Mathematical Forum ISSN: 0972-9852
Vol.28(2), 2020

GOAL PROGRAMMING APPROACH TO SOLVE FRACTIONAL FUZZY TRANSPORTATION PROBLEM USING MODIFIED S-CURVE MEMBERSHIP FUNCTION

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Received on: 08/06/2020

Accepted on: 20/12/2020

Abstract

In this paper we used goal programming approach to solve fuzzy transportation problem. Here we considered multi-objective interval valued fractional transportation problem. We used modified S-curve non-linear membership function to get optimum solution for multi-objective fractional transportation problem where the parameters are fuzzy. The results of using modified S-curve membership functions are very flexible and thus we can explain the vagueness in parameters. Considering the degree of satisfaction and vagueness; the proposed method outstands the other works in revelant field by giving best solution.

Keywords:Goal Programming, S-curve membership function, Vagueness, Fuzzy Parameters.

2010 AMS classification: 90C70, 90C29

1. Introduction

Linear programming problem (LPP) have many branches. Transportation problem is one among these branches which is having a lot of real time applications. Transportation problems follows a classical approach in which the constraints are considered to be equality type. The model developed by Kantorovich [15] for the organizing and planning in productions and model developed by Hitchcock [8] for sources to destinations distribution are few among the earliest transportation models. Introduction of fuzzy set theory by Zadeh [14] was a breakthrough but it was a theoretical concept until Zimmermann [9] solved linear programming problem with many objective functions. He proved the efficiency of solutions solved by fuzzy linear programming. Bit et al [2] used linear membership functions in fuzzy programming and they applied it to find a solution for a transportation problem with multiple objectives.

The decision maker have a tedious task in goal programming. He have to set an aspiration level for each goal. Uncertainties should be considered here to get apt solution. This approach is widely used for modeling, solve and analyze optimization problems with multiple objectives. The concept of goal programming was there from early 60's onwards. The works of Charnes and Cooper [1] gave path way for development of this approach. Narasimhan [19] used membership functions in fuzzy goal programming model and find the optimum solution. Lee and Moore [20] showed that goal programming be used to find solution in multi-objective transportation problem.

Membership functions may have different forms. The method in Zimmermann [9] use membership functions which are linear. Leberling [10] uses a tangent type membership function. Hannan[6] proposes interval linear membership function. Carlsson and Korhonen[4] uses exponential membership function to get optimal solution. Sakawa[16] proposes an inverse tangent membership function and used it in interactive computer programs. Logistic type of membership function is described in the work of Watada[13]. Concave piecewise linear membership function used by Ichihashi and Kume [11] and piecewise linear membership function used by Hu and Fang [5] for solving fuzzy problems. The introduction of hyperbolic membership function to solve vector maximum LPP was done on the work of Leberling [10]. He showed that fuzzy linear programming with non linear membership function like hyperbolic membership function can give efficient solution. Non linear membership functions include a lot of branches. Dhingra and Moskowitz [3] defined exponential, quadratic and logarithmic membership functions. In the paper proposed in [10], these non linear membership functions are applied to get solution for optimal problem. Non linear membership functions like tangent type, exponential, hyperbolic etc results non-linear programming. Non linearity can be eliminated by using linear membership function. But we may face difficulties to select the solution when the membership function is linear. In this paper, we propose modified S-curve membership function which can eliminate shortcomings of a linear membership function. Linear membership function may become restrictive in nature while the modified S-curve form is flexible. The proposed method can explain vagueness in fuzzy parameters.

2. Preliminaries

2.1. Multi-objective Fractional Transportation Model

When the objective function consists of a ratio of functions and simultaneously it deals with optimization problems; the transportation problem can be solve using a special case of non-linear programming called fractional programming. When the value of the objective function lies in an interval, the fractional transportation problem becomes interval valued fractional transportation problem. In multi objective transportation problem, the goal is to minimize the ratios of interval valued fractional objective functions. The fractional transportation problem developed by Swarup has an application in the field of logistics and supply management. Suppose a manufacturing unit have m storehouses at different places. They are selling their product through n outlets at various places. In this case, each storehouse has a specific level of supply and each outlet have specific level of demand. Let a_i be the total supply from the store house i. Let b_j be the total demand for a product in an outlet j. Transportation cost to transfer product from i to j be S_{ij}^t.Let C_{ij}^t be the profit obtained per unit from store house i to outlet j. γ and δ are the fixed costs. The quantity of products transported from store house i to outlet j is denoted by x_{ij}.

Mathematical formulation of multi-objective frational transportation problem in crisp form is stated as follows:

$$Minimize Z_t(x) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} S_{ij}^t x_{ij}+\gamma}{\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^t x_{ij}+\delta}, t = 1,2,\ldots.,T$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, i = 1,2,\ldots,m,$$
$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1,2,\ldots,n,$$
$$x_{ij} \geq 0, i = 1,2,\ldots m, j = 1,2,\ldots n.$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j (equilibrium condition) \qquad (2.1)$$

where $Z(x) = \{Z^1(x), Z^2(x), \ldots, Z^t(x)\}$ be the t objective function vectors. We assume that a_i, $b_j \geq 0$, for all i, j and S_{ij}^t,$C_{ij}^t \geq 0$, for all i, j.

2.2. Fuzzy Multi-objective Fractional transportation problem

The parameters in equation 2.1 are exact. The cost cannot be measured exactly due to the changes in the environment. Let $\tilde{S}_{ij}$, $\tilde{C}_{ij}$ be the fuzzy parameter. Now equation 2.1 in fuzzy parameters is as follows:

$$
\begin{aligned}
& Minimize \tilde{Z}_t(x) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{s}_{ij}^t x_{ij} + \gamma}{\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}^t x_{ij} + \delta}, t = 1,2, \ldots., T \\
& subjectto \sum_{j=1}^{n} x_{ij} = a_i, i = 1,2, \ldots, m, \\
& \sum_{i=1}^{m} x_{ij} = b_j, j = 1,2, \ldots, n, \\
& x_{ij} \geq 0, i = 1,2, \ldots m, j = 1,2, \ldots n. \\
& \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j (equilibriumcondition)
\end{aligned}
$$

where $\tilde{Z}(x) = \{\tilde{Z}^1(x), \tilde{Z}^2(x), \ldots, \tilde{Z}^t(x)\}$ be the t objective function vectors. We assume that a_i, $b_j \geq 0$, for all i, j and $\tilde{S}_{ij}^t$,$\tilde{C}_{ij}^t \geq 0$, for all i, j.

2.3 Model Detailing of Goal Programming

Decision vectors which are controlled by higher level decision making can be found by the use of individual optimal solution. Fuzzy goal level in objectives also can be determined similarly. In order to formulate the proposed fuzzy goal programming models these two are required. Associated membership grade will characterize fuzzy goals. We can transform them into flexible fuzzy membership goals. This can be done by introducing deviational variables which are negative and positive d_t^+ and d_t^-,t=1,2..T. We assign highest membership grade as the level of aspiration for them. In the proposed work we used 0.999 as the highest membership grade for the negative and positive deviational variables. Deviations between achievement and aspiration levels of our goal should be the minimum and this can be achieved by the goal programming. Let G_t be the aspiration level where t=1,2...T. Model formulation of goal programming is given below.

$$
\begin{gathered}
\text{Minimize } Z_t(x) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} [s_t^a, s_t^b] x_{ij} - G_{t1}}{\sum_{i=1}^{m} \sum_{j=1}^{n} [c_t^a, c_t^b] x_{ij} - G_{t2}}, \text{ t=1,2,....,T} \\
\text{subject to } \sum_{j=1}^{n} x_{ij} = a_i, \text{ i=1,2,...,m,} \\
\sum_{i=1}^{m} x_{ij} = b_j, \text{ j=1,2,...,n,}
\end{gathered}
$$

$$x_{ij} \geq 0, \text{ i=1,2,...m, j=1,2,...n.}$$

The equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

Now we assume that $Z_t(x) = d_t^+ - d_t^- + G_t$. Then the above problem cab be stated as follows:

$$\text{Minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} d_t^+ - d_t^-$$
$$\text{s.t } \frac{[s_t^a, s_t^b]x_{ij} - G_{t1}}{[c_t^a, c_t^b]x_{ij} - G_{t2}} = d_t^+ - d_t^-, \text{ t=1,2,...T,}$$
$$d_t^+ - d_t^- \geq 0, \text{ t=1,2,...T.}$$

2.4 Model Formulation using Min-Max Approach

Goal programming model can be solved by different methods. Weighted goal programming method is used in Emre K. Can and Mark H. Houck[7]. Jean-Pierre Crouzeix [12] used preemptive goal programming method to solve generalized fractional programming. Min-max approach is a commonly used method proposed by Zimmerman[9]. Mathematical formulation using Min-Max approach is given below

$$\text{Minimize } \zeta$$
$$\text{subject to } \sum_{j=1}^{n} x_{ij} = a_i, \text{ i=1,2,...,m,}$$
$$\sum_{i=1}^{m} x_{ij} = b_j, \text{ j=1,2,...,n,}$$
$$Z_t(x) + d_t^- - d_t^+ = G_t, t = 1,2,\ldots T, \zeta \geq d_t^+, t = 1,2,..T d_t^+, d_t^- \geq 0 d_t^+ d_t^- = 0, t = 1,2,\ldots T, x_{ij} \geq 0, \forall i,j.$$
$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ (equilibrium condition).}$$

3 Modified S-curve non-linear membership function

3.1 Logistic Function

The α in our proposed method is a fuzzy parameter. It is a value which corresponds to the degree of vagueness. The value of α lies between 0 and ∞. The logistic function for proposed function with the above α and non-linear membership function can be expressed as follows

$$\mu_{\tilde{S}} = \begin{cases} 1 & S < S^a \\ \frac{B}{1+Ce^{\alpha S}} & S^a < S < S^b \\ 0 & S > S^b \end{cases} \tag{3.1}$$

where $\mu_{\tilde{S}}$ lies between 0 and 1 and it is a measure of the membership grade of S. Here S^a represents the upper value of S and S^b represents the lower value of S. Shape of our non-linear membership function is determined by the value of α which is a fuzzy parameter. Values of B and C are constants in logistic function.

3.2 S-curve non-linear membership function

Membership function in linear form is restrictive. This demerit can be rectified by the use of modified S-curve. It is flexible and thus it can give the measure of vagueness. Modified S-curve membership function is a subclass of logistic function where we have to find the specific values of B, C and α. The analytically [17] calculated values are as B=1, C=0.001001001 and α=13.81350 . Equation for the special modified s-curve logistic function is given as,

$$\mu_{\tilde{S}} = \begin{cases} 1 & S_{ij} < S_{ij}^a \\ 0.999 & S_{ij} = S_{ij}^a \\ \frac{B}{1+Ce^{\alpha S}} & S_{ij}^a < S_{ij} < S_{ij}^b \\ 0.001 & S_{ij} = S_{ij}^b \\ 0 & S_{ij} > S_{ij}^b \end{cases} \tag{3.2}$$

where S_{ij}^a represents the upper value of the fuzzy parameter $\tilde{S}_{ij}$ and S_{ij}^b represents the lower value of the fuzzy parameter $\tilde{S}_{ij}$. In the proposed method the fuzzy interval is represented by $[S_{ij}^a, S_{ij}^b)$. Here the first element is crisp whereas the second element is fuzzy . Therefore the range become fuzzy.

After we solve and get the solution, if membership value is included in the interval [0, 1) , there wont be any difference in the solutions depending on the shape of membership function. There wont be any difference in the solutions whether we use a linear membership function or a non linear membership function. In any case, the nonlinear membership function, like S-curve membership function, may conceivably change its shape as per the parameter values. At that point the decision maker can apply his/her system to a transportation problem utilizing these parameters. Along these lines, the nonlinear membership function is substantially more advantageous than the linear ones. This particular range is selected because in transportation problem the available supply and demand need not be 100% of the necessity. In the meantime, the production and transportation costs won't be zero. We have taken the minimum value of $\mu_{\tilde{S}}$ as 0.001 and the maximun value of $\mu_{\tilde{S}}$ as 0.999. This concept have real life applications in transportation problem.

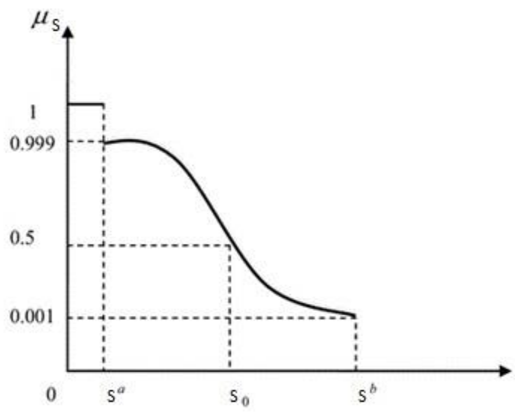

Fig. 1: Modified S-curve membership function

4 Main Result

4.1 Model detailing of Fuzzy Goal programming using modified S-curve non-linear function

Fuzzy goal programming can be used to solve linear programming problems with multiple objectives. This method was introduced by Mohammed[18] where he used linear membership functions. In the proposed method negative deviational variable and positive deviational variable are represented as d_t^- and d_t^+. The positive and negative deviational variables lies between 0.001 and 0.999. The flexible modified S-curve membership grade with deviational variables can be represented as follows:

$$\frac{B}{1+Ce^{\alpha\left(\frac{\tilde{Z}_t-\tilde{Z}_t^L}{\tilde{Z}_t^u-\tilde{Z}_t^L}\right)}}+d_t^- - d_t^+ = 0.999, t = 1,2,..T \qquad (4.1)$$

where $d_t^- d_t^+$=0.001,

B=1, C=0.001001001,$\alpha = 13.81350$. Here μ varies from 0.001 to 0.999 with the interval of 0.0499.

The corresponding min-max approach of goal programming model for the given fuzzy problem using modified S-curve non linear membership grade is given below:

$$\text{Min } \zeta$$

$$\text{s.t } \frac{B}{1+Ce^{\alpha\left(\frac{Z_t - Z_t^L}{Z_t^u - Z_t^L}\right)}} + d_t^- - d_t^+ = 0.999 \text{ ,t=1,2,..T}$$

$$\text{B=1, C=0.001001001, } \alpha = 13.81350$$

$$\zeta \geq d_t^- \text{,t=1,2,...T}$$

$$d_t^+ d_t^- = 0.001$$

$$\sum_{j=1}^{n} x_{ij} = a_i \text{, i=1,2,...,m,}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \text{, j=1,2,...,n,}$$

$$d_t^- , d_t^+ \geq 0$$

$$\zeta \leq 0.999$$

$$\zeta \geq 0.001$$

$$x_{ij} \geq 0 \text{, i=1,2,...m, j=1,2,...n.}$$

5 Algorithm

Step 1: Solve the transportation problem taking single objective at a time muting all other objectives.

Step 2: Repeat step 1 until all objectives get covered.

Step 3: For each and every derived solution, find values for the objectives. Obtained values are tabulated to get pay-off matrix which can be shown as follows

	$Z_1(\mathbb{X}^1)$	$Z_1(\mathbb{X}^2)$	...	$Z_1(\mathbb{X}^t)$
$\mathbb{X}^1$	Z_{11}	Z_{12}	...	Z_{1t}
$\mathbb{X}^2$	Z_{21}	Z_{22}	...	Z_{2t}
.	.	.	...	.
.	.	.	...	.
.	.	.	...	.

$\mathbb{X}^t$	Z_{t1}	Z_{t2}	$\cdots$	Z_{tt}

Table 1: Pay-off matrix

Step 4:The best and worst values for the solutions in each and every objectives where calculated using step 3 . They can be denoted as L_t and U_t respectively for the t^{th} objective function. The level of achievement will come to the aspired level when it is L_t. Achievement will reach the maximum acceptable level at U_t

Step 5: Membership function for the modified S-curve non-linear function is as follows:

$$\mu_{\tilde{Z}} = \begin{cases} 1 & Z_t < L_t \\ 0.999 & Z_t = L_t \\ \dfrac{B}{1+Ce^{13.81350\left(\frac{Z_t - Z_t^L}{Z_t^u - Z_t^L}\right)}} & L_t < Z_t < U_t \\ 0.001 & Z_t = U_t \\ 0 & Z_t > U_t \end{cases} \tag{5.1}$$

t=1,2,..T

Step 6: Formulate equivalent crisp model for the first fuzzy model. This can be done with non-linear S- curve membership functions

Step 7: Crisp model obtained by step 5 can be solved to get compromised solution which is optimal. This can be done by various mathematical tools. In the proposed work we used LINGO 18.0

6 Example

$$Min\widetilde{Z_1}(x) = \frac{[16,17.5)x_{11}+[19,20.5)x_{12}+[12,13.5)x_{13}+[22,23.5)x_{21}+[13,14.5)x_{22}+[19,20.5)x_{23}}{[20,21.5)x_{11}+[25,26.5)x_{12}+[15,16.5)x_{13}+[25,26.5)x_{21}+[18,19.5)x_{22}+[25,26.5)x_{23}}$$

$$\frac{+[14,15.5)x_{31}+[28,29.5)x_{32}+[8,9.5)x_{33}}{+[20,21.5)x_{31}+[35,36.5)x_{32}+[10,11.5)x_{33}}$$

$$Min\widetilde{Z_2}(x) = \frac{[15,16.5)x_{11}+[20,21.5)x_{12}+[18,19.5)x_{13}+[20,21.5)x_{21}+[15,16.5)x_{22}}{[9,10.5)x_{11}+[14,15.5)x_{12}+[12,13.5)x_{13}+[16,17.5)x_{21}+[10,11.5)x_{22}}$$

$$\frac{+[17,18.5)x_{23}+[24,25.5)x_{31}+[25,26.5)x_{32}+[10,11.5)x_{33}}{+[14,15.5)x_{23}+[8,9.5)x_{31}+[20,21.5)x_{32}+[6,7.5)x_{33}}$$

sub to

$$x_{11}+x_{12}+x_{13}=14x_{21}+x_{22}+x_{23}=16x_{31}+x_{32}+x_{33}=12x_{11}+x_{21}+x_{31}=10x_{12}+x_{22}+x_{32}=15x_{13}+x_{23}+x_{33}=17x_{ij}\geq 0,i=1,2,\ldots m,j=1,2,\ldots\ldots,n.$$

Solution:

Step 1: On solving Z_1 and Z_2 we get the solution as follows:
$\mathbb{X}^1 = (x_{11}=0,x_{12}=0,x_{13}=14,x_{21}=0,x_{22}=15,x_{23}=1,x_{31}=10,x_{32}=0,x_{33}=2)$
$\mathbb{X}^2 = (x_{11}=0,x_{12}=3,x_{13}=11,x_{21}=10,x_{22}=0,x_{23}=6,x_{31}=0,x_{32}=12,x_{33}=0)$
Step 2: Values of the objective functions are:
$Z_1(\mathbb{X}^1)$ = 0.7626904, $Z_1(\mathbb{X}^2)$ = 0.8210151, $Z_2(\mathbb{X}^1)$ = 1.677618, $Z_2(\mathbb{X}^2)$ = 1.280166.
Step 3: Now we can find the lower and upper bound of Z_1 and Z_2
Z_1^L = 0.7626904, Z_1^U = 0.8210151, Z_2^L = 1.280166, Z_2^U = 1.677618.
Step 4: Using modified S-curve membership function the corresponding crisp model detailing is as follows:

Minimize ζ

sub to

$$\frac{1}{1+0.001001001e^{\alpha\left(\frac{Z_1-0.7626904}{0.8210151-0.7626904}\right)}}+d_t^- - d_t^+ = 0.999$$

$$\frac{1}{1+0.001001001e^{\alpha\left(\frac{Z_2-1.280166}{1.677618-1.280166}\right)}}+d_t^- - d_t^+ = 0.999$$

$$0<\alpha<\infty$$

$$\zeta \geq d_t^-, t=1,2$$

$$d_t^+ d_t^- = 0.001, t=1,2$$

$$x_{11}+x_{12}+x_{13}=14x_{21}+x_{22}+x_{23}=16x_{31}+x_{32}+x_{33}=12x_{11}+x_{21}+x_{31}=10x_{12}+x_{22}+x_{32}=15x_{13}+x_{23}+x_{33}=17\ d_t^+,d_t^- \geq 0.001$$

$$\zeta \leq 0.999$$

$$\zeta \geq 0.001$$

$$x_{ij} \geq 0, \forall i, j.$$

where

$$Z_1(x) = \frac{[16,17.5)x_{11}+[19,20.5)x_{12}+[12,13.5)x_{13}+[22,23.5)x_{21}+[13,14.5)x_{22}+[19,20.5)x_{23}}{[20,21.5)x_{11}+[25,26.5)x_{12}+[15,16.5)x_{13}+[25,26.5)x_{21}+[18,19.5)x_{22}+[25,26.5)x_{23}}$$

$$\frac{+[14,15.5)x_{31}+[28,29.5)x_{32}+[8,9.5)x_{33}}{+[20,21.5)x_{31}+[35,36.5)x_{32}+[10,11.5)x_{33}}$$

$$Z_2(x) = \frac{[15,16.5)x_{11}+[20,21.5)x_{12}+[18,19.5)x_{13}+[20,21.5)x_{21}+[15,16.5)x_{22}}{[9,10.5)x_{11}+[14,15.5)x_{12}+[12,13.5)x_{13}+[16,17.5)x_{21}+[10,11.5)x_{22}}$$

$$\frac{+[17,18.5)x_{23}+[24,25.5)x_{31}+[25,26.5)x_{32}+[10,11.5)x_{33}}{+[14,15.5)x_{23}+[8,9.5)x_{31}+[20,21.5)x_{32}+[6,7.5)x_{33}}$$

By assigning different values to α and μ, we solve the given problem using LINGO 18.0 software.

Table 2 shows the variation of all variables, deviations and the objective values keeping vagueness parameter, α = 13.81350 and varying degree of satisfaction, μ from 0.001 to 0.999. Here we consider interval steps of μ as 0.0499. Results obtained by varying vagueness and μ is shown in table 3.

The values of x_{13}, x_{21} andx_{22} are found to be zero. Values of x_{23} and x_{33} are found to be 16 and 1 respectively.

The values of objective functions decreases as we increase the degree of satisfaction. It is graphically shown in figure 2 and figure 3. Variations of Z_1 and Z_2 with respect to different vaguenesses and degrees of satisfaction are shown in figure 4 and figure 5 respectively.

μ	ζ	X_{11}	X_{12}	X_{31}	X_{32}	d_1^-	d_1^+	d_2^-	d_2^+	Z_1	Z_2
0.001	0.066804	4.935700	9.064300	5.064300	5.935700	0.066804	0.014969	0.066804	0.014969	0.779598	1.395650
0.0509	0.057388	5.797954	8.202046	4.202046	6.797954	0.057388	0.017425	0.057388	0.017425	0.778469	1.387981
0.1008	0.055922	5.957196	8.042804	4.042804	6.957196	0.055922	0.017889	0.055922	0.017889	0.778256	1.386537
0.1507	0.055048	6.056314	7.943686	3.943686	7.056314	0.055048	0.018177	0.055048	0.018177	0.778123	1.385634
0.2006	0.054406	6.131078	7.868922	3.868922	7.131078	0.054406	0.018393	0.054406	0.018393	0.778023	1.384950
0.2505	0.053870	6.192885	7.807115	3.807115	7.192885	0.053870	0.018553	0.053870	0.018553	0.777939	1.384384
0.3004	0.053432	6.246892	7.753108	3.753108	7.246892	0.053432	0.018715	0.053432	0.018715	0.777866	1.383888
0.3503	0.053046	6.295957	7.704043	3.704043	7.295957	0.053046	0.018866	0.053046	0.018866	0.777800	1.383436
0.4002	0.052673	6.341873	7.658127	3.658127	7.341873	0.052673	0.018990	0.052673	0.018990	0.777738	1.383013
0.4501	0.052324	6.385886	7.614114	3.614114	7.385886	0.052324	0.019112	0.052324	0.019112	0.777678	1.382606
0.5	0.051991	6.429011	7.570989	3.570989	7.429011	0.051991	0.019235	0.051991	0.019235	0.777619	1.382207
0.5499	0.051662	6.472125	7.527875	3.527875	7.472125	0.051662	0.019357	0.051662	0.019357	0.777560	1.381807
0.5998	0.051338	6.516111	7.483889	3.483889	7.516111	0.051338	0.019488	0.051338	0.019488	0.777500	1.381399
0.6497	0.050988	6.561962	7.438038	3.438038	7.561962	0.050988	0.019607	0.050988	0.019607	0.777437	1.380973
0.6996	0.050636	6.610938	7.389062	3.389062	7.610938	0.050636	0.019749	0.050636	0.019749	0.777370	1.380516
0.7495	0.050251	6.664820	7.335180	3.335180	7.664820	0.050251	0.019900	0.050251	0.019900	0.777296	1.380013
0.7994	0.049819	6.726435	7.273565	3.273565	7.726435	0.049819	0.020071	0.049819	0.020071	0.777211	1.379437
0.8493	0.049311	6.800910	7.199090	3.199090	7.800910	0.049311	0.020280	0.049311	0.020280	0.777108	1.378738
0.8992	0.048660	6.899548	7.100452	3.100452	7.899548	0.048660	0.020556	0.048660	0.020556	0.776972	1.377809
0.9491	0.047650	7.057785	6.942215	2.942215	8.057785	0.047650	0.020983	0.047650	0.020983	0.776751	1.376312
0.999	0.043116	7.909590	6.090410	2.090410	8.909590	0.043116	0.023193	0.043116	0.023193	0.775541	1.368088

Table 2: Results obtained for different degrees of satisfaction

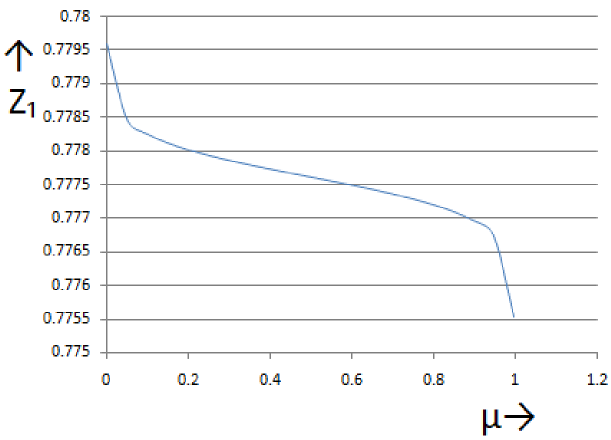

Fig. 2: Objective function, Z_1 versus degree of satisfaction, μ

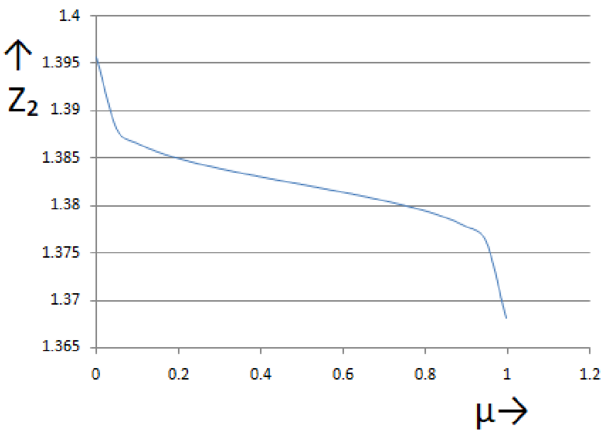

Fig. 3: Objective function,Z_2 versus degree of satisfaction, μ

α	μ	ζ	X_{11}	X_{12}	X_{31}	X_{32}	d_1^-	d_1^+	d_2^-	d_2^+	Z_1	Z_2
5	0.1008	0.033973	2.4644	11.5356	7.5356	3.4644	0.033973	0.029436	0.033973	0.029436	0.782643	1.416343
5	0.2006	0.033830	2.9531	11.0469	7.0469	3.9531	0.033830	0.029560	0.033830	0.029560	0.782062	1.412393
5	0.3004	0.033737	3.2782	10.7218	6.7218	4.2782	0.033737	0.029641	0.033737	0.029641	0.781670	1.409729
5	0.4002	0.033663	3.5445	10.4555	6.4555	4.5445	0.033663	0.029706	0.033663	0.029706	0.781345	1.407522
5	0.5	0.033597	3.7888	10.2112	6.2112	4.7888	0.033597	0.029765	0.033597	0.029765	0.781044	1.405480
5	0.5998	0.033532	4.0327	9.9673	5.9673	5.0327	0.033532	0.029823	0.033532	0.029823	0.780742	1.403424
5	0.6996	0.033462	4.2981	9.7019	5.7019	5.2981	0.033462	0.029885	0.033462	0.029885	0.780409	1.401165
5	0.7994	0.033379	4.6211	9.3789	5.3789	5.6211	0.033379	0.029959	0.033379	0.029959	0.780000	1.398388
5	0.8992	0.033259	5.1045	8.8955	4.8955	6.1045	0.033259	0.030067	0.033259	0.030067	0.779379	1.394167
5	0.999	0.032650	7.9096	6.0904	2.0904	8.9096	0.032650	0.030628	0.032650	0.030628	0.775541	1.368088
7	0.001	0.037408	1.9955	12.0045	8.0045	2.9955	0.037408	0.026733	0.037408	0.026733	0.783192	1.420072
7	0.1008	0.035720	4.0361	9.9639	5.9639	5.0361	0.035720	0.027994	0.035720	0.027994	0.780737	1.403395
7	0.2006	0.035475	4.3826	9.6174	5.6174	5.3826	0.035475	0.028189	0.035475	0.028189	0.780303	1.400442
7	0.3004	0.035317	4.6131	9.3869	5.3869	5.6131	0.035317	0.028315	0.035317	0.028314	0.780010	1.398457
7	0.4002	0.035192	4.8023	9.1977	5.1977	5.8023	0.035192	0.028415	0.035192	0.028415	0.779769	1.396815
7	0.5	0.035080	4.9756	9.0244	5.0244	5.9756	0.035080	0.028507	0.035080	0.028507	0.779546	1.395300
7	0.5998	0.034970	5.1489	8.8511	4.8511	6.1489	0.034970	0.028596	0.034970	0.028596	0.779322	1.393777
7	0.6996	0.034853	5.3374	8.6626	4.6626	6.3374	0.034853	0.028692	0.034853	0.028692	0.779076	1.392108
7	0.7994	0.034715	5.5669	8.4331	4.4331	6.5669	0.034715	0.028806	0.034715	0.028806	0.778775	1.390060
7	0.8992	0.034516	5.9104	8.0896	4.0896	6.9104	0.034516	0.028973	0.034516	0.028972	0.778318	1.386965
7	0.999	0.033527	7.9096	6.0904	2.0904	8.9096	0.033527	0.029827	0.033527	0.029827	0.775541	1.368088

α	μ	ζ	X_{11}	X_{12}	X_{31}	X_{32}	d_1^-	d_1^+	d_2^-	d_2^+	Z_1	Z_2
11	0.001	0.048393	4.1672	9.8328	5.8328	5.1672	0.048393	0.020681	0.048393	0.020681	0.780574	1.402282
11	0.1008	0.043298	5.4543	8.5457	4.5457	6.4543	0.043298	0.023095	0.043298	0.023095	0.778923	1.391067
11	0.2006	0.042585	5.6732	8.3268	4.3268	6.6732	0.042585	0.023481	0.042585	0.023481	0.778634	1.389105
11	0.3004	0.042136	5.8190	8.1810	4.1810	6.8190	0.042136	0.023739	0.042136	0.023739	0.778441	1.387790
11	0.4002	0.041767	5.9386	8.0614	4.0614	6.9386	0.041767	0.023933	0.041767	0.023933	0.778281	1.386706
11	0.5	0.041449	6.0482	7.9518	3.9518	7.0482	0.041449	0.024120	0.041449	0.024120	0.778134	1.385707
11	0.5998	0.041141	6.1578	7.8422	3.8422	7.1578	0.041141	0.024306	0.041141	0.024306	0.777987	1.384705
11	0.6996	0.040813	6.2772	7.7228	3.7228	7.2772	0.040813	0.024502	0.040813	0.024502	0.777825	1.383609
11	0.7994	0.040431	6.4225	7.5775	3.5775	7.4225	0.040431	0.024742	0.040431	0.024742	0.777628	1.382268
11	0.8992	0.039870	6.6402	7.3598	3.3598	7.6402	0.039870	0.025077	0.039870	0.025077	0.777330	1.380243
11	0.999	0.037205	7.9096	6.0904	2.0904	8.9096	0.037205	0.026878	0.037205	0.026878	0.775541	1.368088
13	0.001	0.060041	4.7480	9.2520	5.2520	5.7480	0.060041	0.016655	0.060041	0.016655	0.779839	1.397287
13	0.1008	0.051279	5.8343	8.1657	4.1657	6.8343	0.051279	0.019514	0.051279	0.019514	0.778420	1.387652
13	0.2006	0.050056	6.0192	7.9808	3.9808	7.0192	0.050056	0.019991	0.050056	0.019991	0.778173	1.385973
13	0.3004	0.049271	6.1423	7.8577	3.8577	7.1423	0.049271	0.020296	0.049271	0.020296	0.778008	1.384847
13	0.4002	0.048656	6.2433	7.7567	3.7567	7.2433	0.048656	0.020552	0.048656	0.020552	0.777871	1.383921
13	0.5	0.048110	6.3359	7.6641	3.6641	7.3359	0.048110	0.020786	0.048110	0.020786	0.777746	1.383067
13	0.5998	0.047580	6.4285	7.5715	3.5715	7.4285	0.047580	0.021017	0.047580	0.021017	0.777620	1.382211
13	0.6996	0.047021	6.5293	7.4707	3.4707	7.5293	0.047021	0.021267	0.047021	0.021267	0.777482	1.381276
13	0.7994	0.046371	6.6521	7.3479	3.3479	7.6521	0.046371	0.021576	0.046371	0.021576	0.777314	1.380132
13	0.8992	0.045423	6.8361	7.1639	3.1639	7.8361	0.045423	0.022012	0.045423	0.022012	0.777060	1.378407
13	0.999	0.040955	7.9096	6.0904	2.0904	8.9096	0.040955	0.024423	0.040955	0.024423	0.775541	1.368088

Table 3: Results obtained by varying vagueness and μ

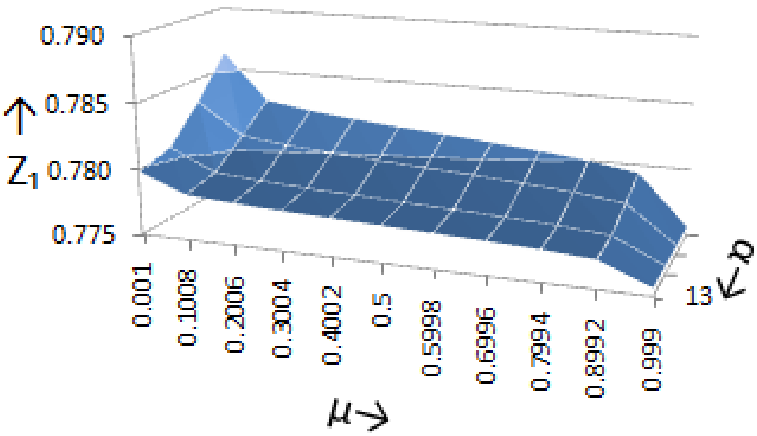

Fig. 4: Ojective function, Z_1 versus degree of satisfaction for different vaguenesses.

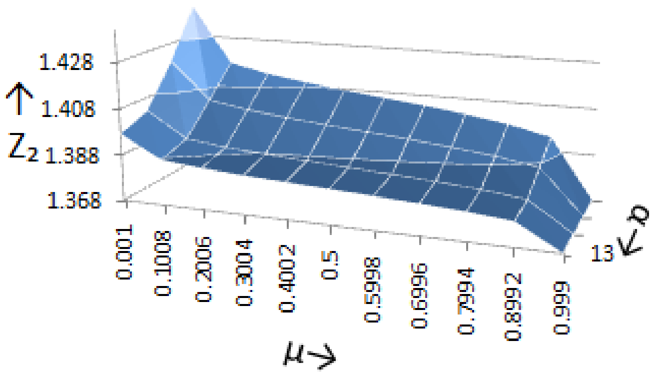

Fig. 5: Ojective function,Z_2 versus degree of satisfaction for different vaguenesses

7 Conclusion

A novel methodology for solving interval valued fractional fuzzy transportation problem using modidifed S-curve membership function is proposed.When information available is little or a little during planning, S-curve membership function will come into picture for solving fractional transportation problems using fuzzy parameters. This flexibility of membership function over linear membership functions enables decision maker to form an apt membership functions upon his judgement.The real effectiveness in using S-curve membership function is shown in this paper. A study on interval valued fractional transportation problem using goal programming approach is not yet done by researchers in relevant field. The obtained results from the above example shows that the decision maker will get two more dimensions, Vagueness,α and degree of satisfaction,μ for making apt decision. In real life problems degree of satisfaction wont be zero or 100 percentage. Hence we are not considering $\mu = 0$ or $\mu = 1$ in our problem. LINGO 18.0 is used to obtain the results.

Acknowledgement: We are thankful to the unknown reviewer for constructive as well as creative suggestions.

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