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DETOUR GLOBAL DOMINATION NUMBER OF CORONA PRODUCT GRAPHS

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Abstract

A subset S of V of a connected graph G= $(V,E)$is a detour global dominating set if S is both detour set and global dominating set of G. The minimum cardinality taken over all detour global dominating sets is called the detour global domination number of G and is denoted by γ_{dg}(G). A detour global dominating set of cardinality γ_{dg} (G) is called a γ_{dg}- set of G. In this paper we determine γ_{dg}(G) for Corona product of some standard graphs.

Keywords: Detour set, detour global dominating set, corona product

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1. Introduction

By a graph G, we mean a finite undirected connected graph without loops or multiple edges. Unless and otherwise stated, the graph G = (V, E) have n = $|V|$ vertices and m = $|E|$edges. For basic definitions and terminologies, we refer [1,5]. For vertices u and v in a graph G, the detour distance D (u, v) is the length of a longest u – v path in G. A u – v path of length D (u, v) is called a u – v detour. The closed detour interval ID$[u , v]$ consists of u, v and all vertices in some u – v detour of G. These concepts were studied by Chartrand et al. [2,3] For S $\subseteq V(G)$, ID $[S] = \bigcup_{u,v \in S}$ ID$[u , v]$. A subset S of V of a graph G is called a detour set if ID $[S] = V(G)$.The detour number dn(G) of G is the minimum cardinality taken over all detour sets in G. These concepts were studied by Chartrand [4].

The concepts of domination number and global domination number of a graph were introduced in [7,12]. A subset S of V of a graph G= (V, E) is called a dominating set of G if every vertex in V − S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G.A subset S of V of a graph G= (V, E) is called a global dominating set (g.d. set) if it is a dominating set of a graph G and its complement $\bar{G}$ of G. The global domination number γ_g(G) of G is the minimum cardinality taken over all global dominating sets in G.

The corona $G_1 \circ G_2$ of two graphs $G_1$and G_2 is defined as the graph H obtained by taking one copy of G_1(which has n_1 points) and n_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

Theorem 1.1: Every end vertex of a connected graph G belongs to every detour global dominating set of G.

Theorem 1.2: If the set S contains only end and full vertices is a detour global dominating set of G, then S is the unique minimum detour global dominating set of G and $\gamma_{dg}(G) = |S|$.

2.Detour Global Domination Number of Corona product of graphs

Theorem 2.1: For the graph $H = P_n \circ K_1 (n \geq 2)$, then $\gamma_{dg}(H) = |P_n|$.

Proof: Let $H = P_n \circ K_1$ has $2n$ vertices $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ and $2n - 1$ edges $v_i v_{i+1}, 1 \leq i \leq n-1$ and $v_i u_i$, $1 \leq i \leq n$. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of P_n and $\{u_i\}$ be the vertex set of $i^{th}(1 \leq i \leq n)$ copy of K_1 .The set D = $\{u_1, u_2, \dots, u_n\}$ is the set of all pendant vertices of H. Then D is the subset of every

$\gamma_{dg} - set$ of H(by Theorem 1.1.) and so $\gamma_{dg}(G) \geq |D|$.It is clear that D is a detour global dominating set of H so that $\gamma_{dg}(H) = n$.

Theorem 2.2: For the graph $H = P_n \circ K_2 (n \geq 2)$, then $\gamma_{dg}(H) = |P_n|$.

Proof: Let $H = P_n \odot K_2$ has $3n$ vertices $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_{2n}\}$ and $4n - 1$ edges. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of P_n and $\{u_{2i-1}, u_{2i}\}$ be the vertex set of $i^{th}(1 \leq i \leq n)$ copy of k_2. Any set S containing at least one vertex from each copy of $k_2$forms a minimum detour dominating set of H. Also, it is a dominating set of $\overline{H}$. Since every vertex in $H - S$ is not adjacent to at least one vertex in S. Therefore, S itself is a minimum detour global dominating set of H. Hence $\gamma_{dg}(H) = n$.

Theorem 2.3: For the graph $H = P_n \circ K_m (n \geq 2)$, then $\gamma_{dg}(H) = |P_n|$.

Proof: Let$H = P_n \circ K_m$. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of P_n and $\{u_{i1}, u_{i2}, \dots\dots, u_{im}\}$ be the vertex set of i^{th} copy of $K_m$which is adjacent to $v_i(1 \leq i \leq n)$.Then the graph H has $n(m + 1)$vertices $\{v_1, v_2, \dots, v_n, u_{11}, u_{12}, \dots u_{1m}, u_{21}, u_{22}, \dots, u_{2m}, \dots, u_{n1}, u_{n2}, \dots, u_{nm}\}$. Obviously, for $j = 1\ to\ m, S_j = \{u_{1j}, u_{2j}, u_{3j}, \dots, u_{(n-1)j}, u_{nj}\}$ are some detour dominating sets of H. Then any subset of detour dominating set of H with cardinality two, dominates all the vertices in the complement $\overline{H}$ of H. Therefore, sets S_j itself is a detour global dominating set of H. Further, no set less than $n(|S_j| = n)$ vertices is a detour global dominating set. Hence, each S_j is a minimum detour global dominating set of H. Hence $\gamma_{dg}(H) = |P_n| = n$.

Example 2.4: $\gamma_{dg}(P_2 \circ K_5) = 2$

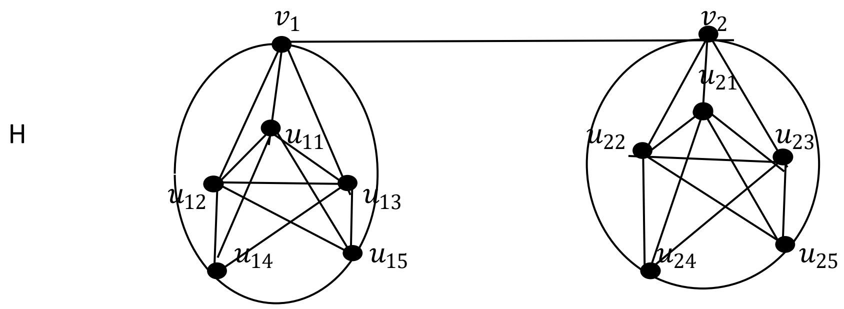

Figure 2.1

In figure 2.1, The sets $\{u_{11},u_{21}\}$,$\{u_{12},u_{22}\}$,$\{u_{13},u_{23}\}$,$\{u_{14},u_{24}\}$,$\{u_{15},u_{25}\}$ are some minimum detour global dominating sets of H. Hence $\gamma_{dg}(P_2 \circ K_5) = 2$.

Theorem 2.5: For the graph $H = C_n \circ K_1 (n \geq 3)$, then $\gamma_{dg}(H) = n$.

Proof: Let $H = C_n \circ K_1$. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of the cycle of order n and let u_i be the vertex set of the i^{th}copy of K_1 which is adjacent to v_i. The graph H has $2n$ vertices $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$.Then the set S $= \{u_1, u_2, \dots, u_{n-1}, u_n\}$ is the set of all end vertices of H. By theorem 1.1, S is a subset of every detour global dominating set of H and so $\gamma_{dg}(H) \geq n$. It is clear that S is a detour global dominating set of H and so S is the unique minimum detour global dominating set of H. Therefore $\gamma_{dg}(H) = n$.

Theorem 2.6: For the graph $H = C_n \circ K_2 (n \geq 3)$, then $\gamma_{dg}(H) = n$.

Proof: Let $H = C_n \circ K_2 (n \geq 3)$. Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of the cycle of order n and $\{u_{i1}, u_{i2}\}$ be the vertex set of the i^{th}copy of K_2 which is adjacent to $v_i (i = 1\ to\ n)$.

Then the graph H has3n vertices$\{v_1, v_2, \dots, v_n, u_{11}, u_{12}, u_{21}, u_{22}, \dots, u_{n1}, u_{n2}\}$.The sets $S_1 = \{u_{11}, u_{21} \dots., u_{(n-1)1}, u_{n1}\}$ and $S_2 = \{u_{12}, u_{22} \dots., u_{(n-1)2}, u_{n2}\}$ are some detour global dominating set of H. There does not exist any $\gamma_{dg} - set$ of cardinality less than n ($|S_1| = |S_2| = n$). Hence $\gamma_{dg}(H) = n$.

Theorem 2.7: For the graph $H = C_n \circ K_m (n \geq 3)$, then $\gamma_{dg}(H) = n$.

Proof: Let $\{v_1, v_2, \dots, v_n\}$ be the vertices of C_n and let $\{u_{i1}, u_{i2}, \dots\dots, u_{im}\}$ be the vertex set of i^{th} copy of $K_m$which is adjacent to $v_i (1 \leq i \leq n)$. Then,$V(H) = \{v_1, v_2, \dots, v_n, u_{11}, u_{12}, \dots u_{1m}, u_{21}, u_{22}, \dots, u_{2m}, \dots, u_{n1}, u_{n2}, \dots, u_{nm}\}$. Obviously, for $j = 1\ to\ m$,$S_j = \{u_{1j}, u_{2j}, u_{3j}, \dots, u_{(n-1)j}, u_{nj}\}$ are some detour dominating sets of H. It is also a dominating set of $\overline{H}$. Since, every vertex in $\{H - S_j / j = 1 to\ m\}$ is not adjacent to at least one vertex in S_j Therefore, sets S_j itself is a detour global dominating set of H. Further, there does not exist any $\gamma_{dg} - set$ of cardinality less than n$(|S_j| = n)$. Hence, each S_j is a minimum detour global dominating set of $C_n \circ K_m$. Hence $\gamma_{dg}(H) = |C_n| = n$.

Example 2.8: $\gamma_{dg}(C_4 \circ K_2) = 4$.

In figure 2.2, $S = \{u_{11}, u_{21}, u_{31}, u_{41}\}$ forms a minimum detour global dominating set of H.

Therefore $\gamma_{dg}(C_4 \circ K_2) = 4$

H:

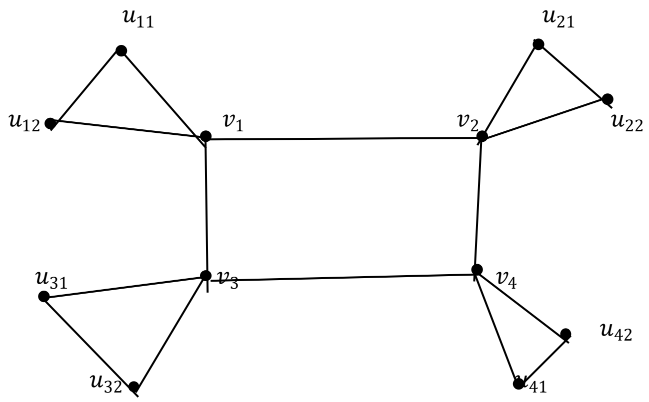

Figure 2.2

Theorem 2.9: For the graph $H = W_n \circ K_1 (n \geq 4)$, then $\gamma_{dg}(H) = n$.

Proof:

Let $H = W_n \circ K_1$. Here $W_n$is a graph formed by connecting a single universal vertex to all vertices of a cycle of order $n - 1$. Let $\{v_1, v_2, \dots, v_{n-1}, v_n\}$ be the vertices of the wheel graph of order n with v_n be a universal vertex and $\{v_1, v_2, \dots, v_{n-1}\}$ be the vertices of an outer cycle. Let u_i be the vertex set of the i^{th}copy of K_1 which is adjoint to v_i. The graph H has 2n vertices $\{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$.Then the set S $= \{u_1, u_2, \dots, u_{n-1}, u_n\}$, being the set of all end vertices of H. By theorem 1.1, S is a subset of every detour global dominating set of H. It is clear that S is a detour set. Also, S dominates all the vertices in H and the complement $\overline{H}$ of H. Therefore, S itself is a detour global dominating set of H. Further, no set less than $n(|S| = n)$ vertices is a detour global dominating set. Hence S is the unique minimum detour global dominating set of H. Therefore $\gamma_{dg}(H) = |S| = n$.

Theorem 2.10: For the graph $H = W_n \circ K_2 (n \geq 4)$, then $\gamma_{dg}(H) = n$.

Proof: Let $H = W_n \circ K_2$. Here $W_n$is a graph formed by connecting a single universal vertex to all vertices of a cycle of order $n - 1$. t $\{v_1, v_2, \dots, v_{n-1}, v_n\}$ be the vertices of the wheel graph of order n with v_n be a universal vertex and $\{v_1, v_2, \dots, v_{n-1}\}$ be the vertices of an outer cycle and let $\{u_{i1}, u_{i2}\}$ be the vertex set of the i^{th}copy of K_2 adjoint to $v_i(i = 1\ to\ n)$. Then the graph H has 3n vertices $\{v_1, v_2, \dots, v_n, u_{11}, u_{12}, u_{21}, u_{22}, \dots, u_{n1}, u_{n2}\}$. The sets $S_1 = \{u_{11}, u_{21} \dots., u_{(n-1)1}, u_{n1}\}$ and $S_2 = \{u_{12}, u_{22} \dots., u_{(n-1)2}, u_{n2}\}$ are some detour global dominating set of H. Further, no set less than n vertices is a detour global dominating set. Therefore sets S_1 and S_2 are minimum detour global dominating set of H. Hence $\gamma_{dg}(H) = n$.

Theorem 2.11: For the graph $H = W_n \circ K_m (n \geq 4)$, then $\gamma_{dg}(W_n \circ K_m) = n$.

Proof: Let $H = W_n \circ K_m$. Here $W_n$is a graph formed by connecting a single universal vertex to all vertices of a cycle of order $n - 1$. Let $\{v_1, v_2, \dots, v_{n-1}, v_n\}$ be the vertices of the wheel graph of order n with v_n be a universal vertex and $\{v_1, v_2, \dots, v_{n-1}\}$ be the vertices of an outer cycle. Let $\{u_{i1}, u_{i2}, \dots\dots, u_{im}\}$ be the vertex set of i^{th} copy of $K_m$adjacent to $v_i(1 \leq i \leq n)$. Then $V(H = W_n \circ K_m) = \{v_1, v_2, \dots, v_n, u_{11}, u_{12}, \dots u_{1m}, u_{21}, u_{22}, \dots, u_{2m}, \dots, u_{n1}, u_{n2}, \dots, u_{nm}\}$. Obviously, for $j = 1\ to\ m$,$S_j = \{u_{1j}, u_{2j}, u_{3j}, \dots, u_{(n-1)j}, u_{nj}\}$ are some detour dominating sets of $W_n \circ K_m$. Then any subset of detour dominating set of H with cardinality two, dominates all the vertices in the complement $\overline{H}$ of H. Therefore, sets S_j itself is a detour global dominating set of H. Further, no set less than $n(|S_j| = n)$ vertices is a detour global dominating set. Hence, each S_j is a minimum detour global dominating set of $W_n \circ K_m$. Hence $\gamma_{dg}(W_n \circ K_m) = |W_n| = n$.

Example 2.12: $\gamma_{dg}(W_4 \circ K_3) = 4$

In figure 2.3, $S = \{u_{12}, u_{22}, u_{32}, u_{42}\}$ forms a minimum detour global dominating set of the graph H. $\gamma_{dg}(W_4 \circ K_3) = 4$.

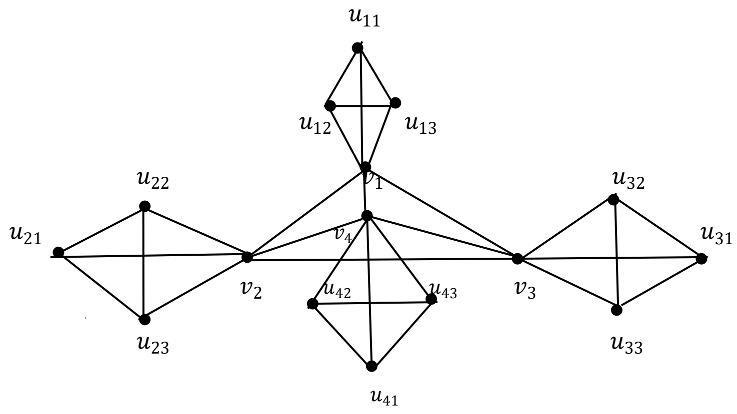

Figure 2.3

Theorem 2.13: For the graph $H = K_{1,n} \circ K_1$, then $\gamma_{dg}(K_{1,n} \circ K_1) = n + 1$.

Proof: Let $H = K_{1,n} \circ K_1$. Let $V(K_{1,n}) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\}$ with v as its root vertex and $\{v_1, v_2, \dots, v_{n-1}, v_n\}$ be the set of end vertices. et u_i be the vertex set of the i^{th}copy of K_1 adjacent to $v_i (1 \le i \le n)$ and w be the vertex of a copy of K_1 adjacent to the root vertex v. The graph H has $2n + 1$ vertices $\{v_1, v_2, \dots, v_n, v, u_1, u_2, \dots, u_n\}$.Then the set S $= \{u_1, u_2, \dots, u_{n-1}, u_n, w\}$ being the set of all end vertices of H. By theorem 1.1, S is a subset of every detour dominating set of H. It is clear that S is a detour set. Also, S dominates all the vertices in the complement $\overline{H}$ of H. Since S contains only end vertices and the graph $H - S$ has no full vertex. Hence S is the unique minimum detour global dominating set of H. Therefore $\gamma_{dg}(K_{1,n} \circ K_1) = |S| = n + 1$.

Theorem 2.14: For the graph $H = K_{1,n} \circ K_2$, then $\gamma_{dg}(K_{1,n} \circ K_2) = n + 1$.

Proof: Let $H = K_{1,n} \circ K_2$. Let $V(K_{1,n}) = \{v_1, v_2, \dots, v_{n-1}, v_n, v\}$ with v as its root vertex and $\{v_1, v_2, \dots, v_{n-1}, v_n\}$ be the set of end vertices and let $\{u_{i1}, u_{i2}\}$ be the vertex set of the i^{th}copy of K_2 which are adjacent to $v_i (i = 1 \; to \; n)$ and $\{w_1, w_2\}$ be the vertex set of a copy of K_2 which are adjacent to the root vertex v. Then the graph H has $3n + 1$ vertices $\{v_1, v_2, \dots, v_n, v, u_{11}, u_{12}, u_{21}, u_{22}, \dots, u_{n1}, u_{n2}\}$. The sets $S_1 = \{u_{11}, u_{21} \dots, u_{(n-1)1}, u_{n1}, w_1\}$ and $S_2 = \{u_{12}, u_{22} \dots, u_{(n-1)2}, u_{n2}, w_2\}$ are some detour global dominating set of H. There does not exist any $\gamma_{dg} - set$ of cardinality less than n ($|S_1| = |S_2| = n$). Hence $\gamma_{dg}(K_{1,n} \circ K_2) = n$.

Theorem 2.15: For the graph $H = K_{1,n} \circ K_m$, then $\gamma_{dg}(K_{1,n} \circ K_m) = n + 1$.

Proof: Let $H = K_{1,n} \circ K_m$. Let $V(K_{1,n}) = \{v_1, v_2, \ldots, v_{n-1}, v_n, v\}$ with v as its root vertex and $\{v_1, v_2, \ldots, v_{n-1}, v_n\}$ be the set of end vertices. Let $\{u_{i1}, u_{i2}, \ldots\ldots, u_{im}\}$ be the vertex set of i^{th} copy of $K_m$which are adjacent to $v_i (1 \le i \le n)$ and $\{w_1, w_2, \ldots, w_m\}$ be the vertex set of a copy of $K_m$which are adjacent to the root vertexv.Then $V(H) =$
$\{v_1, v_2, \ldots, v_n, v\} \cup \{u_{11}, u_{12}, \ldots u_{1m}, u_{21}, u_{22}, \ldots, u_{2m}, \ldots, u_{n1}, u_{n2}, \ldots, u_{nm}\}\}$.
Obviously, for $j = 1\ to\ m$, $S_j = \{u_{1j}, u_{2j}, u_{3j}, \ldots, u_{(n-1)j}, u_{nj}, w_j\}$ are some detour dominating sets of $K_{1,n} \circ K_m$. Then any subset of detour dominating set of H with cardinality two dominates all the vertices in the complement $\overline{H}$ of H. Therefore, sets S_j itself is a detour global dominating set of H. Further, no set less than $n(|S_j| = n)$ vertices is a detour global dominating set. Hence, each S_j is a minimum detour global dominating set of $K_{1n} \circ K_m$. Hence $\gamma_{dg}(K_{1,n} \circ K_m) = |K_{1,n}| = n + 1$.

Example 2.16: $\gamma_{dg}(K_{1,4} \circ K_3) = 5$.
In Figure 2.4, $S = \{u_{13}, u_{23}, u_{33}, u_{43}, w_3\}$ forms a minimum detour global dominating set of the graph H. Hence $\gamma_{dg}(K_{1,4} \circ K_3) = 5$

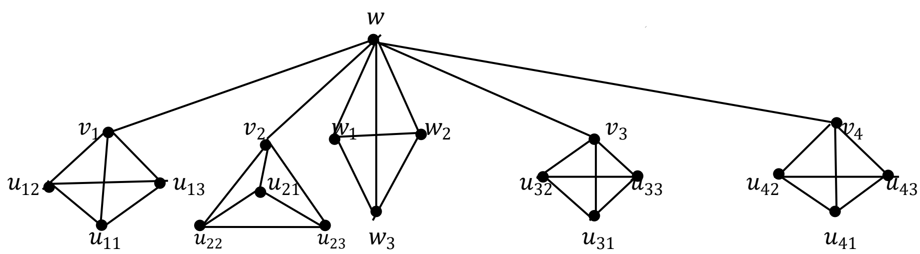

Figure 2.4

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