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FLOW AND HEAT TRANSFER AROUND A CIRCULAR CYLINDER

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ABSTRACT

The temperature distribution within the thermal boundary layer region due to the flow of a non-Newtonian fluid around a heated circular cylinder, maintained at a constant temperature higher than that of the fluid at infinity is considered. The flow problem is solved by the method used by Meksyn. The graphs of the Nusselt number for different Prandtl numbers and for the same Eckert number have been plotted for various non-Newtonian parameters.

1. *Introduction* :

The two-dimensional thermal boundary layer equation for the flow of a second-order fluid past a heated flat wall has been derived by Srivastava (1967). He has obtained the temperature distribution for the flow near a stagnation point occurring on a flat plate maintained at a constant temperature higher than that of the fluid at infinity. Srivastava and Maiti (1966) have discussed the flow of a second-order fluid past a cylinder by expanding the flow functions in series and obtaining the first four terms by Kármán-Pohlhausen method. Srivastava and Saroa (1970) have studied the heat transfer in a second-order fluid for flow around a circular cylinder by Kármán-Pohlhausen method.

It has been observed that when only skin friction, etc., at the wall is needed, the Meksyn method is more useful as it has been found to give better results. In this paper we have reworked the problem of flow of a second-order fluid around a circular cylinder by series expansion used by Meksyn (1956). We have found the point of separation for the Newtonian case comes out to be 111·2° where as the exact value is

109·6°. Srivastava and Saroa (1970) have obtained the separation point at 110·8° by taking the boundary layer thickness to be variable along the cylinder. The corresponding point of separation obtained by Srivastava and Maiti (1966) by Kármán-Pohlhausen method is at 116·5°. Thus the method used here is expected to give more correct results.

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll (1960) is

$$\tau_{ij}=-\tilde{p}\delta_{ij}+\mu_1 D_{ij}+\mu_2 E_{ij}+\mu_3 D_{ik} D_j^{k}$$

where

$$D_{ij}=v_{i,j}+v_{j,i}$$

and

$$E_{ij}=a_{i,j}+a_{j,i}+2v_{m,i}v^{m}{}_{,j} \tag{1}$$

The v_i and a_i are the components of velocity and acceleration respectively. τ_{ij} is a stress tensor $\tilde{p}$ is an indeterminate hydrostatic pressure which differs, in general, from the mean pressure $p=\frac{1}{3}(\tau_{11}+\tau_{22}+\tau_{33})$; μ_1, μ_2, μ_3 are material constants of the fluid, and a comma denotes a covariant differentiation. The case $\mu_2=\mu_3=0$ corresponds to an incompressible Newtonian fluid. On thermodynamic considerations μ_2 is found to be negative. The material constants have been determined experimentally for solutions of poly-isobutylene in cetane of various concentrations by Markovitz and Brown.

2. *Boundary Layer Equations* :

Consider a stream of an incompressible second-order fluid moving with a uniform velocity U_∞ at infinity in presence of a fixed circular cylinder of radius l maintained at a constant temperature T_w. Let T_∞ be the temperature of the fluid at infinity and assume that $T_w > T_\infty$. We use cylindrical polar coordinates (r, θ, z) with z-axis coinciding with the axis of the cylinder. The flow is two-dimensional in r and θ directions.

The two-dimensional velocity boundary layer equations for the fluid governed by equations (1) are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu_1\frac{\partial^2 u}{\partial y^2}+\nu_2\Big[\frac{\partial^3 u}{\partial t\partial y^2}$$
$$+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\Big] \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

where $y=r-l$, $x=l\theta$ and u, v are the components of velocity in the x and y directions respectively, U is the velocity in the main stream, ρ is the density of the fluid and $\nu_i=\mu_i/\rho$, t is the time. The equation (2) is independent of μ_3 but the pressure involves both μ_2 and μ_3, and is not constant over the boundary layer region.

The thermal boundary layer equation for the incompressible second-order fluid against a heated wall has been given by Srivastava (1967) as

$$\rho c\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\left(\mu_1+\tfrac{1}{2}\mu_2\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial y}\right)^2$$
$$+\mu_2\left(u\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^2 u}{\partial y^2}\right)\frac{\partial u}{\partial y} \tag{4}$$

where c is the specific heat, k is the thermal conductivity and T is temperature. This equation is valid within the boundary layer over both a flat wall and a curved wall when x is taken in the tangential direction and y along the normal to the surface.

The boundary conditions on u, v and T are

$$u=0,\ v=0,\ T=T_w \text{ at } y=0,$$
$$u\to U,\ T\to T_\infty \text{ as } y\to\infty \tag{5}$$

The velocity distribution U outside the velocity boundary layer region created by the cylinder is given by

$$U(\theta)=2U_\infty\sin\theta=2U_\infty\left[\theta-\frac{1}{3!}\theta^3+\frac{1}{5!}\theta^5-\frac{1}{7!}\theta^7+\cdots\right] \tag{6}$$

The form (6) suggests that the velocity components u and v within the velocity boundary layer region and the temperature T within the thermal boundary layer region should be taken in the forms

$$u(\theta,\eta)=2U_\infty[\theta f_1'(\eta)-\frac{4}{3!}\theta^3 f_3'(\eta)+\frac{6}{5!}\theta^5 f_5'(\eta)-\frac{8}{7!}\theta^7 f_7'(\eta)+\cdots] \tag{7}$$

$$v(\theta,\eta)=-\left(\frac{2\nu_1 U_\infty}{l}\right)^{1/2}[f_1(\eta)-\frac{4}{2!}\theta^2 f_3(\eta)+\frac{6}{4!}\theta^4 f_5(\eta)-\frac{8}{6!}\theta^6 f_7(\eta)+\cdots] \tag{8}$$

$$T^*(\theta,\eta)=\frac{T-T_\infty}{T_w-T_\infty}=4E[T_1(\eta)-\theta^2 T_3(\eta)+\theta^4 T_5(\eta)-\theta^6 T_7(\eta)+\cdots] \tag{9}$$

where $\eta=y\left(\frac{2U_\infty}{\nu_1 l}\right)^{1/2}$ and the Eckert number $E=\frac{U_\infty^2}{C(T_w-T_\infty)}$. A prime denotes $\partial/\partial\eta$. We shall confine ourselves to terms up to f_7 and T_7 only.

The forms (7) and (8) of u and v satisfy the equation (3) of continuity. The boundary conditions of f_i and T_i, $i=1, 3, 5, 7$, etc., are

$$\begin{aligned} &f_1=f_3=f_5=f_7=0,\ f_1'=f_3'=f_5'=f_7'=0, \\ &\qquad T_1=\frac{1}{4E},\ T_3=T_5=T_7=0 \text{ at } \eta=0, \\ &f_1'\to 1,\ f_3'\to \tfrac{1}{4},\ f_5'\to \tfrac{1}{6},\ f_7'\to \tfrac{1}{8},\ T_1=T_3=T_5=T_7=0 \text{ at } \eta\to\infty. \end{aligned} \tag{10}$$

Substituting the expressions for U, u, v and T from (6) to (9) into (2) and (4) and equating the coefficients of like powers of θ on both sides of the equations, we get the following two sets of ordinary differential equations :

$$f_1'''+f_1f_1''=-1+f_1'^2+\alpha(-2f_1'f_1'''+f_1f_1^{iv}+f_1''^2), \tag{11}$$

$$\begin{aligned} f_3'''+f_1f_3''=-1+4f_1'f_3'-3f_1''f_3+\alpha[-4(f_1'f_1'''+f_1'''f_3') \\ +(f_1f_3^{iv}+3f_1^{iv}f_3)+4f_1''f_3''], \end{aligned} \tag{12}$$

$$\begin{aligned} f_5'''+f_1f_5''=-\tfrac{8}{8}+6f_1'f_5'-5f_1''f_5+\tfrac{40}{3}(f_3'^2-f_3f_3'') \\ +\alpha[-6(f_1'f_5'''+f_1'''f_5')+(f_1f_5^{iv}+5f_1^{iv}f_5) \\ +6f_1''f_5''+\tfrac{80}{8}(-2f_3'f_3'''+f_3f_3^{iv}+f_3''^2), \end{aligned} \tag{13}$$

$$\begin{aligned} f_7'''+f_1f_7''=-8+8f_1'f_7-7f_1''f_7+168f_3'f_5'-63f_3f_5'' \\ -105f_3''f_5+\alpha[-8(f_1'f_7'''+f_1'''f_7') \\ -168(f_3'f_5'''+f_3'''f_5')+(f_1f_7^{iv}+35f_1^{iv}f_7) \\ +(105f_3^{iv}f_5+63f_3f_5^{iv})+168f_3''f_5'' \\ +8f_1''f_7''), \end{aligned} \tag{14}$$

and

$$\frac{1}{P}T_1''+f_1T_1'=0, \tag{15}$$

$$\frac{1}{P}T_3''+f_1T_3'=2f_1'T_3-2f_3T_1'+f_1''^2+\alpha[f_1''(f_1'f_1''-f_1f_1''')], \tag{16}$$

$$\begin{aligned} \frac{1}{P}T_5''+f_1T_5'=4f_1'T_5-2f_3T_3'+\tfrac{4}{3}f_3'T_3-\tfrac{1}{4}f_5T_1'+\tfrac{4}{3}f_1''f_3'' \\ +\tfrac{2}{3}\alpha[f_1''(f_1''f_3'+4f_1'f_3'')-f_1''(f_1f_1'''-f_3'''+3f_1'''f^3) \\ -f_1f_1'''f_3''], \end{aligned} \tag{17}$$

$$\begin{aligned} \frac{1}{P}T_7''+f_1T_7'=6f_1'T_7-2f_3T_5'+\tfrac{8}{8}f_3'T_5-\tfrac{1}{4}f_5T_3'+\tfrac{1}{10}f_5'T_3 \\ -\tfrac{1}{90}f_7T_1'+\tfrac{1}{10}f_1''f_5''+\tfrac{4}{9}f_3''^2+\alpha[\tfrac{1}{20}f_1''(-f_1f_5'''+6f_1'f_5'' \\ +f_1''f_5'-5f_1'''f_5)-\tfrac{4}{9}f_3''(f_1f_3'''+3f_1'''f_3-4f_1''f_3') \\ +\tfrac{1}{60}(80f_1'f_3''^2-80f_1''f_3f_3'''-3f_1f_1'''f_5'')], \text{ etc.} \end{aligned} \tag{18}$$

where $\alpha=\dfrac{2U_\infty\nu_2}{l\nu_1}$ is the non-Newtonian parameter and $P=\dfrac{\rho\nu_1 c}{k}$ is the Prandtl number.

3. *Solutions of Equations* :

The equations (11) to (18) subject to boundary conditions (10) can be solved by any numerical method. We solve the equations by the method of series expansion followed by the method of Laplace. For this purpose, we express the functions $f_i(\eta)$ and $T_i(\eta)$ in power series of η as

$$f_i(\eta)=\frac{1}{2\,!}A_i\eta^2+\frac{1}{3\,!}B_i\eta^3+\frac{1}{4\,!}C_i\eta^4+\frac{1}{5\,!}D_i\eta^5+\frac{1}{6\,!}E_i\eta^6+\cdots \quad (19)$$

$$\left.\begin{aligned} T_1(\eta)&=\frac{1}{4E}+a_1\eta+\frac{1}{2\,!}b_1\eta^2+\frac{1}{3\,!}c_1\eta^3+\frac{1}{4\,!}d_1\eta^4+\frac{1}{5\,!}e_1\eta^5+\cdots \\ T_j(\eta)&=a_j\eta=\frac{1}{2\,!}b_j\eta^2+\frac{1}{3\,!}c_j\eta^3+\frac{1}{4\,!}d_j\eta^4+\frac{1}{5\,!}e_j\eta^5+\cdots \end{aligned}\right\} \quad (20)$$

where $i=1, 3, 5, 7$ and $j=3, 5, 7, \cdots$

The forms (19) and (20) of f_i, T_1 and T_j satisfy the boundary conditions (10) at $\eta=0$. The expansions (19) and (20) are valid only for sufficiently small values of η. Substituting f_i, T_1 and T_j from (19) and (20) into (11) to (18), and equating the coefficients of different powers of η to zero, we obtain the constants B_i, C_i, D_i, E_i, etc., and b_i, c_i, d_i, e_i, etc., as functions of A_i's and a_i's only.

Thus, if A_i and a_i are known, the velocity profile and the temperature distribution are completely determined.

The constants A_i, a_i, etc., can be determined by using the condition (10) as $\eta\to\infty$. We write the equations (11) to (14) and (15) to (18) in the following forms

$$f_i'''+f_1f_i''=H_i(\eta),\ i=1, 3, 5, 7 \quad (21)$$

and

$$T_i''+Pf_1T_i'=PM_i(\eta),\ i=1, 3, 5, 7 \quad (22)$$

where $H_i(\eta)$ and $M_i(\eta)$ are the right-hand sides of the equations (11) to (14) and (15) to (18) respectively.

Letting

$$\left.\begin{aligned} F(\eta)&=\int_0^\eta f_1(\eta)\,d\eta \\ G(\eta)&=P\int_0^\eta f_1(\eta)\,d\eta \end{aligned}\right\} \quad (23)$$

and

$$\phi_i(\eta)=A_i+\int_0^\eta e^F H_i(\eta)\,d\eta$$

$$\psi_i(\eta)=a_i+P\int_0^\eta e^G M_i(\eta)\,d\eta \tag{24}$$

we get the following by integrating twice the equations (21) and (22) respectively :

$$f_i'(\eta)=\int_0^\eta e^{-F}\phi_i(\eta)\,d\eta \tag{25}$$

$$T_i(\eta)=a_0+\int_0^\eta e^{-G}\psi_1(\eta)d\eta \tag{26}$$

The coefficients A_i and a_i, $i=1, 3, 5, 7$ are given by

$$\int_0^\infty e^{-F}\phi_1(\eta)d\eta=1$$

$$\int_0^\infty e^{-F}\phi_3(\eta)d\eta=\frac{1}{4}$$

$$\int_0^\infty e^{-F}\phi_5(\eta)d\eta=\frac{1}{6}$$

$$\int_0^\infty e^{-F}\phi_7(\eta)d\eta=\frac{1}{8},\ \text{etc.} \tag{27}$$

and

$$\int_0^\infty e^{-G}\psi_1(\eta)d\eta=-\frac{1}{4E}$$

$$\int_0^\infty e^{-G}\psi_3(\eta)d\eta=0$$

$$\int_0^\infty e^{-G}\psi_5(\eta)d\eta=0$$

$$\int_0^\infty e^{-G}\psi_7(\eta)d\eta=0,\ \text{etc.} \tag{28}$$

These integrals can be evaluated asymptotically by Laplace's method. Putting $F=G=\tau$, transforming the equations (27), (28) to the variable τ and integrating in the gamma functions, we find

$$L_i=\tfrac{1}{3}[P_{1i}\Gamma_{(1/3)}+P_{2i}\Gamma_{(2/3)}+P_{3i}\Gamma_{(1)}+P_{4i}\Gamma_{(4/3)}+P_{5i}\Gamma_{(5/3)}] \tag{29}$$

$$l_i=\tfrac{1}{3}[p_{1i}\Gamma_{(1/3)}+p_{2i}\Gamma_{(2/3)}+p_{3i}\Gamma_{(1)}+p_{4i}\Gamma_{(4/3)}+p_{5i}\Gamma_{(5/3)}] \tag{30}$$

where

$$L_1=1,\ L_3=\tfrac{1}{4},\ L_5=\tfrac{1}{6},\ L_7=\tfrac{1}{8}$$

$$l_1=0,\ l_3=0,\ l_5=0,\ l_7=0$$

$$
\begin{aligned}
p_{1i} &= k_1 a_i \\
p_{2i} &= k_2 a_i + k_1^2 b_i \\
p_{3i} &= k_3 a_i + \tfrac{3}{2} k_1 k_2 b_i + \tfrac{1}{2} k_1^3 c_i \\
p_{4i} &= k_4 a_i + b_i(\tfrac{1}{2} k_2^2 + \tfrac{4}{3} k_1 k_3) + c_i(k_1^2 k_2) + (d_i + a_i A_1)\tfrac{1}{3} k_1^4 \\
p_{5i} &= k_5 a_i + b_i(\tfrac{5}{4} k_1 k_4 + \tfrac{5}{6} k_2 k_3) + c_i(\tfrac{5}{6} k_1^2 k_3 + \tfrac{5}{3} k_1 k_2^2) + (d_i + a_i A_1)\tfrac{5}{12} k_1^3 k_2 \\
&\quad + (e_i + a_i B_1 + 3 b_i A_1 + P b_i A_1)\tfrac{1}{24} k_1^5 \qquad (31)
\end{aligned}
$$

and

$$
\begin{aligned}
k_1 &= \left(\frac{6}{PA_1}\right)^{1/3} \\
k_2 &= -\frac{B_1}{6A_1}\left(\frac{6}{PA_1}\right)^{2/3} \\
k_3 &= \frac{6}{PA_1}\left(-\frac{C}{20A_1} + \frac{B_1^2}{16A_1^2}\right) \\
k_4 &= \frac{1}{3PA_1}\left(\frac{6}{PA_1}\right)^{1/3}\left(-\frac{D_1}{5A_1} + \frac{7B_1C_1}{10A_1^2} - \frac{35B_1^3}{72A_1^3}\right) \\
k_5 &= \frac{1}{3PA_1}\left(\frac{6}{PA_1}\right)^{2/3}\left(-\frac{E}{28A_1} + \frac{B_1D_1}{6A_1^2} + \frac{C_1^2}{10A_1^2} - \frac{11B_1^2C_1}{28A_1^3}\right. \\
&\quad \left. + \frac{385}{1728}\frac{B_1^4}{A_1^4}\right) \qquad (32)
\end{aligned}
$$

The constants P_{ni} can be obtained from p_{ni} by replacing a_i, b_i, c_i, d_i, e_i, etc., in p_{ni} by A_i, B_i, C_i, D_i, E_i, etc , respectively and by putting $P=1$ in k_i

The unknowns A_i and a_i are determined by the conditions (29) and (30) respectively. The series (29) and (30) are in general divergent. So, we use Euler's transformation

$$\sum_{n=0}^{\infty} (-1)^n A_n = \sum_{n=0}^{\infty} (-1)^n \frac{\Delta^n A_0}{2^{n+1}} \qquad (33)$$

where

$$\Delta A_m = A_{m+1} - A_m, \quad \Delta^2 A_m = \Delta A_{m+1} - \Delta A_m$$

To determine A_1, A_3, A_5, A_7 from the condition (29), we took five terms of the series and applied Euler's transformation. Similarly, a_1, a_3, a_5, a_7 are determined from the condition (30). The values of A_i, $i=1, 3, 5, 7$ have been determined for $\alpha=0, -\cdot06, -\cdot10$ and the values are given in table (1). The values of a_i, $i=1, 3, 5, 7$ are determined for

$\alpha=0, -\cdot 06, -\cdot 10$, Eckert number $E=0\cdot 1$ and Prandtl number $P=5$, 25, 50 respectively. The values of the constants a_i for $P=5$, 25, 50 are given in the tables (2*a*), (2*b*) and (2*c*) respectively.

4. *Discussion* :

The shearing stress τ_0 on the wall of the cylinder is given by

$$\tau_0=\mu_1\left(\frac{\partial v}{\partial y}\right)_{y=0} \tag{34}$$

The location of the point of separation can be found from the condition that the velocity gradient normal to the wall and hence the shearing stress at the wall vanishes there. The condition that the shearing stress τ_0 at the surface vanishes is given for $\alpha=0$, $\alpha=-\cdot 06$ and $\alpha=-\cdot 10$ respectively by

$$0\cdot 033192X^3-0\cdot 360508X^2+2\cdot 748296X-7\cdot 021791=0 \tag{35}$$

$$0\cdot 144717X^3-0\cdot 636374X^2+3\cdot 263831X-8\cdot 302649=0 \tag{36}$$

$$0\cdot 457042X^3-1\cdot 251123X^2+3\cdot 487458X-8\cdot 715832=0 \tag{37}$$

where $X=\theta^2$

Solving the cubics (35), (36) and (37), we find the acceptable roots as $X=3\cdot 773$, $X=3\cdot 096$ and $X=2\cdot 611$ respectively. Thus the separation points occur at $\theta=111\cdot 2°$, $\theta=100\cdot 8°$ and $\theta=92\cdot 5°$ for $\alpha=0$, $\alpha=-\cdot 06$ and $\alpha=-\cdot 10$ respectively.

This shows that the effect of second-order parameter in the constitutive equation on the position of the separation point is to advance it towards the forward stagnation point. The second-order effect is exhibited through the non-dimensional parameter $\alpha=\frac{2U_\infty\mu_2}{\mu_1 l}$. Thus the point of separation depends on the material constants μ_1 and μ_2 and also on the flow parameters U_∞, l.

Next, the heat flux g from the cylinder to the fluid is given by

$$g=k\left(\frac{\partial T}{\partial y}\right)_{y=0}=-\frac{4kE}{l}\sqrt{\mathrm{Re}}\,(T_w-T_\infty)[T_1'(0)-\theta^2T_3'(0)+\theta^4T_5'(0)-\theta^6T_7'(0)] \tag{38}$$

where $\mathrm{Re}=\frac{2U_\infty\rho}{l\mu_1}$ is the Reynolds number. Defining the Nusselt number

$\mathrm{Nu}=(lg)/k(T_w-T_\infty)$,

we have $\mathrm{Nu}=-4E\sqrt{\mathrm{Re}}\,[T_1'(0)-\theta^2T_3'(0)+\theta^4T_5'(0)-\theta^6T_7'(0)]$ (39)

Taking the Eckert number $E=0{\cdot}1$, the graphs of $(\mathrm{Nu}/\sqrt{\mathrm{Re}})$ against θ have been plotted for $\alpha=0, -{\cdot}06, -{\cdot}10$ and Prandtl number $P=5, 25, 50$ respectively in the figures (1), (2) and (3). The angle θ_0 for which $\mathrm{Nu}=0$ corresponds to the point beyond which the frictional heating effects dominate and in that region, the temperature of the fluid in the immediate neighbourhood of the cylinder becomes higher than that of the cylinder, so the heat transfer occurs from that fluid to the cylinder. Figures (1), (2) and (3) show that the effect of the non-Newtonian parameters is to shift this critical point towards the forward stagnation point. In the figures (1), (2) and (3), we see that the curves corresponding to $\alpha=0, -{\cdot}06, -{\cdot}10$ intersect at $\theta=18°$, $18{\cdot}5°$ and $19{\cdot}3°$ respectively, which indicate that the effects of the non-Newtonian parameters in the constitutive equation of the fluid are to increase the heat flux from the forward stagnation point to the points corresponding to $\theta=18°$, $18{\cdot}5°$ and $19{\cdot}3°$, and to decrease it beyond these points. The points corresponding to $\theta=18°$, $18{\cdot}5°$ and $19{\cdot}3°$ on the cylinder for $P=5$, 25 and 50 are some special points, since the heat flux at these points are unaffected by the non-Newtonian terms in the constitutive equation of the fluid.

The effect of the Prandtl number P on the Nusselt number is to increase it near the forward stagnation point and to decrease it away from it. This effect is reversed at $\theta=18°$, $18{\cdot}5°$ and $19{\cdot}3°$ for $P=5$, 25 and 50 respectively.

Table 1

Values of A_i, $i=1, 3, 5, 7$ for different values of α

α	0	$-{\cdot}06$	$-{\cdot}10$
A_1	1·241289	1·467715	1·540756
A_3	·728752	·865454	·924752
A_5	1·370821	2·249922	4·423389
A_7	3·696526	16·117027	50·900498

Table 2a

Values of a_i, $i=1, 3, 5, 7$ for $P=5$ and different values of α

α	0	$-\cdot 06$	$-\cdot 10$
a_1	$-2\cdot 747038$	$-2\cdot 924363$	$-2\cdot 967116$
a_3	$-2\cdot 190759$	$-2\cdot 919243$	$-4\cdot 518575$
a_5	$-1\cdot 647793$	$-1\cdot 639249$	$-1\cdot 652486$
a_7	$-\cdot 607418$	$-1\cdot 055053$	$-\cdot 616854$

Table 2b

Values of a_i, $i=1, 3, 5, 7$ for $P=25$ and different values of α

α	0	$-\cdot 06$	$-\cdot 10$
a_1	$-4\cdot 826255$	$-5\cdot 12187$	$-5\cdot 200781$
a_3	$-6\cdot 982098$	$-9\cdot 252398$	$-10\cdot 576516$
a_5	$-4\cdot 074602$	$-4\cdot 917089$	$-5\cdot 749960$
a_7	$-1\cdot 487565$	$-2\cdot 341636$	$-2\cdot 249602$

Table 2c

Values of a_i, $i=1, 3, 5, 7$ for $P=50$ and different values of α

α	0	$-\cdot 06$	$-\cdot 10$
a_1	$-6\cdot 12601$	$-6\cdot 496594$	$-6\cdot 597995$
a_3	$-11\cdot 407323$	$-15\cdot 102021$	$-16\cdot 546527$
a_5	$-6\cdot 551364$	$-7\cdot 966250$	$-9\cdot 709789$
a_7	$-2\cdot 314563$	$-3\cdot 646994$	$-3\cdot 646027$

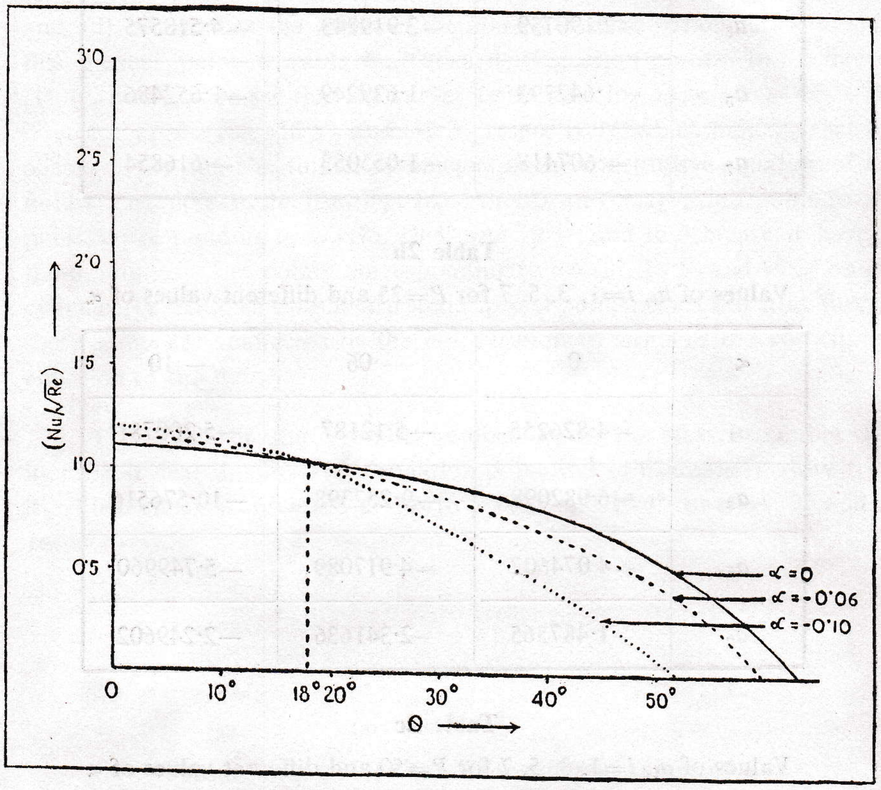

Fig 1. Variations of $Nu/\sqrt{Re}$ against θ for $P=5$ and $\alpha=0, -\cdot 06, -\cdot 10$

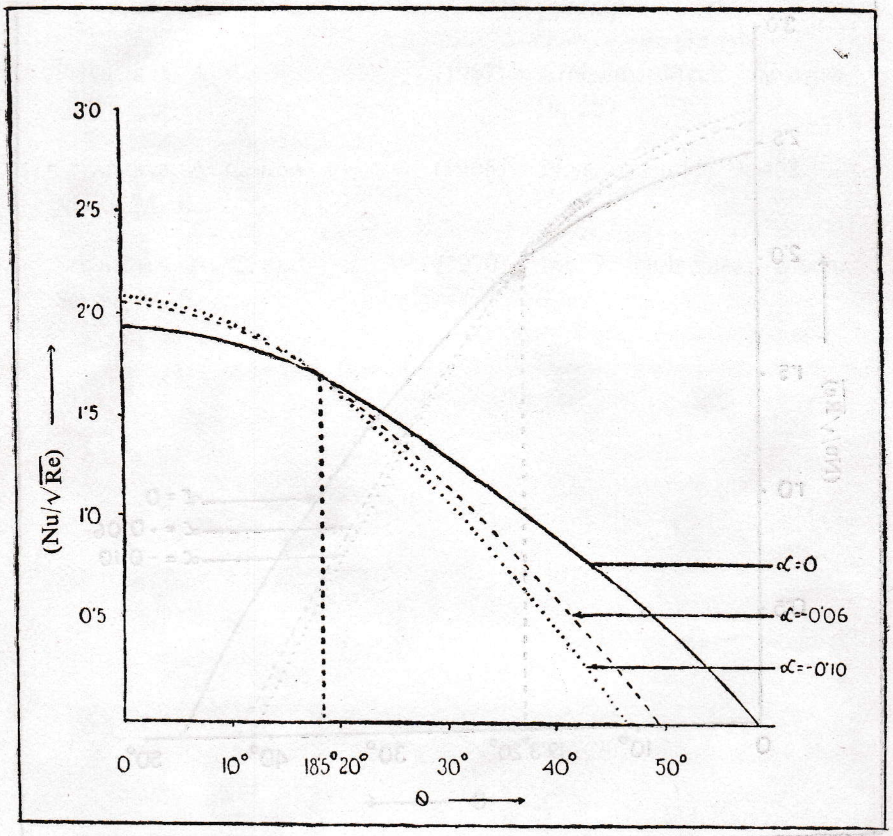

Fig 2. Variations of $(Nu/\sqrt{Re})$ against θ for $P=25$ and $\alpha=0, -\cdot 06, -\cdot 10$

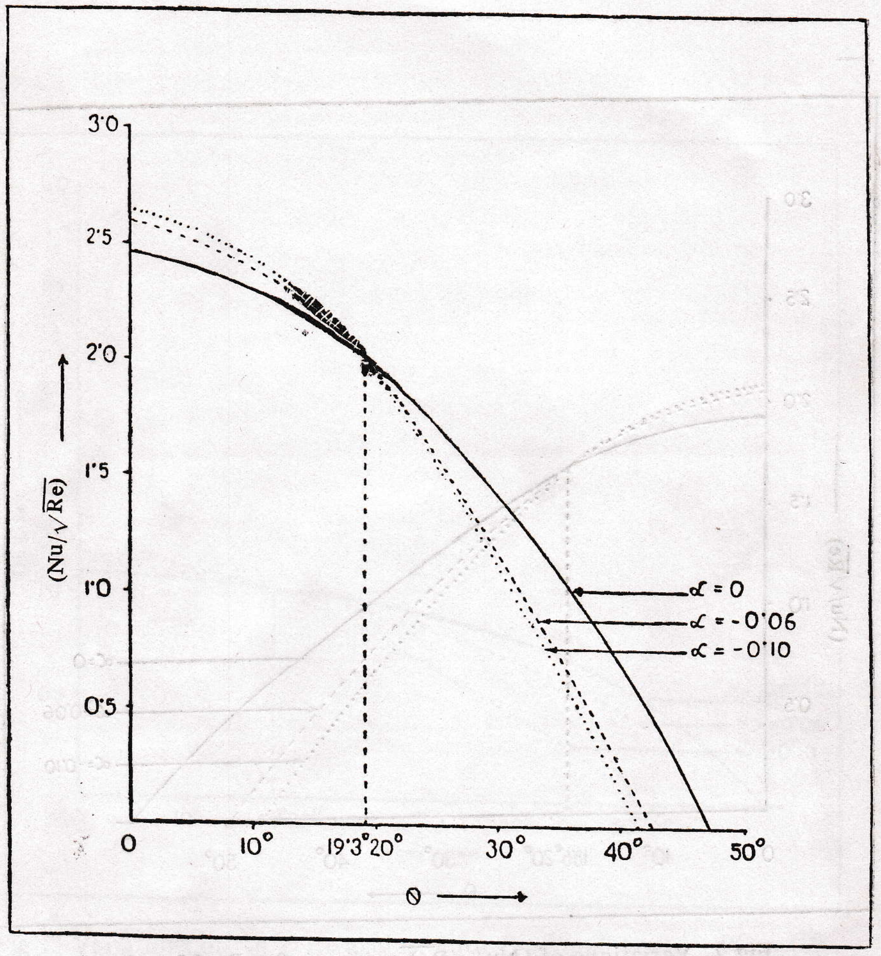

Fig 3. Variations of $Nu/\sqrt{Re}$ against θ for $P=50$ and $\alpha=0, -\cdot06, -\cdot10$

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