Mathematical Forum ISSN: 0972-9852
Vol.28(1), 2020

UNSTEADY HYDROMAGNETIC ELECTRICALLY CONDUCTING FLOW PAST A CONTINUOUSLY MOVING INCLINED PLATE

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Received on: 10/08/2020
Accepted on: 29/10/2020

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Abstract

An exact analysis of free convection flow of an electrically conducting incompressible fluid over a continuously moving inclined plate embedded on a porous medium is presented. The flow becomes unsteady as the periodic transverse suction velocity is applied to the surface. The visco-elastic fluid flow is characterized by Walters liquid (model B'). Analytical solution of the problem is obtained by using multi-parameter perturbation technique. The expressions for velocity field, temperature field, concentration field, shearing stress at the plate are derived analytically. It is noticed that the momentum and thermal fields are strongly affected in the flow by the visco-elastic parameters. Again, the concentration field is significantly affected in case of Newtonian fluid but not affected by visco-elastic parameter.

Keywords: Free convection, Heat source, Hydromagnetic flow, Porous medium, Suction.

2010 AMS classification: 76Wxx, 76Sxx

1. Introduction

The development of heat and mass transfer with chemical reaction under the influence of magnetohydrodynamic flow are encountered in many branches of science and engineering which attracted many researchers. They have shown the immense interest and got indulged to this significant application which has the potential to solve engineering problems such as power generation, crystals growing, liquid metals fluid, electromagnetic pumps, MHD couples and bearing, in aerodynamics the control of periphery layer, heat exchangers, catalytic reactors, food processing and cooling towers. Flows in permeable medium have numerous applications in geothermal and oil tanks.

Unsteady hydro magnetic flows sent via a porous medium have studied by Raptis and Valhos [11]. Kameswaran et al. [6] have experimented transmit of heat and mass in nano fluids over widening sheet with respect to dissipation and chemical reaction impacts. Similarity transformations to study MHD dissipative fluid which was sent through an inclined surface was examined by Ramana Reddy et al. [10]. Raju, Varma, Seshaiah [9] investigated about heat transfer effects by the impact of MHD viscous dissipative flow over an upright plate. Hamad and Pop [5] presented MHD nano-fluid flow past an upright penetrable smooth plate subject to heat source in a revolving frame. Satyanarayana, Venkateswarlu [12] have elaborated the heat transfer of MHD flow in a vertical porous plate. Srikanth, Sureshbabu, Srinivas [15] analyzed MHD fluid with the effect of chemical reaction under the transmission of heat and mass. The results for unsteady convection flow with variable temperature over an endless erect permeable moving plate presented by Nasser El-Fayez [8]. Recently Selvarani and Govindarajan.[13] have thoroughly examined the free convective nano fluid flow past over a tilted plate with transmit of heat and mass.

Non-Newtonian fluid flow plays important roles in several industrial manufacturing processes. An example of such non-Newtonian fluid includes drilling mud, polymer solution or melts, certain oils and greases and many other emulsions. Some of the typical application of non-Newtonian fluid flow are noticed in the drilling of oil and gas oils, molten polymer, many solid suspensions, glass fiber and paper production, in blood flow, drawing of plastic films, waste fluids etc. The application of the mechanisms of elastico-viscous fluid flows in modern technology and industries have attracted the researcher like Kelly et al. [7], Abel et al [1], Sonth et al. [14]. Choudhury and Das [2], Choudhury and Dey [3], Choudhury and Purkayastha [4] also investigated some problems in the area of non-Newtonian fluid flow.

In this paper, an analysis is carried out to study the two dimensional unsteady hydromagnetic boundary layer flow of elastico-viscous over a continuously moving inclined surface in presence of heat and mass transfer. The present study can help to increase the process rates in atomic power engineering, chemical engineering, space research and various branches of industry and agriculture as heat and mass transfer theory has many practical importance in all these fields. The visco-elastic fluid characterized by Walters liquid (Model $B^{/}$**)** was developed to simulate viscous fluids possessing short memory elastic effects and can simulate accurately many complex,

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polymetric bio-technological fluids. The Walters fluid (Model B′) has therefore been studied extensively in many fluid flow problems.

Walters [16] has showed that for liquids with short memories, the equation of state can be simplified to

$$\tau^{ik} = 2\eta_0 e^{(1)ik} - 2k_0 \frac{\delta}{\delta t} e^{(1)ik} \quad (1.1)$$

$\eta_0 = \int_0^\infty N(\lambda) d\tau$ is the limiting viscosity at small rate of shear, $k_0 = \int_0^\infty \lambda N(\lambda) d\tau$.

The convected derivative of any contravariant tensor is given by

$$\frac{\delta b^{ik}}{\delta t} = \frac{\partial b^{ik}}{\partial t} + v^m \frac{\partial b^{ik}}{\partial x^m} - \frac{\partial v^k}{\partial x^m} b^{im} - \frac{\partial v^i}{\partial x^m} b^{mk} \quad (1.2)$$

where v_i is the velocity vector. This idealized model is a valid approximation of Walters liquid (Model B′) taking very short memories into account so that the terms involving $\int_0^\infty \lambda^n N(\lambda) d\lambda, n \geq 2$ have been neglected.

2. Mathematical Formulation

Consider the unsteady free convective flow of a viscous incompressible electrically conducting fluid over a continuously moving inclined surface in presence of constant suction and heat flux and transverse magnetic field. The x´-axis is taken along the plate and y´-axis is normal to the plate directed into the fluid. The inclination of the angle is assumed to so small that $sin\alpha = 0$.

The induced magnetic field is neglected as the magnetic Reynolds number of the flow is very small. Since the plate is considered infinite in the x´-direction, hence all the fluid properties are independent of x´. Let u' and v' be the fluid velocities along x' and y' axes respectively and the plate temperature T' is oscillating about a non-zero plate temperature $T_w{}'$.

Within the framework of the above stated assumption, applying Boussinesq's approximation and with reference to the generalized equations described before the equation relevant to the transient two-dimensional problems are governed by the following system of coupled non-linear differential equations.

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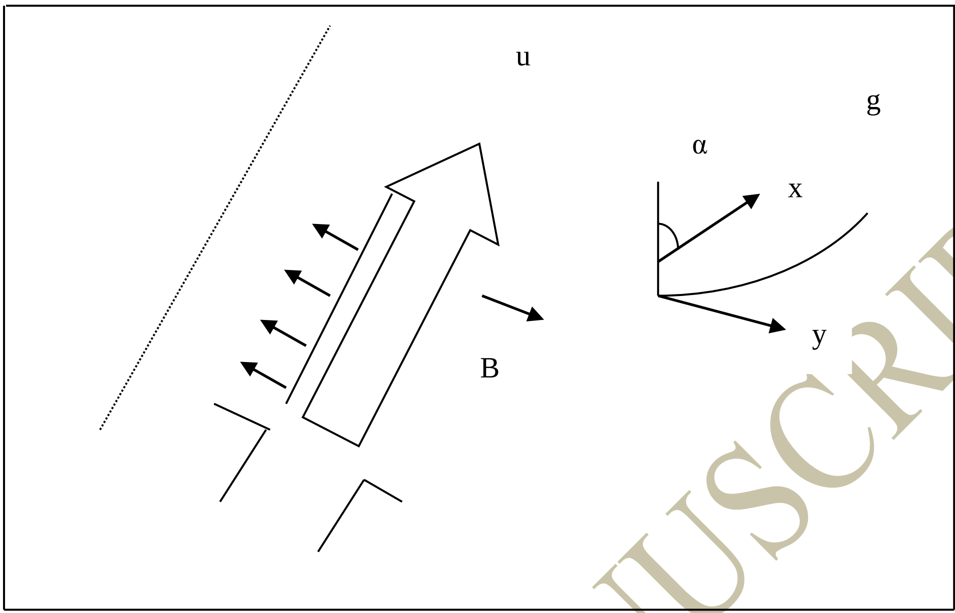

Fig.1: Flow configuration

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \Rightarrow v' = -v_0{}' \tag{2.1}$$

Momentum equation

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta\left(T' - T_\infty{}'\right)\cos\alpha + g\beta'\left(C' - C_\infty{}'\right)\cos\alpha + \nu\frac{\partial^2 u}{\partial y'^2} - \frac{\sigma B_0{}^2 u'}{\rho} - \frac{\nu}{K'}u'$$

$$-\frac{k_0}{\rho}\left[\frac{\partial^3 u'}{\partial t' \partial y'^2} + v'\frac{\partial^3 u'}{\partial y'^3} - 2\frac{\partial v'}{\partial y'}\frac{\partial^2 u'}{\partial y'^2} - 3\frac{\partial u'}{\partial y'}\frac{\partial^2 v'}{\partial y'^2}\right] \tag{2.2}$$

Energy equation

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = k\frac{\partial^2 T'}{\partial y'^2} + S'\left(T' - T_\infty{}'\right) \tag{2.3}$$

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Concentration equation

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D\frac{\partial^2 C'}{\partial y'^2}-K_r\left(C'-C_\infty{}'\right) \tag{2.4}$$

The boundary conditions of the problem are :

$$y'=0: u'=0, v'=-v_0{}', T'=T_w{}'+\varepsilon\left(T_w{}'-T_\infty{}'\right), C'=C_w{}'+\varepsilon\left(C_w{}'-C_\infty{}'\right)$$

$$y'\to\infty: u'\to 0, T'\to T_\infty{}', C'\to C_\infty{}' \tag{2.5}$$

We now introduce the following non-dimensional quantities:

$$y=\frac{y'v_0{}'}{\nu}, t=\frac{t'v_0{}'^2}{4\nu}, \omega=\frac{4\nu\omega'}{v_0{}'^2}, u=\frac{u'}{v_0{}'}, \nu=\frac{\eta_0}{\rho}, M=\left(\frac{\sigma B_0{}^2}{\rho}\right)\frac{\nu}{v_0{}'^2}, Kp=\frac{v_0{}'^2K'}{\nu^2}$$

$$T=\frac{T'-T_\infty{}'}{T_w{}'-T_\infty{}'}, \phi=\frac{C'-C_\infty{}'}{C_w{}'-C_\infty{}'}, \Pr=\frac{\nu}{k}, Gr=\frac{\nu g\beta\left(T_w{}'-T_\infty{}'\right)}{v_0{}'^3}, Gm=\frac{\nu g\beta\left(C_w{}'-C_\infty{}'\right)}{v_0{}'^3}$$

$$S=\frac{4S'\nu}{v_0{}'^2}, Sc=\frac{\nu}{D}, r=\frac{Kr\,\nu}{v_0{}'^2}, \quad K=\frac{k_0v_0{}^2}{\rho\nu^2} \tag{2.6}$$

where acceleration due to gravity, density, electrical conductivity, coefficient of kinematic viscosity, volumetric coefficient of expansion for heat transfer , volumetric coefficient of expansion for mass transfer , angular frequency, coefficient of viscosity, thermal diffusivity, temperature , temperature at the plate, temperature at infinity, concentration parameter, Prandtl number, Grashof number for heat transfer, heat source parameter, permeability parameter, magnetic parameter, chemical reaction parameter and Schmidt number are represented by $g,\rho,\sigma,\nu,\beta,\beta',\omega,\eta_0,k,T,T_w,T_\infty,\phi,\Pr,Gr,S,Kp,M,r$ and Sc respectively.

The non-dimensional form of (2.2) to (2.4) are as follows:

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=GrT\cos\alpha+Gm\phi\cos\alpha+\frac{\partial^2 u}{\partial y^2}-\frac{u}{Kp}-Mu-K\left[\frac{1}{4}\frac{\partial^3 u}{\partial t\partial y}-\frac{\partial^3 u}{\partial y^3}\right] \tag{2.7}$$

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$$\frac{1}{4}\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2}+\frac{1}{4}ST \qquad (2.8)$$

$$\frac{1}{4}\frac{\partial \phi}{\partial t}-\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-r\phi$$

(2.9)

The corresponding boundary conditions are:

$$y=0: u=0, T=1+\varepsilon e^{i\omega t}, C=1+\varepsilon e^{i\omega t}$$

$$y\to\infty: u\to 0, T\to 0, C\to 0 \qquad (2.10)$$

3. Method of solution

To solve equations (2.7) to (2.9), we assume ε to be very small and the velocity, temperature and concentration in the neighbourhood of the plate as

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y) \qquad (3.1)$$

$$T(y,t)=T_0(y)+\varepsilon e^{i\omega t}T_1(y) \qquad (3.2)$$

$$\phi(y,t)=\phi_0(y)+\varepsilon e^{i\omega t}\phi_1(y) \qquad (3.3)$$

Substituting equations (3.1) to (3.3) in equations (2.7), (2.8) and (2.9) respectively, equating the harmonic and non harmonic terms and neglecting the coefficients of ε^2, we get

Zeroth order equations:

$$Ku_0'''+u_0''+u_0'-\left(M+\frac{1}{Kp}\right)u_0=-GrT_0\cos\alpha-Gm\phi_0\cos\alpha \qquad (3.4)$$

$$T_0''+\Pr T_0'+\frac{1}{4}\Pr ST_0=0 \qquad (3.5)$$

$$\phi_0''+Sc\phi_0'-rSc\phi_0=0 \qquad (3.6)$$

subject to boundary conditions are :

$$y=0: u_0=0, T_0=1, \phi_0=1$$

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$$y \to \infty : u_0 = 0, T_0 = 0, \phi_0 = 0 \qquad (3.7)$$

First order equations:

$$Ku_1''' + \left(1 - \frac{Ki\omega}{4}\right)u_1'' + u_1' - \left(M + \frac{1}{Kp} + \frac{i\omega}{4}\right)u_1 = -GrT_1\cos\alpha - Gm\phi_1\cos\alpha \qquad (3.8)$$

$$T_1'' + \Pr T_1' - \frac{1}{4}\Pr(i\omega - S)T_1 = 0 \qquad (3.9)$$

$$\phi_1'' + Sc\phi_1' - \frac{i\omega Sc}{4}\phi_1 - rSc\phi_1 = 0 \qquad (3.10)$$

with relevant boundary conditions are,

$$y = 0 : u_1 = 0, T_1 = 1, \phi_1 = 1$$

$$y \to \infty : u_1 = 0, T_1 = 0, \phi_1 = 0 \qquad (3.11)$$

Solutions of (3.6) and (3.10) subject to the boundary conditions (3.7) and (3.11) are obtained.

Using multi-parameter perturbation technique and taking K<< 1, we assume

$$u_0 = u_{00} + Ku_{01} \qquad (3.12)$$

$$u_1 = u_{10} + Ku_{11} \qquad (3.13)$$

Now using equations (3.12), (3.13) in equations (3.4) and (3.8) and equating the coefficients of like powers of K and neglecting the higher power of K, we get the following set of differential equations:

Zeroth order :

$$u_{00}'' + u_{00}' - \left(M + \frac{1}{Kp}\right)u_{00} = -GrT_0\cos\alpha - Gm\phi_0\cos\alpha \qquad (3.14)$$

$$u_{10}'' + u_{10}' - \left(M + \frac{1}{Kp} + \frac{i\omega}{4}\right)u_{10} = -GrT_1\cos\alpha - Gm\phi_1\cos\alpha \qquad (3.15)$$

The modified boundary conditions are:

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$$y = 0 : u_{00} = 0, T_0 = 1, u_{10} = 0, T_1 = 1$$

$$y \to \infty : u_{00} = 0, T_0 = 0, u_{10} = 0, T_1 = 0 \qquad (3.16)$$

First order:

$$u''_{01} + u'_{01} - \left(M + \frac{1}{Kp} \right) u_{01} = -u'''_{00} \qquad (3.17)$$

$$u''_{11} + u'_{11} - \left(M + \frac{1}{Kp} + \frac{i\omega}{4} \right) u_{11} = -u'''_{10} + \frac{i\omega}{4} u''_{10} \qquad (3.18)$$

The corresponding boundary conditions are,

$$y = 0 : u_{01} = 0, T_0 = 0, u_{11} = 0, T_1 = 0$$

$$y \to \infty : u_{01} = 0, T_0 = 0, u_{11} = 0, T_1 = 0 \qquad (3.19)$$

Solving equations (3.4) and (3.7) subject to the boundary conditions (3.5) and (3.8); (3.14), (3.15), (3.17), (3.18) subject to boundary conditions (3.16) and (3.19) we obtained the solutions of T_0, T_1, u_0, u_1. Solutions and constants are not given due to brevity.

4. Discussions and results

The non dimensional shearing stress, heat flux and rate of mass transfer in terms of τ and Nusselt number Nu and Sherwood number Sh at the plate is given by

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0} - K \left(\frac{1}{4} \frac{\partial u}{\partial y} - \frac{\partial^2 u}{\partial y^2} \right)_{y=0}, Nu = \left(\frac{\partial T}{\partial y} \right)_{y=0}, Sh = \left(\frac{\partial \phi}{\partial y} \right)_{y=0} \qquad (4.1)$$

The purpose of the present study is to investigate the effects of elastico-viscous parameter on free convective boundary layer flow past an inclined moving surface in presence of chemical reaction. The non-dimensional parameter K represents the visco-elastic effect. The equivalent results for Newtonian fluid are obtained by setting K=0.

In order to get physical insight into the problem, the fluid velocity u is depicted against y in the Figures 1-3. The variation of the shearing stress τ against various flow parameters viz. S, Gr, M is illustrated in the figures 4-6. The numerical

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calculations are to be carried out for $Gr = 7, Gm = 5, S = 0.2, Kp = 4, M = 1, \Pr = 5, \omega t = \pi / 2, \varepsilon = 0.01, \omega = 5$ throughout the discussion.

Figures 2-4 represent that the fluid velocity boosts up considerably in the neighbourhood of the plate but as the distance increases, the speed diminishes after attaining the greatest speed. Both Newtonian and Visco-elastic fluid flows followed the same trend. Enhancement of velocity of complex fluid system is also observed with the escalation of visco-elasticity factor (K=0.05, 0.1) in-comparison with the simple Newtonian fluid. The effects of Grashof number, heat source parameter, magnetic parameter are described in figure 2, 3 and 4 respectively. In all the cases fluid velocity at first amplifies and then subdues with the increasing distance.

Figure 5, 6 and 7 represent the skin friction profile for various values of the flow parameters. In Figure 5, it is detected that as the degree of coolness (Gr>0) increases, the shearing stress or viscous drag will decelerate in various fluid flow mechanisms. The growths of heat source parameter decelerate the magnitude of skin friction along with the increasing values of visco-elastic parameter.

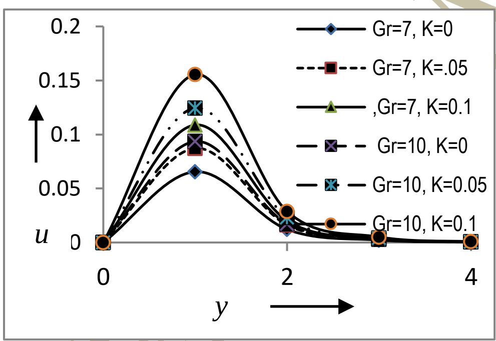

Fig. 2: Variation of transient velocity u against y for different values of Grashof number.

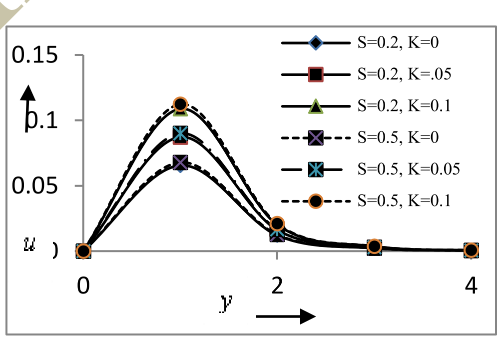

Fig. 3: Variation of transient velocity u against y for different values of heat source parameter.

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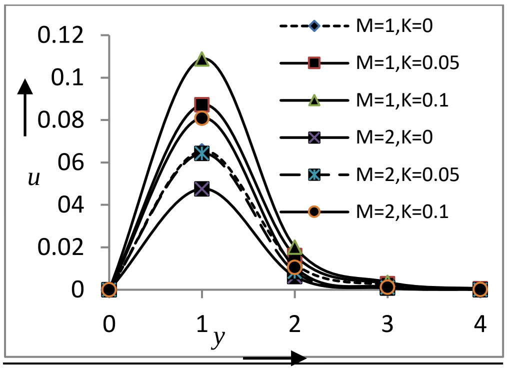

Fig. 4: Variation of transient velocity u against y for different values of magnetic parameter.

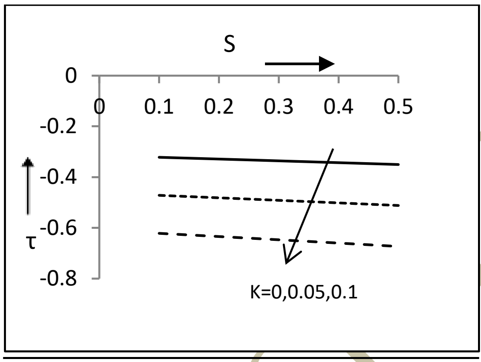

Fig. 6: Variation of skin friction τ at the wall against S.

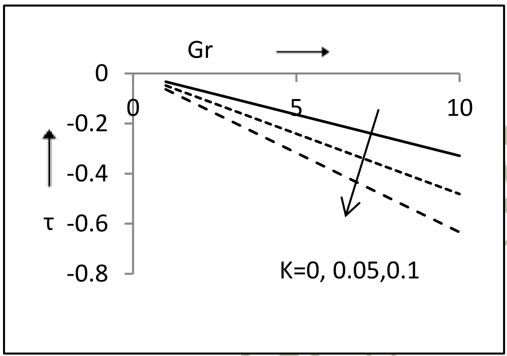

Fig. 5: Variation of skin friction τ at the wall against Gr.

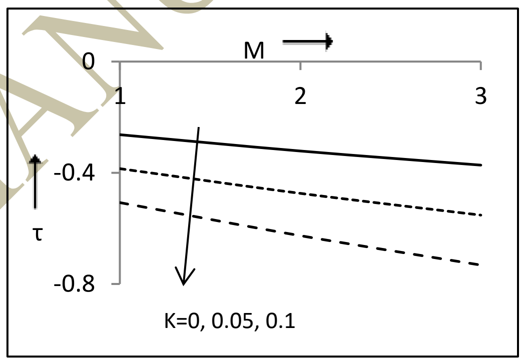

Fig. 7: Variation of skin friction τ at the wall against M.

5. Conclusion

This paper brings out following cases:

(i) The fluid flow is accelerated and then retarded slowly during the enhancement of visco- elastic parameter.

(ii) The fluid is accelerated with the growing values of visco elastic parameter in contrast with the Newtonian fluid.

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(iii) The viscous drag formed by both Newtonian and non-Newtonian fluid experienced a declined trend in case of increasing values of Grashof number Gr, heat source parameter S and magnetic parameter M.

(iv) Rate of heat transfer and mass transfer are not affected significantly during the changes made in visco-elasticity of the fluid flow.

Acknowledgement: I am thankful to the unknown reviewer for constructive as well as creative suggestions.

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