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STUDY OF AN ELECTRICAL CIRCUIT USING NODAL ANALYSIS AND GRAPH THEORETIC APPROACH

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Abstract

In Mathematics, Graph Theory is a branch where we study about some points or nodes and the defined connections among the points. In electrical circuit, electrical elements are connected in a particular way for passing current. An electrical circuit consists of nodes and branches which obeys the current laws such as Kirchhoff's Current Law, Kirchhoff's Voltage Law etc. The circuits can be simplified by using many well known theorems of Electrical Engineering. The same process can be done with the help of Graph Theory and matrices also. From circuits we can find matrices by different rules. There are different matrices such as branch impedance matrix, branch admittance matrix, tie-set matrix etc which can be obtained from electrical circuits. In this paper, an electrical circuit with one unknown resistance is analysed with the help of graph theory and nodal analysis of electrical circuits. The main objective of this paper is to study about branch current and loop current with the help of graph theory through matrix equilibrium equations obtained from current laws.

Keywords: Graph Theory, Electrical Circuits, Tie-set matrix, Branch current, Loop current.

2010 AMS classification: 05C50

1. Introduction

Graph Theory is a branch of Mathematics that deals in some points and the connections between them. With the help of graph theory many mathematical models have been formulated to solve real life problems in different fields. Different kinds of optimization can be done with the help of graph theory in Physics, Chemistry, Computer Science, Communication Science, Electrical Engineering etc. It has also close relations with other branches of Mathematics such as Group Theory, Matrix Theory, Topology etc. In Network Analysis, an electrical network is defined as the collections of some interconnected electrical elements. Their adjacency and direction defines the behaviour of the circuit. Flowing current through the circuits obeys different laws such as Kirchhoff's current laws, Kirchhoff's voltage laws etc. In recent times Graph theory is broadly used in electrical circuit analysis. Many researchers have done remarkable work on this topic. In 1980, Vandewalle [22] applied coloured branch theorem in circuit theory. In the same year Satoru Fujishige developed an algorithm to solve graph realization problem using fundamental circuit matrix or graphic matrix [14]. In 1983 Istvan Fary developed some algorithms on generalised circuits [13]. In 1985, Karl Gustafson studied graph and networks using vector calculus and developed some results relating to divergence and curl [16]. In 1992, Brooks, Smith, Stone and Tutte expressed current and potential differences using determinant [7]. In 1997, Gunther and Hoscheck adapted ROW method for simulation of electric circuits [15]. Lamb, Asher and Woodball [18] studied about the relations of bond graphs and electrical networks. In 2005 R.B. Bapat [3] studied incidence matrix, Laplacian and distance matrix for a tree with attached graphs. In 2012, Li and Xuan [19] used improved adjacency matrix for calculation of distribution network flow. In the same year bond graph and Matlab was used to solve an electrical model by obtaining the system equations [20]. Harper [17] studied morphisms for resistive electrical networks to solve Kirchhoff's problem in 2014. Alman, Lian and Tran [1] found a new result on circular planner electrical network in 2015. Thus research in this field is growing day by day and many optimization models have been developed. We will find branch currents of an electrical circuit with one unknown resistance using network equilibrium equations formed by matrices and then this graph theoretic approach will be verified by nodal analysis. Our main objective is to study the currents of an electrical circuit with the help of graph theory and matrices. This matrix method is very helpful to study the currents of electrical circuits containing many branches. In complex circuits, it is difficult to find the individual branch currents all at a time by using the existing methods of electrical circuit analysis. This difficulty can be overcome by using matrix method.

1.1. Graph: A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$ is defined by a particular relation. The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. So in a graph a vertex set is a set of points $\{x_1, x_2, x_3 \dots . x_n\}$and edge is a line which connects two points$x_i \; and \; x_j$.Such graphs are called undirected graphs. A directed graph is similar to an undirected graph except the edge set $E \in V \times V$.

1.2. **Electrical Circuits**: An electrical circuit consists of internally connected elements such as resistors, capacitors, inductors, diodes, transistors etc. The behaviour of electrical circuits generally depends upon the characteristic of each of internally connected elements and the rule by which they are connected together. This rule gives a relation between electrical circuits with graph theory.

1.3. **Kirchhoff's Current Law (KCL):** For any lumped electrical network, at any time the net sum (taking into account the orientations) of all the currents leaving any node or vertex is zero.

1.4. **Kirchhoff's Voltage Law (KVL):** For any lumped electrical network, at any time the net sum (taking into account the orientations) of the voltages around a loop (i.e. circuit) is zero.

2. From Circuit to Graph:

A graph can be obtained from a circuit. We identify the graph G= (V, E) where V is the set of vertices and E is the set of edges. The edge between i^{th} and j^{th} vertices can be denoted by {i,j} ignoring the direction. Similarly the notation (i,j) can be used for oriented edges, where i is the start vertex and j is the end vertex. In general current source and voltage source are replaced by open circuit and short circuit respectively. It may be observed that a branch may contain more than one element. The choice of branches and nodes with respect to network element is flexible.

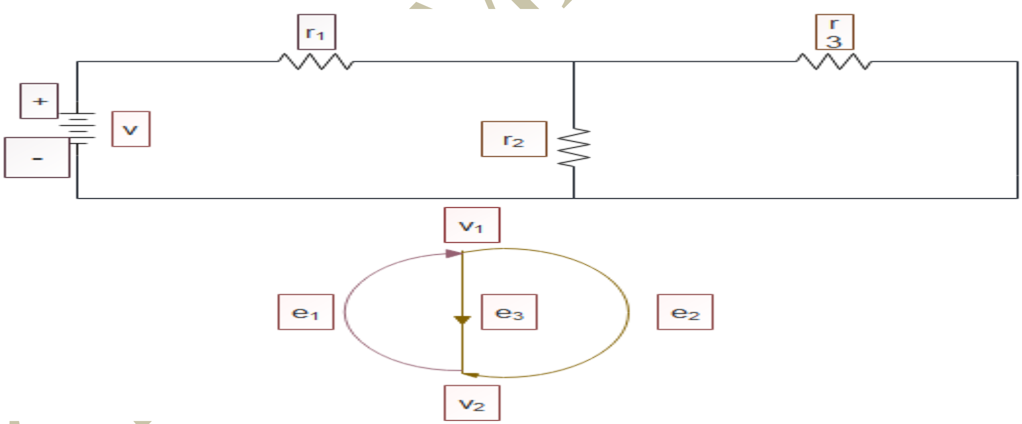

Fig.1: A circuit and its corresponding graph

2.1. Matrices Associated to a Graph:

2.1.1. Fundamental Tie Set Matrix (Fundamental Loop Matrix) :

This matrix is associated to a fundamental loop i.e. a loop formed by only one link (branch that does not belong to a particular tree) associated with other twigs (branch of trees). Here, all loops obtained from a particular tree forms the rows and all branches form the columns such that

$$b_{ij} = \begin{cases} 1, \; if \; branch \, b_j \; in \; loop \, i \; are \; in \; the \; same \; direction \\ -1, \; if \; branch \, b_j \; in \; loop \, i \; are \; in \; the \; opposite \; direction \\ 0, \; if \; branch \; b_j \; is \; not \; in \; loop \, i \end{cases}$$

2.1.2. Branch Impedance Matrix [Z_b]: It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal entry is zero.

3. Application of Graph Theory in Network Equilibrium Equations

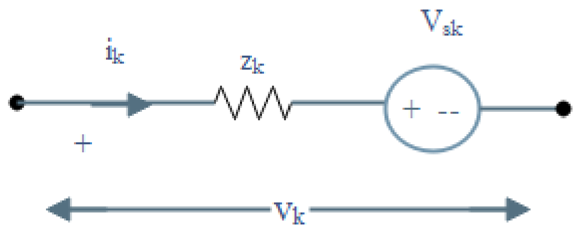

Fig. 2: Voltage source in series with resistance

If V_{SK} be the voltage source in a branch k having impedance z_k and carrying current i_k, then the branch voltage $v_k = z_k i_k + V_{SK}$

If we consider this equation for the whole circuit then matrix form of the above equation is $[V_b] = [Z_b][I_b] + [V_S]$

where $[Z_b]$ is the branch impedance matrix, $[I_b]$ is the column vector of branch currents and $[V_S]$ is the column vector of source voltage. Now Kirchhoff's Voltage law in matrix form is given by

$$[B][V_b] = 0$$

$$\Rightarrow [B]([Z_b][I_b] + [V_S]) = 0 \qquad (1)$$

$$\Rightarrow [B][Z_b][I_b] = -[B][V_S]$$

Also the branch current matrix equation is $[I_b] = [B^T][I_L]$, where $[I_L]$ is the loop current matrix and $[B^T]$ is the transpose of [B].

So, we get $$[B][Z_b][B^T][I_L] = -[B][V_S] \qquad (2)$$

We will use this equation to analyze the following circuit.

4. Finding Branch Current using Graph Theoretic Approach:

Consider the following circuit with one unknown resistance 'r'

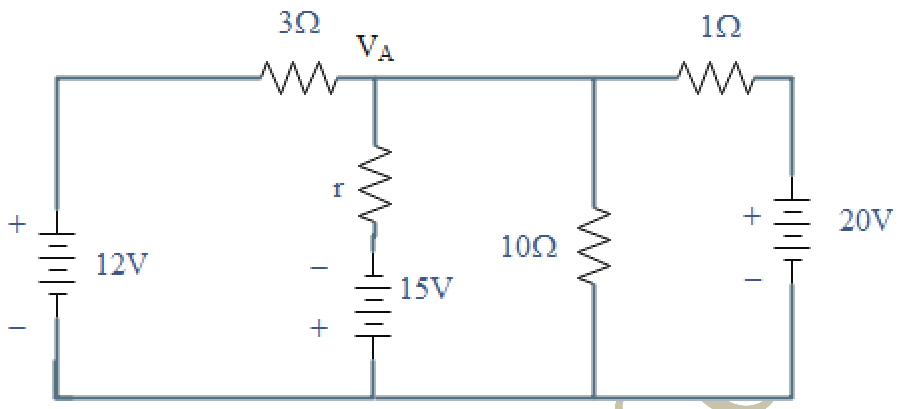

Fig. 3: A circuit with unknown resistance

The graph from the circuit is given by

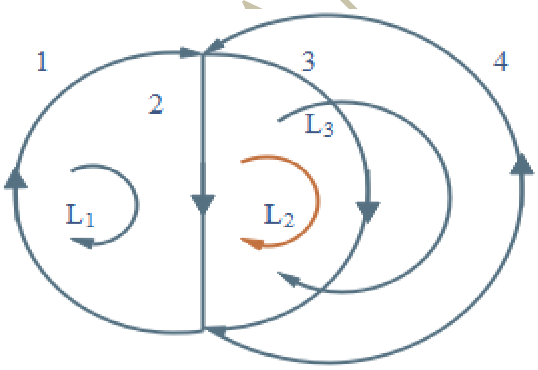

Fig. 4: Graph from Circuit

From the figure we see that the graph of the circuit contains two nodes, four branches and three loops. Using the definitions we form the following matrices

$$B=\begin{bmatrix}1 & 1 & 0 & 0\\ 0 & -1 & 1 & 0\\ 0 & -1 & 0 & -1\end{bmatrix},\quad Z_b=\begin{bmatrix}3 & 0 & 0 & 0\\ 0 & r & 0 & 0\\ 0 & 0 & 10 & 0\\ 0 & 0 & 0 & 1\end{bmatrix},$$

$$B^T=\begin{bmatrix}1 & 0 & 0\\ 1 & -1 & -1\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix},\quad I_L=\begin{bmatrix}L_1\\ L_2\\ L_3\end{bmatrix},\quad V_s=\begin{bmatrix}12\\ 15\\ 0\\ 20\end{bmatrix}$$

So putting in $[B][Z_b][B^T][I_L]=-[B][V_s]$ and simplifying we have

$$\begin{bmatrix}3+r & -r & -r\\ -r & r+10 & r\\ -r & r & r+1\end{bmatrix}\begin{bmatrix}L_1\\ L_2\\ L_3\end{bmatrix}=\begin{bmatrix}-27\\ 15\\ 35\end{bmatrix}$$

From this we find the loop currents as

$$L_1=\frac{68r-270}{43r+30}A,\ L_2=\frac{-72r+45}{43r+30}A,\ L_3=\frac{140r+1050}{43r+30}A\ ,Where\ 43r+30\neq 0$$

Again to find the individual branch currents we have $\left[I_b\right]=\left[B^T\right][I_L]$ where $\left[I_b\right]$ is the column matrix of individual branch currents, $\left[B^T\right]$ is the transpose of [B] and $[I_L]$ is the column matrix of loop currents. On simplification we have the branch currents

$$\begin{bmatrix}i_1\\ i_2\\ i_3\\ i_4\end{bmatrix}=\begin{bmatrix}\dfrac{68r-270}{43r+30}\\ \dfrac{-1365}{43r+30}\\ \dfrac{-72r+45}{43r+30}\\ \dfrac{-140r-1050}{43r+30}\end{bmatrix}\qquad (3)$$

5. Verification of the Result by Nodal Analysis:

Now we will verify the above results using nodal analysis. A node is a point where two or more branches join. In nodal analysis node potential is assumed to be higher than the other voltage source appearing in the circuit. Using this assumption current equation is formed applying KCL. If the current value is found negative after calculation then it means that at that branch node voltage is not the leading voltage.

Now in figure 2, let the nodal voltage at A be V_A and considering this as leading voltage from Kirchhoff's current law we have

$$\frac{V_A-12}{3}+\frac{V_A+15}{r}+\frac{V_A}{10}+\frac{V_A-20}{1}=0$$

$$\Rightarrow V_A=\frac{720r-450}{43r+30}$$

So the branch currents are given by

$$i_{3\Omega}=\frac{\frac{720r-450}{43r+30}-12}{3}=\frac{68r-270}{43r+30}A \qquad i_{r\Omega}=\frac{\frac{720r-450}{43r+30}+15}{r}=\frac{1365}{43r+30}A$$

$$i_{10\Omega}=\frac{\frac{720r-450}{43r+30}}{10}=\frac{72r-45}{43r+30}A \qquad i_{1\Omega}=\frac{\frac{720r-450}{43r+30}-20}{1}=\frac{-140r-1050}{43r+30}A$$

(4)

So from equation (3) and (4) we have

$i_1=i_{3\Omega},\quad i_2=i_{r\Omega},\quad i_3=i_{10\Omega},\quad i_4=i_{1\Omega}$

6. Result:

We see that individual branch currents are same in both processes. In case of i_2 and i_3 negative sign is just showing that the direction of current is opposite.

7. Conclusion:

From the above study we observed that the result obtained by Graph theoretic approach is same as the result obtained by nodal analysis in a circuit containing an unknown resistance. Thus the graph theoretic approach is verified by nodal analysis. We can construct many electrical circuits for different values of resistance 'r'. In graph theoretic approach, values of currents can be obtained simply by network equilibrium equation without knowing the nodal and mesh analysis of electrical circuits. Graph theoretic approach is very beneficial in the case when an electrical circuit contains many branches. In such cases it is laborious to use nodal analysis to find branch currents and loop currents though it is helpful in calculating currents in simple circuit. This difficulty can be solved with the help of matrix method because in this method we can easily multiply n^{th} order matrices with the help of computer programming. Thus with the help of Graph Theory and matrix, branch currents of circuits containing n loops can be obtained.

References:

[1] Alman Joshua, Lian Carl, TranBrandon(2015), Circular planar electrical networks: Posets and positivity, Journal of Combinatorial Theory, Series A 132 (2015) 58–101.

[2] Banerjee Anirban, Jost Jurgen (2008). On the spectrum of the normalized graph Laplacian, Linear Algebra and its Applications, Elsevier, 428 (2008) 3015–3022.

[3] Bapat R.B.(2005), Distance matrix and Laplacian of a tree with attached graphs , Linear Algebra and its Applications 411 (2005) 295–308, Elsevier.

[4] Bapat R.B. (2014), Graph and Matrices, Second edition, Springer, 2014.

[5] Berdewar O. K., Dr. Deo S.D (2016). Application of Graph Theory in Electrical Network, International Journal of Science and Research, Volume 5, Issue 3, March , 2016.

[6] Boruah Chanchal, Gogoi Krishna, Chutia Chandra (2017). Analysis of Some Electrical Circuits with the Help of Graph Theory Using Network Equilibrium Equations, International Journal of Innovative Research in Science,Engineering and Technology,Vol. 6, Issue 1, January 2017.

[7] Brooks R.L., Smith C.A.B., Stone A.H., Tutte W.T.(1992), Determinants and current flows in electric networks, Discrete Mathematics 100 (1992) 291-301.

[8] Bryant Randal E (1990). Symbolic Simulation—Techniques and Applications. Computer Science Department, Paper 240.

[9] Chakrabarti Abhijit. Circuit Theory Analysis and Synthesis, Dhanpat Rai & Co.

[10] Deo Narsingh. Graph Theory with Applications to Engneering and Computer Science, Phi Learning Private Limited, New Delhi.

[11] Dorfler Florian, Simpson-Porko John W, Bullo Francesc.(2018).Electrical Networks and Algebraic Graph Theory: Models, Properties and Applications, Proceedings of the IEEE, Vol. 106, No.5, May 2018.

[12] Erdos P, Meir A, SOS V.T, Turan P.((2006) ,On some applications of graph theory, Discrete Mathematics 306 853–866, Elsevier.

[13] FARY Istvan (1983), Generalized Circuits. Elements Of Mathematical Theory, Journnl of Pure and Applied Algebra 29 241-283.

[14] Fujishigi Satoru (1980), An Efficient PQ-Graph Algorithm for Solving the Graph-Realization Problem, Journal Of Computer And System Sciences 21, 63-86.

[15] Gunther M., Hoschek M. (1997), ROW methods adapted to electric circuit simulation packages, Journal of Computational and Applied Mathematics 82159-170.

[16] Gustafson Karl, Harary Frank (1985), The Curl Of Graphs And Networks, Mathematical Modelling, Vol. 6, pp 145-155.

[17] Harper L.H. (2014), Morphisms for resistive electrical networks, Discrete Applied Mathematics 163, 181–193, Elsevier.

[18] Lamba J.D., Asher G .M., Woodall D.R.(1997), Bond graphs III: Bond graphs and electrical networks, Discrete Applied Mathematics 73 (1997) 21 l-250, Elsevier.

[19] Li LIU, Xuan ZHAO(2012), Application of Improved Adjacency Matrix Multiplication in Distribution Network Flow Calculation, Energy Procedia 14(2012) 983-989.

[20] Šarga Patrik, Hroncová Darina, Curilla Miloslav, Gmiterko Alexander(2012), Simulation of electrical system using Bond Graphs and MATLAB/Simulink, Procedia Engineering 48, 656 – 664, Elsevier.

[21] Satish(2013). Introduction to Graph Theory, IJREAS Volume 3, Issue 6,June (2013), ISSN: 2249-3905.

[22] Vandewalle Joos(1980), The Coloured Branch Theorem and Its Applications in Circuit Theory, IEEE Transactions On Circuits And Systems . VOL. CAS- 27, NO 9, September.

[23] Zhang Xiaoling, Zhang Heping(2009), Some graphs determined by their spectra, Linear Algebra and its Applications , Elsevier ,431 1443–1454.