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GENERAL TYPE WAVES PROPAGATING AT THE INTERFACE BETWEEN HEAT-FLUX DEPENDENT MICROPOLAR THERMO-ELASTIC HALF SPACE AND A COMPRESSIBLE NON-VISCOUS FLUID

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ABSTRACT

The problem of general type waves propagating at the interface between micropolar thermo-elastic half space and a compressible non-viscous fluid is studied. It is found that the waves are not necessarily plane waves and the motion is not necessarily confined to a plane perpendicular to the interface. We get additional waves which are counterpart of classical elastic waves. The characteristic equation for the waves obtained in this case is similar to the characteristic equation for Stoneley-type (plane) waves but is effected by micropolar and thermal effects.

1. *Introduction* :

The micropolar theory of elasticity formulated by Eringen [1, 2] is a mathematical model of solids with an effective microstructure. This theory is expected to find applications in the treatment of the mechanics of granular materials with elongated rigid grains and composite fibrous materials. It explains the behaviour of solid propellant grains, polymeric materials and fiber glass in which the classical theory is inadequate.

Several attempts have been made in recent years to develop generalized theory of thermoelasticity. Kaliski [3] and Lord and Shulman [4] have developed a generalized theory of thermoelasticity which involves heat-flux rate. Green and Lindsay [5] have developed another generalized theory which involves temperature rate among the constitutive variables. Recently, Chandrasekhariah [6] have formulated a generalized theory of micropolar thermoelasticity by including heat-flux among the constitutive variables. There also exists some experimental evidence in favour of thermal waves (second sound) propagating with finite, though quite large, speeds. Ackerman *et al* [7] and Ackerman and Overton [8] performed such experiments on solid helium and concluded that such phenomena do exist, though the frequency range of such thermal excitation in which thermal waves can be directed is extremely narrow.

Chadrasekhariah [9] studied waves of general type propagating at the interface between an elastic half space and a compressible fluid. We have discussed here the same problem in the context of the generalized theory of micropolar thermo-elasticity [6]. As in [9], our analysis also shows that the particles of the solid and fluid vibrate in different planes and the orientation of each of these planes relative to the interface varies from point to point, in general and only under a particular condition, the planes become perpendicular to the interface. The characteristic equation for phase velocity of the waves obtained in this case involve thermal as well as micropolar effects and we have seen that this equation holds irrespective of whether the waves are unidirectional or not.

2. *Statement of the Problem and Basic Equations* :

We assume that non-viscous compressible fluid is free to slide parallel to the interface of the micropolar thermo-elastic solid half space. We choose the Cartesian coordinate axes (x_1, x_2, x_3) such that the micropolar thermo-elastic solid occupies the region $x_3>0$ and the fluid the region $x_3<0$. The linearised equations of heat-flux dependent micropolar thermo-elasticity of a homogeneous and isotropic solid in the absence of body forces and heat sources, are [6]

$$t_{ij}=\lambda\delta_{ij}u_{k,k}+\mu(u_{i,j}+u_{j,i})+k'(u_{j,i}-\epsilon_{ijk}\xi_k)-\beta_T\delta_{ij}\theta \tag{2.1}$$

$$m_{ij}=\alpha'\xi_{r,r}\delta_{ij}+\beta'\xi_{i,j}+\gamma'\xi_{j,i} \tag{2.2}$$

$$(\mu+k')\nabla^2\vec{u}+(\lambda+\mu)\text{ grad }(\text{div }\vec{u})$$
$$+k'\text{ curl }\vec{\xi}-\beta_T\text{ grad }\theta=\rho\ddot{\vec{u}} \tag{2.3}$$

$$K\nabla^2\theta-\theta_0(1+\tau\partial/\partial t)(c^*\dot{\theta}+\beta_T\text{ div }\dot{\vec{u}})=0 \tag{2.4}$$

$$\gamma'\nabla^2\vec{\xi}+(\alpha'+\beta')\text{ grad }(\text{div }\vec{\xi})+k'\text{ curl }\vec{u}-2k'\vec{\xi}=\rho J\ddot{\vec{\xi}} \tag{2.5}$$

where, t_{ij} is the force stress tensor, m_{ij} is the couple stress tensor, $\vec{u}$ is the displacement, $\vec{\xi}$ is the microrotation vector, λ and μ are Lame constants, $\beta_T=(3\lambda+2\mu+k')a_t{}^*$, $a_t{}^*$ being the coefficient of the linear thermal expansion, ρ is the mass density of solid, K is the thermal conductivity, θ is the temperature deviation, $(\alpha', \beta', \gamma', k')$ are micropolar constants, J is the rotational inertia, θ_0 is the initial uniform temperature, τ is the thermal relaxation time and c^* is the specific heat at constant volume. A superposed dot denotes differentiation with respect to time t.

We take non-viscous compressible fluid which is governed by equations

$$\rho_0 \ddot{\vec{d}} = -\text{grad } p_0 \tag{2.6}$$

$$p_0 = -\rho_0 c_0^2 \text{ div } \vec{d} \tag{2.7}$$

where, ρ_0 is the mass density of fluid, p_0 is the pressure in the fluid, c_0 is the speed of sound in the fluid and $\vec{d}$ is the displacement vector.

Since we assume that the fluid is free to slide parallel to the interface $x_3=0$, the conditions to be satisfied are

$$(i)\ u_3=d_3\ ;\ (ii)\ t_{33}=-p_0\ ;\ (iii)\ t_{31}=0\ ;\ (iv)\ t_{32}=0\ ;\ (v)\ m_{31}=0\ ;$$
$$(vi)\ m_{32}=0\ ;\ (vii)\ \theta_{,3}+h\theta=0 \text{ at } x_3=0 \tag{2.8}$$

Here h is the heat transfer coefficient at the interface.

3. *Solution* :

We take for solid

$$\vec{u} = \text{grad } \Phi + \text{curl } \vec{\Psi} \tag{3.1}$$

and for liquid

$$\vec{d} = \text{grad } \phi_0 \tag{3.2}$$

where, Φ and Ψ are the Lame potentials and ϕ_0 is the displacement potential in the liquid.

Using (3.1)and (3.2) in (2.3)–(2.7), we get

$$\left(\nabla^2 - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}\right)\Phi = \frac{\beta_T \theta}{\lambda+2\mu+k'} \tag{3.3}$$

$$\left(\nabla^2 - \frac{1}{b^2}\frac{\partial^2}{\partial t^2}\right)\vec{\Psi} = -\frac{k'}{\mu+k'}\vec{\xi} \tag{3.4}$$

$$\left[\left\{\nabla^2 - \frac{\theta_0}{K^*\rho}\frac{\partial}{\partial t}\left(1+\tau\frac{\partial}{\partial t}\right)\right\}\left\{\nabla^2 - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}\right\} - \frac{e}{K^*}\nabla^2\frac{\partial}{\partial t}\left(1+\tau\frac{\partial}{\partial t}\right)\right]\Phi = 0 \tag{3.5}$$

$$\left[\left\{(\alpha'+\beta'+\gamma')\nabla^2 + (\alpha'+\beta')\text{ curl curl} - 2k' - \rho J\frac{\partial^2}{\partial t^2}\right\}\right.$$
$$\left.\left\{\nabla^2 - \frac{1}{b^2}\frac{\partial^2}{\partial t^2}\right\} - \frac{k'^2}{\mu+k'}\text{curl curl}\right]\vec{\Psi} = 0 \tag{3.6}$$

$$c_0^2 \nabla^2 \phi_0 = \partial^2 \phi_0 / \partial t^2 \tag{3.7}$$

$$P_0=-\rho_0\frac{\partial^2\phi_0}{\partial t^2} \tag{3.8}$$

where

$$a^2=a^{*2}+k'/\rho,\ b^2=b^{*2}+k'/\rho$$
$$e=\beta_T{}^2\theta_0/\rho c^*(\lambda+2\mu+k')$$
$$a^{*2}=(\lambda+2\mu)/\rho,\ b^{*2}=\mu/\rho,\ K^*=K/\rho c \tag{3.9}$$

We consider the displacement potentials, the temperature and microrotation of the form

$$\Phi=f(x_3)P(x_1,\ x_2)e^{i\omega t} \tag{3.10}$$
$$\vec{\Psi}=g(x_3)\vec{Q}(x_1,\ x_2)e^{i\omega t},\ \vec{Q}=(Q_1,\ Q_2,\ 0) \tag{3.11}$$
$$\phi_0=h_0(x_3)P_0(x_1,\ x_2)e^{i\omega t} \tag{3.12}$$
$$\theta=\theta^*(x_1,\ x_2,\ x_3)e^{i\omega t} \tag{3.13}$$
$$\vec{\xi}=\vec{\xi}^*(x_1,\ x_2,\ x_3)e^{i\omega t} \tag{3.14}$$

where, ω is the angular frequency and $P(x_1,\ x_2)$, $Q(x_1,\ x_2)$, $P_0(x_1,\ x_2)$ satisfying the 2-dimensional reduced wave equation

$$(\partial^2/\partial x_\alpha\partial x_\alpha+\omega^2/c^2)F=0,\ \alpha=1,\ 2 \tag{3.15}$$

Obviously, the waves are interfacial in nature and propogate with speed c.

Also, $f(x_2)$, $g(x_3)$ and $h_0(x_3)$ satisfy the conditions

$$f(x_3)\to 0,\quad g(x_3)\to 0 \text{ as } x_3\to\infty$$

and

$$h_0(x_3)\to 0 \text{ as } x_3\to-\infty \tag{3.16}$$

Substituting (3.10)–(3.12) into (3.5–3.7) and using (3.15) and (3.16), we get

$$f(x_3)=e^{-mx_3}+Ae^{-nx_3} \tag{3.17}$$
$$g(x_3)=e^{-qx_3}+De^{-q'x_3} \tag{3.18}$$
$$h_0(x_3)=e^{lx_3} \tag{3.19}$$

where, A and D are arbitrary constants, m and n are roots satisfying the equations

$$m^2+n^2=[(2p^2-r^2)+(i\omega\theta_0/K^*\rho)\{1+i\omega\tau\}\{1+\rho e/\theta_0\}]$$
$$m^2n^2=p^2[(p^2-r^2)+(i\omega\theta_0/K^*\rho)\{1+i\omega\tau\}\{1-r^2/p^2+e\rho/\theta_0\}] \tag{3.20}$$

Similarly q and q' are roots satisfying the equations

$$q^2+q'^2=[(p^2-s^2)+(\alpha'+\beta'+\gamma')p^2/\gamma'-\{\rho J\omega^2-2k'+k'^2/(\mu+k')\}/\gamma']$$
$$q^2q'^2=(p^2-s^2)[(\alpha'+\beta'+\gamma')p^2/\gamma'-(\rho J\omega^2-2k')/\gamma'] \tag{3.21}$$

In these equations, we have put

$$l^2=p^2-\omega^2/c_0^2,\ p^2=\omega^2/c^2,\ r^2=\omega^2/a^2,\ s^2=\omega^2/b^2 \tag{3.22}$$

With the aid of equations (3.17)–(3.19), the equations (3.10)–(3.12) become

$$\Phi=(e^{-mx_3}+Ae^{-nx_3})P(x_1,x_2)e^{i\omega t} \tag{3.23}$$
$$\vec{\Psi}=(e^{-qx_3}+De^{-q'x_3})\vec{Q}(x_1,x_2)e^{i\omega t} \tag{3.24}$$
$$\phi_0=P_0(x_1,x_2)e^{lx_3+i\omega t} \tag{3.25}$$

With the aid of equations (3.3), (3.4), (3.15), (3.23) and (3.24), we obtain

$$\theta=\eta_1P(x_1,x_2)e^{i\omega t} \tag{3.26}$$
$$\vec{\xi}=\eta_1'\vec{Q}(x_1,x_2)e^{i\omega t} \tag{3.27}$$

where

$$\eta_1=\frac{\rho a^2(m^*e^{-mx_3}+An^*e^{-nx_3})}{\beta_T}$$
$$\eta_1'=-\frac{(\mu+k')(q^*e^{-qx_3}+Dq'^*e^{-q'x_3})}{k'}$$
$$m^*=m^2-p^2+r^2,\ n^*=n^2-p^2+r^2$$
$$q^*=q^2-p^2+s^2,\ q'^*=q'^2-p^2+s^2 \tag{3.28}$$

With the aid of equations (2.1), (3.1), (3.2), (3.8), (3.15), (3.23)–(3.27), the conditions (i), (ii) and (vii) yield

$$(m+An)P+(1+D)X+lP_0=0 \tag{3.29}$$
$$B'(1+A)P+C'(q+Dq')X+\rho_0\omega^2P_0=0 \tag{3.30}$$
$$(h-m)m^*+(h-n)n^*A=0 \tag{3.31}$$

where

$$B'=C'p^2-(\mu+k')s^2,\ C'=(2\mu+k')$$
$$X=Q_{1,2}-Q_{2,1} \tag{3.32}$$

The conditions (iii), (iv) and (v), (vi) taken in the forms

$$t_{31,1}+t_{32,2}=0 \text{ on } x_3=0 \tag{3.33}$$

and

$$m_{31}+m_{32}=0 \text{ on } x_3=0 \tag{3.34}$$

yield

$$qq^*+q'q'^*D=0 \tag{3.35}$$
$$C'(m+An)p^2P+(1+D)XB'=0 \tag{3.36}$$

The equations (3.31) and (3.35) determine the constants A and D respectively.

On using (3.1), (3.2), (3.23)–(3.26), (3.29) and (3.36), we may express the displacement $\vec{u}$, $\vec{d}$ and temperature θ in terms of X and $\vec{Q}$ as follows

$$\vec{u}=(F_1X_{,1}+F_3Q_2,\ F_1X_{,2}-F_3Q_1,\ G_1X)e^{i\omega t} \tag{3.37}$$

$$\vec{d}=(F_0\partial/\partial x_1,\ F_0\partial/\partial x_2,\ G_0)Xe^{i\omega t} \tag{3.38}$$

$$\theta=\eta_1F_1Xe^{i\omega t}/(e^{-mx_3}+Ae^{-nx_3}) \tag{3.39}$$

while $\vec{\xi}$ is given by equation (3·27) in terms of $\vec{Q}$.
In these equations we have put

$$F_0=-(1+D)\,\rho\omega^2e^{lx_3}/lC'p^2,\quad G_0=lF_0$$

$$F_1=-(e^{-mx_3}+Ae^{-nx_3})(1+D)B'/(m+An)C'p^2$$

$$F_3=qe^{-qx_3}+Dq'e^{-q'x_3}$$

$$\begin{aligned}G_1=\{B'/(m+An)C'p^2\}[&(1+D)(me^{-mx_3}+Ane^{-nx_3})\\&-C'p^2(m+An)(e^{-qx_3}+De^{-q'x_3})/B']\end{aligned} \tag{3.40}$$

The functions X and $\vec{Q}$ are in general complex, we may therefore set

$$X=\Omega(x_1,x_2)e^{i\xi'(x_1,x_2)} \tag{3.41}$$

$$Q_\alpha=\bar{\Omega}_{(\alpha)}(x_1,x_2)e^{i\bar{\xi}_{(\alpha)}(x_1,x_2)} \tag{3.42}$$

Substituting (3.41) and (3.42) into (3.37)–(3.38) and taking only real parts of RHS, we get

$$\begin{aligned}
u_1&=F_1(\Omega_{,1}\cos\zeta-\Omega\xi'_{,1}\sin\zeta)+F_3\bar{\Omega}_2\cos\bar{\zeta}_2\\
u_2&=F_1(\Omega_{,1}\cos\zeta-\Omega\xi'_{,2}\sin\zeta)-F_3\bar{\Omega}_1\cos\bar{\zeta}_1\\
u_3&=G_1\Omega\cos\zeta\\
d_1&=F_0(\Omega_{,1}\cos\zeta-\Omega\xi'_{,1}\sin\zeta)\\
d_2&=F_0(\Omega_{,2}\cos\zeta-\Omega\xi'_{,2}\sin\zeta)\\
d_3&=G_0\Omega\cos\zeta
\end{aligned} \tag{3.43}$$

where

$$\zeta=\xi'+\omega t \text{ and } \bar{\zeta}_\alpha=\bar{\xi}_\alpha+\omega t \tag{3.44}$$

Similarly from (3.27) and (3.39), we obtain

$$\vec{\xi}=(\eta_1'\bar{\Omega}_1\cos\bar{\zeta}_1,\ \eta_1'\bar{\Omega}_2\cos\bar{\zeta}_2,\ 0) \tag{3.45}$$

$$\theta=\eta_1F_1\Omega\cos\zeta/(e^{-mx_3}+Ae^{-nx_3}) \tag{3.46}$$

Eliminating ζ from equations (3.43) in two different ways, we get the following two sets of equations connecting components of $\vec{u}$ and $\vec{d}$

$$G_1\Omega(\xi',{}_2u_1-\xi',{}_1u_2)+J'F_1u_3=(F_3G_1\Omega/\eta_1')(\xi',{}_2\xi_2+\xi',{}_1\xi_1) \tag{3.47}$$

$$G_0\Omega(\xi',{}_2d_1-\xi',{}_1d_2)+J'F_0d_3=0 \tag{3.48}$$

$$[(u_1-F_3\bar{\Omega}_2\cos\bar{\zeta}_2)\Omega,{}_2-(u_2+F_3\bar{\Omega}_1\cos\bar{\zeta}_1)\Omega,{}_1]^2/F_1{}^2\Omega^2J'^2+(u_3)^2/G_1{}^2\Omega^2=1 \tag{3.49}$$

$$(d_3/G_0\Omega)^2+[(d_1\Omega,{}_2-d_2\Omega,{}_1)/J'\Omega F_0]^2=1 \tag{3.50}$$

where, $J'=\Omega,{}_2\xi',{}_1-\Omega,{}_1\xi',{}_2$ (3.51)

The components of $\vec{\xi}$ are related by the relation

$$\xi_2=\bar{\Omega}_2\xi_1\cos\bar{\zeta}_2/\bar{\Omega}_1\cos\bar{\zeta}_1 \tag{3.52}$$

In the particular case when Ω=constant and $\xi'=\pm\gamma x_1$, equations (3.47) and (3.48) yield

$$u_2=-F_3\bar{\Omega}_1\cos\bar{\zeta}_1$$
$$d_2=0 \tag{3.53}$$

Substituting equations (3.53) in (3.49) and (3.50), we get

$$[(u_1-F_3\bar{\Omega}_2\cos\bar{\zeta}_2)/\gamma\Omega F_1]^2+(u_3/G_1\Omega)^2=1$$
$$d_1{}^2/\gamma^2\Omega^2F_0{}^2+d_3{}^2/G_0{}^2\Omega^2=1 \tag{3.54}$$

If we eliminate the functions P, X and P_0 from equations (3.29), (3.30) and (3.36) and use (3.22) and (3.32), we get

$$\{2-c^2/b^2+(k'/\mu)(1-c^2/b^2)\}^2$$
$$=[(m+An)/p(1+A)][(2+k'/\mu)^2(q+Dq')/(1+D)p-(\rho_0c^4/\rho b*^4)(1-c^2/c_0{}^2)^{-\frac{1}{2}}] \tag{3.55}$$

This is the characteristic equation determining the phase velocity c of the waves. The analysis of this equation is quite complicated, hence we consider the following special cases of equation (3.55).

Case I

Suppose solid half space is purely (non-thermal) elastic. Then we have $\theta=0$, with the help of equations (3.3), (3.10), (3.15), the equation (3.55) becomes

$$\{2-c^2/b^2+(k'/\mu)(1-c^2/b^2)\}^2$$
$$=(1-c^2/a^2)^{\frac{1}{2}}[(2+k'/\mu)^2(q+Dq')/(1+D)p-(\rho_0c^4/\rho b*^4)(1-c^2/c_0{}^2)^{-\frac{1}{2}}] \tag{3.56}$$

This equation gives the phase velocity of waves when micropolar thermo-elastic half space is replaced by a micropolar elastic half space.

Case II

Suppose that micropolar effect is not taken into account, then $k' \to 0$, with the help of equations (3.4), (3.11), (3.15), the equation (3.55) becomes

$$(2-c^2/b^{*2})^2\, p\eta^* = [4(1-c^2/b^{*2})^{\frac{1}{2}} - (\rho_0 c^4/\rho b^{*4})(1-c^2/c_0^2)^{-\frac{1}{2}}] \quad (3.57)$$

where

$$\eta^* = \frac{(mm^* - nn^*) + h(n^* - m^*)}{mn(m^* - n^*) + h(mn^* - m^* n)}$$

Case III

Suppose that both micropolar and thermal effects are not taken into account, then equation (3.55) becomes

$$(2-c^2/b^{*2})^2 = (1-c^2/a^{*2})^{\frac{1}{2}}[4(1-c^2/b^{*2})^{\frac{1}{2}} - (\rho_0 c^4/\rho b^{*4})(1-c^2/c_0^2)^{-\frac{1}{2}}] \quad (3.58)$$

From equations (3.29), (3.36), it follows that P and P_0 are expressible in terms of the single function X which in turn involves only $\vec{Q}$. Consequently, all five functions Φ, $\vec{\Psi}$, ϕ_0, $\vec{\xi}$ and θ are expressible in terms of $\vec{Q}$ and $\vec{Q}$ can be determined from equation (3.15). Hence problem is solved.

4. *Discussion* :

From equations (3.47) and (3.48), it follows that $\vec{u}$ and $\vec{d}$ lie in different planes and as such the particles of the solid half space and the fluid vibrate in different planes. As the coefficients in (3.47) and (3.48) depend on x_i so the orientation of each of these planes vary from point to point, in general. The planes become perpendicular to the interface if and only if the Jacobian in (3.51) vanishes identically. The planes obtained are similar to the planes obtained in [9], but the equations of the planes are different due to the thermal and micropolar effects. Since $\vec{u}$ and $\vec{d}$ satisfy the equations (3.49) and (3.50), the paths of particles of the solid half space and the fluid are curves of intersections of the planes (3.47), (3·48) and the cylinders (3.49), (3.50). Since the cylinders (3.49) and (3·50) are elliptic, so the particles of the solid half space and the fluid move in different elliptic orbits, the planes and sizes of which vary from one point to another in general. The elliptic orbits obtained in this case are similar to that obtained by Chandrasekhariah [11], but the centre is shifted to the point $(F_3\bar{\Omega}_2 \cos \bar{\zeta}_2, -F_3\bar{\Omega}_1 \cos \bar{\zeta}_1, 0)$ instead of origin. If we neglect micropolar effect, we get same orbits which are obtained in [11].

In particular case we have seen that in solid the waves propagate along x_1 and x_2 directions and the motions of the solid half space is in all the three planes but the motion of the fluid is confined to the x_2-plane only. We get additional waves in x_2 direction in solid due to micropolar effect. If $k' \to 0$, then we get waves only propagating in x_1 direction as obtained in [9]. The orbits of the particles in this case are given by (3.54). The sizes of these orbits also vary from one point to another.

The characteristic equation (3.55) involves micropolar and thermal constants which shows that phase velocity of the waves is also effected by thermal as well as micropolar effects. Equation (3.57) gives phase velocity of the waves when solid half space is thermo-elastic and this equation is identical to the equation (4.10) of [11]. Again equation (3.58) gives phase velocity of the waves when solid half space is purely elastic and this equation is identical to equation (4.11) of [11] and (16) of [9].

References

1. A. C. Eringen, Journal of Mathematics and Mechanics, 15, 909 (1966).
2. A. C. Eringen, Fracture II (Ed) Liebowitz, H. Academic Press, New York (1968).
3. S. Kaliski, Bull. Acad. Poln. Sci. Tech., 13, 253 (1965).
4. H. W. Lord and Y. Shulman, J. Mech. Phys. Solids, 15, 299 (1967).
5. A. E. Green and K. A. Lindsay, J. Elasticity, 2, I (1972).
6. D. S. Chandrasekhariah, Int. J. Engng. Sc., vol. 24, No. 8, pp. 1389–1395 (1986).
7. Ackerman, C. C., Bentman, B., Fairbank, H. A., and Guyer, R. A., 'Second sound in solid helium', Phys. Rev. Letters, vol. 16, 789 (1966).
8. Ackerman, C. C. and Overton, Jr., W. C., 'Second sound in solid helium—3', Phy. Rev. Letters, vol. 22, p. 764 (1969).
9. D. S. Chandrasekhariah, Journal of elasticity, 13 (1983), pp. 83–86.
10. I. Tolstoy, Wave propagation, McGraw-Hill, New York (1973).
11. D. S. Chandrasekhariah, Int. J. Engng. Sc., vol. 22, pp. 301–310 (1984).