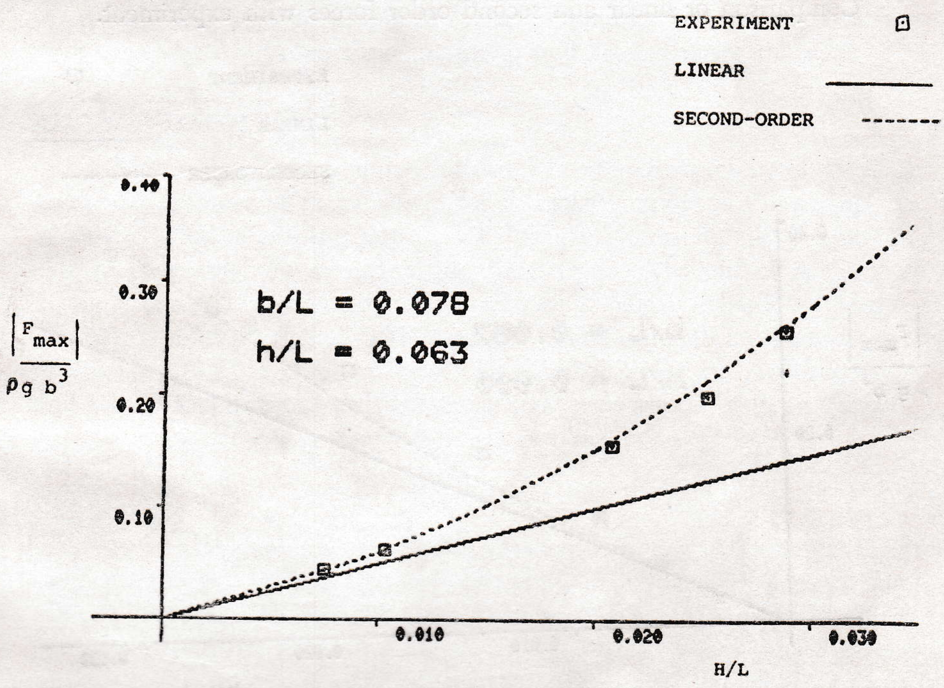

Fig. 2
Comparison of linear and second order forces with experiment.

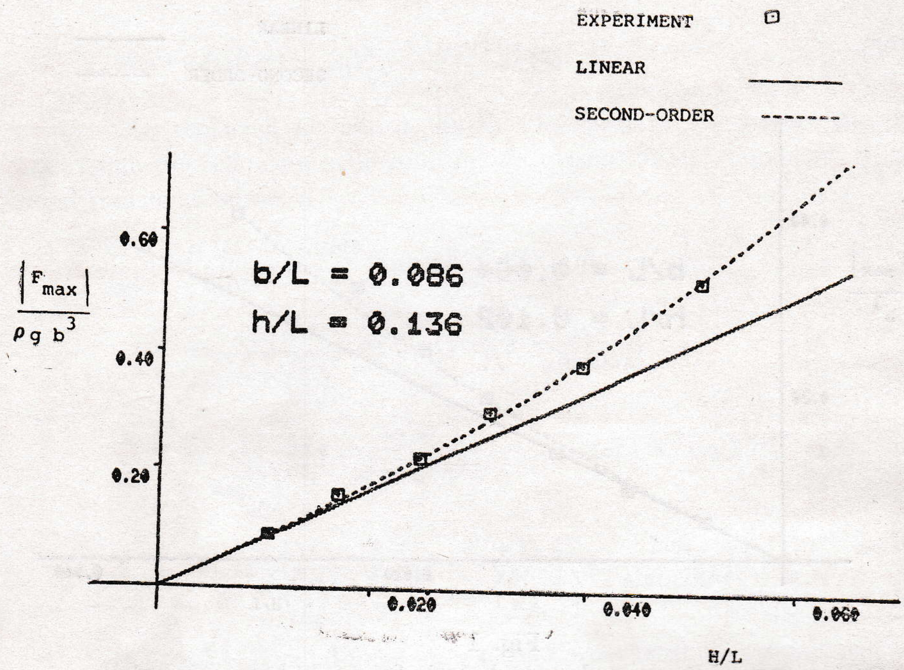

Fig. 3
Comparison of linear and second order forces with experiment.

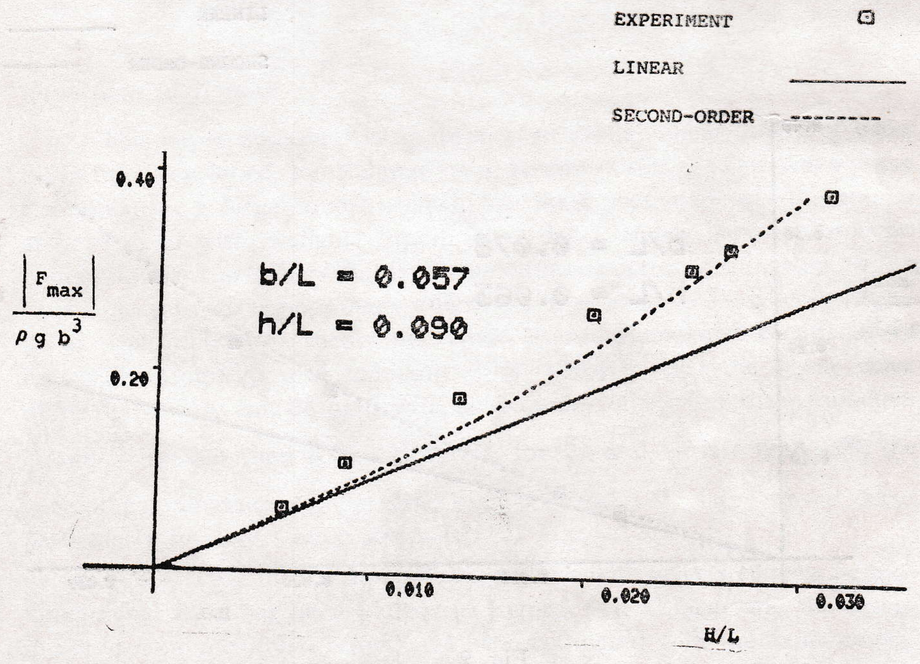

Fig. 4
Comparison of linear and second order forces with experiment.

data of Mogridge and Jamieson (1976) for various dimensionless parameters. The experimental results for $\frac{b}{L}$ and $\frac{h}{L}$ greater than 0.09 were found to agree well with the linear diffraction theory for wave steepness up to 0.09. However, if either $\frac{b}{L}$ or $\frac{h}{L}$ is less than 0.09, linear theory appears to be inapplicable because of the large differences between the theory and the experiment as reported by Mogridge and Jamieson (1976). It is believed that these differences may be partly due to nonlinearity of the waves near the structures and partly due to the effects of viscosity introducing drag forces of considerable magnitude.

The second order solution presented in this paper are compared with force measurements obtained by Mogridge and Jamieson (1976) in Figures 1-4 for various parameter ranges. The data for wave forces is generally seen to agree well with the second order predictions than the first order.

Conclusions

Second order diffraction effects have been included in the calculation of the maximum horizontal force on a vertical surface piercing square caisson. The automatic computer generated values of the maximum forces have been displayed in graphical forms in Figures 1-4 showing considerable correlation between the experiment and second order theory.

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