Mathematical Forum ISSN: 0972-9852
Vol.27, 2015-2019

ORIGIN OF EVOLUTION OF MATHEMATICAL CONCEPTS IN ANCIENT INDIA

Kripasindhu Chaudhuri, F. N. A. Sc. (India), F.I.M.A. (United Kingdom),
Emeritus Fellow (UGC & AICTE),
Department of Mathematics, Jadavpur University
Kolkata – 700 032, West Bengal, India
Email: chaudhuriks@gmail.com

(Invited Article)

Abstract

This article takes a brief look into the history of evolution of mathematical concepts in Ancient India and the directions in which mathematical ideas developed over the ages. The prime approach to mathematical culture in the ancient Indian civilization as compared to that in the Babylonian and Egyptian civilizations is to be reckoned with. The connection between ancient Greek and Indian civilizations in the field of Mathematics is to be briefly delineated.

Keywords: Invention of sunnya (zero); asima (infinity); integral and decimal system of numbers; Sulba-sutras; Upanishada; Chhanda-sutra; Archa Shastra; Atharva Veda, Rig Veda; Yajur Veda; Euclidean Geometry; Brahmasphuta-siddhanta; Taittiriya and Maitreyani Samhita.
2010 AMS Classification: 97A30

1. **Introduction**

It is believed that mathematical ideas developed hand-in-hand with the progress of human civilization in Ancient India. Roots of this impression lie in the *Vedas*, *Samhitas* and *Shastras* of ancient India. Mathematics seems to have occupied a prominent place from the very beginning of *Vedic* civilization. Mathematics was described as *chakshu:* (*eye of the Vedas*) in the *Vashistha Siddhanta*. The following *sloka* in *Archa Shastra* shows how much importance was given to mathematics in those days:

Yatha sikha mayuranang naganang manayo yatha

Tatbat bedanga shashtranang ganitang murdhanisthitam

In this *sloka*, mathematics has been compared with the dazzling tail of a peacock and the gem on a snake's head. It is also declared that the place of mathematics is on the head of the *Vedas* and *Shastras*.

That the scholars of those days were well acquainted with the concept of *integral numbers* or *whole numbers* is evidenced from various *shastras*. One may refer to the following *sloka* in *Rig Veda*:

Indro dadhicho asthivirbrityana pratikrista

Jaghana nabatirnaba

Indra, the king of the heaven, hit the demons ninety nine times with the bones of the sage **Dadhichi**. The integer 99 has been used here to glorify the power of **Indra**. One *sloka* in *Yajur Veda* states:

Trayastringshata stabatu bhutany ya: shamayan

Prajapati parameshthyadhipati rasit

This means that the great Lord administers all the material and living beings with the help of thirty three gods. The integer 33 has been used in this *sloka*.

The first ever census in human civilization was perhaps conducted to count the number of deities as it is revealed in the following *sloka* in *Rig Veda*:

Trini shata tri sahasranyagnim

Stringsachha deba naba chasapayrjan

This *sloka* states that 3339 deities worshipped **Agnideba**.

The four basic operations in arithmetic, namely, *addition*, *subtraction*, *multiplication* and *division*, were extensively used in the Vedic literature. Many complicated calculations involving arithmetic, geometry and astronomy are found in *Archa Jyotish* and *Sulba*-*sutras* composed by **Katyayana**; these concepts were unknown to the other parts of the world at that time. The root meaning of the word *Sulva* is to measure, and in due course, the word came to mean the rope or cord. **Katyayana** is believed to be born in the 2nd century BC some where in the Indian subcontinent but it cannot be estimated when he died. He is known mainly for two compositions. One of them is *Vartikka* which is an elaboration on **Panini** grammar. Along with Mahabhasya of **Patanjali**, this text became a core part of the *Vyakarana* canon. This was one of the six *Vedangas* and it constituted a source of compulsory education for students in the following twelve centuries. He also composed one of the later *Sulba-sutras*, a series of nine texts on the geometry of alter constructions dealing with rectangles, right-angled triangles, rhombuses, etc. Nothing definite is known about when **Panini** lived or even the century in which he lived in the Indian subcontinent. Most scholars suggest that he possibly lived in the 4th or 5th century BC. Nothing certain is known about Pāṇini's personal life. According to the Mahābhāṣya of Patanjali, his mother's name was Dākṣī. Patañjali calls Pāṇini *Dākṣīputra* (meaning *son of Dākṣī*) at several places in the Mahābhāṣya. According to some ancient sources, the name of his father was *Paṇina*, from which the name *Pāṇini* was derived.

Panini probably lived in Salatura of ancient Gandhara, which likely was near Lahur, a town at the junction of Indus and Kabul rivers. According to the memoirs of 7th-century Chinese scholar Xuanzang, there was a town called *So-lo-tu-lo* on River Indus, where Rishi Panini was born, and he composed *Chingming-lun* (Sanskrit: Vyakarana). The *Aṣṭādhyāyī* is the central part of Pāṇini's grammar, and by far the most complex. The *Ashtadhyayi* is the oldest surviving complete linguistic and grammar text of Sanskrit, and Pāṇini refers to previous texts and authors such as the *Unadisutra*, *Dhatupatha*, and *Ganapatha* some of which have not survived. It complements the Vedic ancillary sciences such as the *Niruktas*, *Nighantus*, and *Shiksha*.

Patanjali is the author of the *Mahābhāṣya*, an ancient treatise on Sanskrit grammar and linguistics, based on the *Aṣṭādhyāyī* of Pāṇini. Patañjali's life is dated to mid 2nd century BC by both Western and Indian scholars. This text was titled as a *bhasya* or "commentary" on Katyayana-Panini's work by Patanjali, but is so revered in the Hindu traditions that it is widely known simply as *Maha-bhasya* or "Great commentary". Patanjali has been the authority as the last grammarian of classical Sanskrit for more than 2,000 years, with Panini and Katyayana preceding him. Their ideas on structure, grammar and philosophy of language have also influenced scholars in other Indian religions such as Buddhism and Jainism.

Srinivasiengar [1] writes: "*Only seven of the Sulba-sutras are known at present. They are known by the names Bodhayana, Apasthamba, Katyayana, Manava,* *Maitrayana,Varaha and Vadhula after the names of the Rishis or sages who wrote them*". The concepts of *zero* and *infinity* were known to the medieval India. There is a reference to the word *Sunya* (*zero*) in *Chhanda-sutra* composed by **Pingala** in about 200 BC. This was proved by **Prof**. **G. B. Halstead** in an article [2] published in the *American Mathematical Monthly*. **Prof**. **Halstead** [3] writes that:

"*The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habitation and a name, a picture, a symbol but helpful power, is the characteristic of the Hindu race whence it sprang. It is like coining the Nirvana into dynamos. No single mathematical creation has been more potent for the general on-go of intelligence and power."* However, the arithmetical use of the number *zero* started much later. It is an established fact that the ancient Greeks and Romans were not aware of the number *zero*.

Pingala is the traditional name of the author of the ***Chhanda-sastra*** (also known as *Chhanda-sutra*), the earliest known Sanskrit treatise on prosody.

Little is known about Pingala himself. In later Indian literary tradition, he is variously identified either as the younger brother of Pāṇini or Patañjali.

The *Chhanda-sutra* is a work of eight chapters in the late Sūtra style, not fully comprehensible without a commentary. It has been dated to either the final centuries BC or the early centuries AD, at the transition between Vedic meter and the classical meter of the Sanskrit epics. This would place it close to the beginning of the Common Era, likely post-dating Mauryan times. The 10th century AD mathematician Halayudha wrote a commentary on the *Chanda-sutra* and expanded it. **Halayudha** was a 10th-century Indian mathematician who wrote the ***Mṛtasañjīvanī***, a commentary on Pingala's *Chhanda-śāstra*. The latter contains a clear description of Pascal's triangle (called *meru-prastaara*). Halayudha originally resided at the Rashtrakuta capital Manyakheta, where he wrote under the patronage of emperor Krishna III. His *Kavi-Rahasya* eulogizes Krishna III. Later, he migrated to Ujjain in the Paramara kingdom. There, he composed *Mṛta-Sañjīvanī* in honour of the Paramara king Munja.

Basham [4] remarked:

> *"The mathematical implications of zero and infinity were fully understood in medieval India. The unknown man who devised zero and decimal system was from the world point of view, after Buddha, the most important son of India.*

> *His achievement was the work of an analytical mind of the first order and he deserves much more honour than he has, so far, received".*

Bertrand Russel [5], on the other hand, writes:

> "*As for 0*, *it is a very recent addition*; *even the Greeks and Romans had no such digit*".

The concept of *asima* (infinity) was deeply imbedded in the Indian philosophy since the dawn of Vedic civilization. The Vedantic meaning of *infinity* was beautifully described in a famous *sloka* of *Rig Veda* stated below:

> *Oong purnamada: purnamidam purnat purnamudachhayte*
>
> *Purnasya purnamadaya purnamebabashishyate*

This is the Whole; that is the Whole; the Whole evolves from the Whole. If the Whole is taken out from the Whole, the Whole remains. Here the *Whole* stands for *infinity* and symbolizes the ultimate Lord the **Brahamma**.

The confluence of the concepts of *zero* and *infinity* has taken place in the following *sloka* of *Kathopanishada*:

> *Yada sarbe pramuchayante kama jechhasya hridi srita:*
>
> *Atha martyohamrito bhabatyatra brahama samashnute*

When the mind of a human being gets free from all the earthly passions, the mortal soul unifies with **Lord Brahmma**. This means that a human being having *zero* desire achieves *infinite* realization of mind.

The following *sloka* in *Upanishada* has unified the *God* and the *soul* in a surprisingly mathematical connotation:

> *Balagrashatabhagasya shatadha kalpitasya cha*
>
> *Bhago jiba: sa bingeya: sa chanantaya kalpate*

The soul is like that infinitesimal portion which is obtained by dividing the tip of a hair into ten thousand parts and again it is where the God resides.

2. Mathematics in Ancient Times

In the ancient days, culture of mathematics started in two distinct directions: (1) *geometric approach* and (2) *arithmetical* and *algebraic* approach. In the pre-Greek period, both these approaches were followed in India. On the other hand, arithmetical approach was mainly practiced in the Babylonian and Egyptian cultures. In the *sulba-sutras* of **Baudhayana**, **Apastamba** and **Katyana**, one finds the mathematical ideas followed in the construction of various *Vedic* altars. The **Baudhayana sūtras** are a group of Vedic Sanskrit texts which cover dharma, daily ritual, mathematics, etc. They belong to the *Taittiriya* branch of the Krishna Yajurveda school and are among the earliest texts of the *sutra* genre, perhaps compiled in the 8th to 7th centuries BC. ***Āpastamba Dharmasūtra*** is a Sanskrit text and one of the oldest Dharma-related texts of Hinduism that have survived into the modern age from the 1st-millennium BCE. It is one of three extant Dharmasutras texts from the Taittiriya school of Krishna Yajurveda, the other two being *Baudhayana Dharmasutra* and *Hiranyakesin Dharmasutra*. Katyana was born in the Vedic period of estimated 2nd century BC somewhere in the Indian sub-continent. He was mainly a Sanskrit grammarian having interest in mathematics. His notable works include Vartikka, Vyakarana and later sulba-sutras.

The statement and also an outline of the proof of the famous theorem of **Pythagoras** are found in the *sulba-sutras*. At that time, a fair knowledge about the various properties of similar geometrical figures like triangle, rectangle, square, trapezium and circle existed in India. Many theories of the *Euclidean geometry* were known to the Indians a few centuries ahead of **Euclid** and **Pythagoras**. There are ample evidences to believe that the Indians at that time also possessed much computational skill. *Mensuration*, *fractions*, *surds*, etc., were used in many applications; determination of values of *surds* up to a desired level of accuracy and determination of *square roots* and *cube roots* of real numbers were discussed those days by the Indian mathematicians. The Greeks made significant contribution to mathematics in the era spanning from 600 BC to 300 AD. Axiomatic approach took shape, *Euclidean geometry* was born and studies in *trigonometry* and *number theory* were initiated in Greece during this period. **Pythagoras**, **Euclid** and **Archemedes** (287-212 BC) were the three great scholars of contemporary Greece. **Pythagoras** was born in about 570 BC in the Greek island of Samos. He was taught mathematics by Thales who brought mathematics to the Greeks from ancient Egypt. **Euclid** is the most prominent mathematician of antiquity best known for his treatise on mathematics *The Elements*.

The long lasting nature of *The Elements* makes Euclid the leading mathematician of all times. However, very little is known of Euclid's life except that he taught at Alexandria in Egypt. **Archimedes** was probably born in 287 BC in Syracuse, Sicily and died in 212 BC. He probably spent some time in Egypt early in his career but he resided for most of his life in Syracuse, the principal Greek city-state in ancient Sicily. He is considered to be the most famous mathematician and inventor in ancient Greece. After this golden Greek period, creative thinking in mathematics virtually stopped in the whole of Europe and this dark period of European culture continued for a number of centuries until this darkness was lightened by the ray of modern mathematics in the seventeenth century. On the other hand, the contribution of Indians in mathematics continued unabated from the very beginning of civilization up to the sixteenth century, especially in the fields of arithmetic, algebra and trigonometry.

The *decimal* system was a priceless gift of ancient India to the world. It is an incomparable example of combining abstract thought with utilitarian approach. The real power of the decimal system lies in the concept of *place-values* of digits in a number and also in the notion of '*zero'* as a number. The decimal system of numeration was found in the excavations of the *Mohenjo Daro* and *Harappa* civilizations. **Srinivasiengar** [1] writes: "*Writers on the Mohenjo Daro and Harappa civilizations which may be of the period 3000 BC or so, briefly refer to the decimal system of numeration found in those excavations".* It is recognized that the ancient Indian mathematicians could master many arithmetical concepts involving *fractions, surds*, *ratio-proportion*, *rule of three*, *square root*, *cube root*, etc., due to their knowledge of the decimal system. Mathematicians of the Vedic era had a fascination of handling large numbers. The following powers of the number 10 were named in *Taittiriya Samhita* of *Yajur Veda***:**

10^0 =ek

10^1=dasha

10^2=shata, 10^3=Sahasrha,

10^4=Ayuta, 10^5=Niyuta,

10^6= Prayuta, 10^7=Arbuda,

10^8=Nyarbuda, 10^9=Samudra,

10^{10}=Madhya, 10^{11}=Anta,

10^{12}=Parardha, 10^{13}=Usas,

10^{14}=Vyusti, 10^{15}=Desyat,

10^{16}=Udyat, 10^{17}=Udit,

10^{18}=Suvarga, 10^{19}=Vaher

The *Maitreyani Samhita* and other sections of the *Vedas* give slight variations. However, according to the later Sanskrit literatures, *arbuda* means 10^8 and not 10^7. Jaina works use *Eka*, *Dasa*, *Shata*, *Sahasra*, *Dasa Sahasra*, *Dasa Sahata Sahasra*, *Koti*, *Dasa Koti*, etc.

A number as large as 10^{55} finds mention in the *Ramayana*. The number 10^8 was used in the following *sloka* in *Atharva Veda*:

Brihate jalang brihata indra shura

Sahasrardhasya shatabirjasya

Tena shatang sahasramajutang nayarbudang

Jaghana shatro dasyunamabhidaya senaya

The great fighter **Indra** killed *shata* (10^2), *sahasra* (10^3), *ayut* (10^4) and *nyarbud* (10^8) demons. Large numbers were also used in the contemporary Jain and Buddhist literatures. The Jaina religious works, dating from 500 BC to about 100 BC, use large numbers in the decimal system. In the Buddhistic work entitled *Lalita Vistara* of the first century BC, **Buddha (Bodhi Satva)** enumerates to a mathematician Arjuna the system of numerals in multiples of 100, starting from *Koti* (10^7) up to 10^{53}. No evidence of such large numbers is found in the other parts of the world at that time. The Greeks did not use any number larger than *ayut* (10^4) and the greatest number used by Romans was *sahasra* (10^3). The concept of the decimal system enabled the Indians to deal with the large numbers. The Arabs learnt the decimal system from the Hindus, and Europe had it from the Arabs. The knowledge of the decimal system created a great impact in trade and commerce and it is believed to play a key role in the European renaissance. **Swami Vivekananda** described invention of the decimal system to be "*the very cornerstone of all modern civilizations*".

Algebra began to evolve in India as a separate branch of mathematics from the days of Aryabhata. Indian mathematicians started using mathematical symbols to deal with unknown quantities with the help of the *four basic operations* of Arithmetic. Although some uses of mathematical symbols are found in the book of **Diophantus** in 275 AD, algebraic theories were developed to a great extent in India only. The use of mathematical symbols has revolutionized the progress of mathematics. The solution of linear Diophantine equation was first discussed in the book *Brahmasphuta-siddhanta* written by **Brahmagupta** in 628 AD. In this connection, **Boyer** [6] writes "..... *One admires his mathematical attitude even more first one finds that apparently he was the first one to give a general solution of the linear Diophantine equation ax+by = c where a, b, c, are integers".*

The Indians developed many algebraic theories regarding surds, arithmetic and geometric progressions, equations, permutation and combination. The Hindu mathematician **Sridhara Acharya** was probably born in the 8^{th} century AD and the year of his death is not known. **Sridhara** was a mathematician, Sanskrit pandit and philosopher. He was born in Bhurishresti (Bhurisristi or Bhurshut) village in South Radha (at present day Hughli) in the 8th Century AD. His father Baladev Acharya was a Sanskrit pandit (scholar).

Sridhara discussed the solution of a quadratic equation $ax^2 + bx + c = 0$. Development of algebra reached its peak in India with the discussion of the methods for solving linear and quadratic indeterminate equations. He contributed a lot in mathematics. He was known for two treatises: *Trisatika* (also called the *Patiganitasara*) and the *Patiganita*. His major work *Patiganitasara* was named *Trisatika* because it was written in three hundred slokas. The book discusses counting of numbers, measures, natural number, multiplication, division, zero, squares, cubes, fraction, rule of three, interest- calculation, joint business or partnership and mensuration. He gave an exposition on the zero. He wrote, "*If zero is added to any number, the sum is the same number; if zero is subtracted from any number, the number remains unchanged; if zero is multiplied by any number, the product is zero*". However, his exposition about division by zero is incorrect.

Although studies in trigonometry were initiated by the Greeks, they could not master expertise in this field due to their poor knowledge about algebraic processes. On the other hand, Indians showed their skill in this area by proving many of the identities and formulas that are used in modern trigonometry; they were aware of the *sine* function and computed the *sine table* with a great degree of accuracy. Madhavacharya (1340-1425 AD) exhibited great expertise in trigonometry; he found the expansions of some trigonometric functions in power series. The expansion of $tan^{-1}x$ in an infinite power series of x was derived by the Scottish mathematician **James Gregory** in 1667 and it is known as *Gregory series*. **James Gregory** (also spelled **James Gregorie**)

FRS (1638 –1675 AD) was a Scottish mathematician and **astronomer**. However, **Madhavacharyya** discussed a similar series three hundred years before him. He also computed the value of π correct to eleven decimal places and found the infinite power series expansions of *sine x* and *cosine x* three centuries before **Newton** (1642-1727 AD) did it. He used these series expansions to compute *sine* and *cosine* tables, the values of which are seen to be correct up to eight decimal places.

3. Mathematicians Of Ancient India

The first of the great mathematicians in ancient India appears to be the older **ARYABHATA** or **ARYABHATA I** (476-550 AD) ([7], [8], [9], [10]). He was the first in the line of the major mathematician-astronomers in ancient India. The two most famous authoritative compositions of Aryabhata are *Aryabhatiya* and *Arya-Siddhanta*. It is mentioned in Aryabhatiya that it was composed 3,630 years into the *Kali Yuga* when his age was 23 years [10]. This corresponds to 499 AD which implies that he was born in 476 AD. According to Aryabhata, the Kali Yuga started in 3102 BC. He finished writing his treatise Aryabhatiya in 499 AD in which he gives the exact year of the beginning of Kali Yuga. According to him, he wrote the treatise in the "year 3600 of the Kali Yuga" when he was 23 years old [11]. Accepting that Aryabhata was born in 476 AD, the beginning of Kali Yuga comes out to [3600 – (476 + 23) + 1] = 3102 BC as only one year elapses between 1 BC and 1 AD. Kali Yuga actually means "age of the demon Kaali" or "age of vice" and it is the last of four stages the world goes through as part of the cycle of Yuga described in the Sanskrit scripts within the present Mahayuga. The other three *yugas* are called *Satya Yuga*, *Treta Yuga* and *Dvapar (Dwapar) Yuga*. He was probably born in the present day Kodungallur which happened to be the historical capital city of *Thiruvanchikkulam* of ancient Kerala according to some archeological evidence. Aryabhata's works do not provide any information about his place of birth. Bhaskara I describes Aryabhata as *asmakiya* belonging to the *asmaka* region. During the time of Lord Bhuddha, a branch of the *asmaka* people settled in the region lying between Narmada and Godavari rivers in Central India. Aryabhata is believed to have been born there ([11], [12]). *Asmaka* actually means "*stone*" in Sanskrit language. It is believed that the present day Kodungallur is actually the ancient place *kotunallur* known as Kotum-Kal-l-ur which means the "*city of hard stones*"; however, old records suggest that the city was actually known as Kotum-kol-ur which means the "*city of strict governance*". The fact that many commentaries on the composition

Aryabhatiya originated from Kerala has been used to suggest that Kerala was the main place of Aryabhata's life and activity. On the contrary, several other commentaries on Aryabhatiya came from outside Kerala and most significantly, the composition Arya-Siddhanta is believed to be completely unknown in ancient Kerala [13]. There is an alternative opinion about the place of birth of Aryabhata. It is believed that he was born in *Taregna*, a small town in Bihar, about 30 kilometers away from Patna which was known as *Pataliputra* in ancient times. Aryabhata set up an Astronomical Observatory in the Sun Temple in Taregna. It is fairly certain that he had been to *Kusumapura* for advanced studies and he lived there for some time. Kusumapura is identified as Pataliputra by Bhaskara I (629 AD) and also by Hindu and Buddhist traditions. It is also speculated that Aryabhata might have been the head of an institution (*kulapa*) at Kusumapura and since Nalanda University was at Pataliputra at that point of time, it is believed that he was the head of this university as well [13].

Aryabhata is believed to be the author of several treatises on mathematics and astronomy, some of which are lost. His major work Aryabhatiya had been extensively referred to in the Indian mathematical-astronomical literature and even it survives in modern times. The mathematical part of Aryabhatiya deals with arithmetic, algebra, plane trigonometry and spherical trigonometry. It has 108 verses and 13 introductory verses which are divided into the following four *padas* or chapters:

1. *Gitikapada* (13 verses) dealing with large units of time (*kalpa*, *manvantra*, *yoga*) which describe a cosmology different from earlier texts such as Lagadha's *Vedanga Jyotisha* (1st century BC). A table of sines (*jya*) is also given in a single verse. The duration of the planetary revolutions during a *mahayuga* is estimated to be 4.32 million years.
2. *Ganitapada* (33 verses) covers mensuration (*Ksetra vyavahara*), arithmetic and geometric progressions, gnomon / shadow (*shanku-chhaya*), simple, quadratic, simultaneous and indeterminate equations (*kuttaka*).
3. *Kalakriyapada* (25 verses) dealing with different units of time and a method for determining the positions of planets for a given day, calculations concerning the intercalary month (*adhikamasa*), *Kshaya-tithis*, and a seven day week with names of the days.

4. *Golapada* (50 verses) covering geometric / trigonometric aspects of the celestial sphere, features of the ecliptic, celestial equator, node and shape of the earth, cause of day and night, rising of zodiacal signs on horizon, etc. In addition, some versions cite a few colophons added at the end, extolling the virtues of the work, etc.

The *Arya-siddhanta*, a work on astronomical computations, is known through the compositions of Varahamihira, Aryabhata's contemporary, and later through several mathematicians and commentators including Brahmagupta and Bhaskara I. It contains a description of several astronomical instruments also. **Varahamihira** (505-587 AD) was an Indian astronomer, mathematician and astrologer who was born and lived in Ujjain. He was born in the Avanti region, roughly corresponding to modern-day Malwa. His father Adityadasa was himself an astronomer. **Brahmagupta**, born in the city of Bhinmal in Northwest India, was an ancient Indian astronomer and mathematician who lived probably from 597 AD to 668 AD. His father, whose name was Jishnugupta, was an astrologer. **Bhaskara** (600-680 AD), also called **Bhaskara I** to avoid confusion with the 12th century mathematician **Bhaskara II**, was the first to write numbers in the Hindu decimal system with a circle for *zero* and gave a unique and remarkable approximation of the sine function in his commentary on Aryabhata's work.

The Khwarezmian Iranian scholar and polymath **Abu Rayhan Muhmmad Al-Biruni** whose life span covers the period from 4/5 September 973 to 13 December 1048 AD, briefly known as Al-Biruni, is regarded as one of the greatest scholars of the medieval Islamic era well versed in mathematics, physics, astronomy and natural sciences who distinguished himself as a historian, chronologist and linguist. He travelled to the Indian subcontinent and authored *Tarikh Al-Hind* (History of India) after exploring the Hindu faith practiced in India.

Al-Biruni mentioned two mathematicians of the same name **Aryabhata**. It is a matter of controversy not yet settled and the date of the younger **Aryabhata** is not yet settled. However, according to reliable historical sources, **Aryabhata II** (920-1000 AD) was an eminent ancient Indian mathematician and astronomer who was the author of the book *Maha-Siddhanta*. The treatise consists of eighteen chapters written in the form of Sanskrit *verses*. The initial twelve chapters deal with the topics related to mathematical astronomy and covers the topics longitudes of the planets, lunar and solar eclipses, estimation of eclipses, the lunar crescent, the rising and setting of the planets, association of the planets with each other and with the stars. The next six chapters of the book include topics such as geometry, geography and algebra which were applied to calculate the longitudes of the planes. In about twenty verses in the treatise, he discusses elaborate rules to solve the indeterminate equation $by = ax + c$. These rules have been applied to a number of different cases such as when $c>0$ and $c<0$, etc., when the number of quotients is an even number, when this number is an odd number, etc. He also played a vital role by constructing a *sine* table which was accurate to five decimal places.

There were two astronomers of the same name **Varahamihira**. The younger **Varahamihira** lived in the 2nd century AD and his details are unknown. The elder **Varahamihira** is the most celebrated of all the writers on astronomy in early India who was born in a family of Brahmins settled at Kapittha, a village near Ujjain. His exact date of birth could not be determined. However, one can infer his date of birth from a reference in Al Beruni's work. **Al Biruni** wrote in 1030 AD that the famous book *Panchasiddhantika* of **Varahamihira** was written 526 years prior to him; this implies that *Panchasiddhantika* was written in 504 AD Taking the age of **Varahamihira** to be 25 years at that time, his year of birth is round about 479 AD**. In a commentary on the book *Khandakhadyaka* (665 AD**) of **Brahmagupta**, **Amaraja** writes

> *Nabadhika: panchashatasankhyashake barahamihiracharyo*
>
> *dibang gata:*

This means that **Acharya Varahamihra** passed away in *Saka* 509 (587 AD) at the age of 108 years. The Gupta period is considered to be the golden age in ancient India in respect of scientific and cultural developments which were greatly encouraged and patronized by the benevolent Gupta empires. Almost all the mathematical developments and their applications up to the medieval period were either the achievements of Gupta period or the genesis of that period. A school of mathematics was established at Ujjain under the patronage of the Gupta Emperor **Vikramaditya** under the leadership of **Varahamihira**. **Boyer** [6] writes:

"Varahamihira was one among the Navaratnas (nine gems) of Vikramaditya. A school of mathematics was originated and flourished at Ujjain. It is also Varahamihir school. Pataliputra school of mathematics started its decline by this time. The Ujjain school flourished from the time of Varahamihira to the time of Bhaskar II. Varahamihira, Kalyanbarman, Brahmagupta, Mahaveeracharya and Bhaskar II are the prominent scholars of this school".

Although **Varahamihira** wrote on many branches of sciences, his book *Pancha Siddhantika* is considered to be the best treatise on astronomy written at that time. Three Greek Siddhantas - *Romaka*, *Paulisha* and *Surya*, were incorporated in *Pancha Siddhantika*. In this treatise, **Varahamihira** not only presented new theories but also suggested modifications of the theories on astronomy that existed then. The most prominent Hindu mathematician of 7th century was Brahmagupta who was born in 598 AC near Ujjain and died after 665 AD. He also worked in the Varahamihir school of mathematics at Ujjain. At the age of thirty, he wrote the astronomical treatise entitled *Brahmasiddhanta* (also called the *Brahma-sphuta-siddhanta* meaning "Doctrine of Brahma" written in 628 AD) consisting of 21 chapters; there were two special chapters in it, called *Ganitadhaya* and *Kutakhadyaka*. *Ganitadhya* begins with a definition of *ganaka*, that is, a calculator capable of studying astronomy. *Kutakhadhyaka* applies algebra to astronomical calculations. He also wrote another treatise named *Khandakhadyaka* meaning "Edible Bites" in 665 AD. **Brahmagupta** was also interested in the solution of indeterminate equations which were earlier studied by the elder **Aryabhata**. He also considered the equation $x^2 = 1 + py^2$ which is known as the *Pell equation* by the name of **John Pell** (1611-1685 AD).

Another great Jaina mathematician of the era 9th century from Bihar was **Mahavira** or **Mahaviracharya** who wrote the *Ganita-sara-sangraha* in 850 AD which revised the book Brahma-sphuta-siddhanta. He was a member in the court of **Amoghavarsha Nrupathunga** (800-878 AD), one of the greatest Rashtrakuta monarchs, who ruled over the region which later came to be known as the kingdom of Mysore. In *Ganita-sara-sangraha*, **Mahavira** tried to improve upon the works of his predecessors. We have already referred to some contributions of **Mahavira** in trigonometry. In arithmetic, he first discussed multiplication and then considered in order the topics of division, squaring, square root, cubing, cube root and the summation of series. In dealing with series, he considered arithmetic and geometric progressions and also *Vyutkalita*, that is, the summation of a series after a certain number of initial terms (*ista*) is cut off, a theory which drew the attention of **Aryabhatta II** also. He also worked with fractions, quadratic and radical equations, measurement of areas, sphere, etc. The work of **Mahavira** is perhaps the most noteworthy of the Hindu contributions to mathematics, possibly excepting that of **Bhaskara II** who lived three centuries later.

The order of the topics discussed in Sridhara’s book *Trisatika* bears very close resemblance to the one followed later by **Bhaskara II** in his book *Lilavati*. Bhaskara II was aware of Sridhara's work as he himself admitted in his book *Bija-Ganita.*

Bhaskara II (1114-1185 AD), another great Hindu mathematician, was born at *Bidur* in Karnataka, but worked at Ujjain. In his works, he mainly dealt with astronomy, Arithmetic, mensuration and algebra. His most celebrated work is the *Lilavati*, a treatise based upon Sridhara's *Trisatika* and relating to arithmetic and mensuration. It is said that **Lilabati** was the name of Bhaskara's daughter who could never marry due to astrological prohibitions. To console her daughter **Lilavati**, **Bhaskara II** wrote a book in her name and predicted that the book would acquire eternal fame. The book *Lilabati* contains notations, operations with integers and fractions, *rule of three*, the most common commercial rules, interest, series, alligation, permutation, combination, mensuration and a little algebra. **Bhaskara II** also wrote the *Bija-Ganita*, a work on algebra. This book contains discussions on directed numbers, imaginary quantities, surds, irrationals, simple and quadratic equations. A third work of importance written by **Bhaskara II** is the *Siddhanta Siromani* which contains *Goladhia* (theory of sphere).

Mathematics played an important role in the religious works of the Hindus. Likewise, it was important in the development of the Jaina religion. **Srinivasiengar** [1] writes:

“*The Jainas went to the extent of regarding mathematics as an integral part of their religion. A section of their religious literature was named Ganitanuyoga (literally, the system of calculation). Mahavira, the founder of the Jaina religion, was well versed in mathematics”.*

With this brief introduction to the ancient Indian mathematics, this article may be aptly concluded by quoting **Sri Aurovindo Ghose (**15 August 1872 – 5 December 1950**)** about the surprising development of knowledge in ancient India. **Sri Aurovindo** writes [14]:

“*The mere mass of intellectual production during the period from Asoka well into Mahomedan epoch is something truly prodigious, as can be seen at once if one studies the account which recent scholarship gives of it, and we must remember that that scholarship as yet deals with a fraction of which is still lying extant and what is the extant is only a small percentage of what was once written and known. There is* *no historical parallel for such an intellectual labour and activity before the invention of printing and the facilities of modern science."*

References

[1] C. N. Srinivasiengar (1967) *The History of Ancient Indian Mathematics* (The World Press Private Ltd., Calcutta, India).

[2] G. B. Halstead (1926) "Rational Geometry", *American Mathematical Monthly*, Vol.33, pp. 449 - 454.

[3] G. B. Halstead (1912) *On the Foundation and Technique of Arithmetic*, p.20 (Chicago, USA).

[4] A. L. Basham (1962) *The Wonder that was India* (Grove Press, Inc., New York, USA).

[5] Bertrand Russel (1952) *Introduction to the Mathematical Philosophy* (New Haven, Yale University Press, USA).

[6] C. B. Boyer (1968) *A History of Mathematics* (John Wiley & Sons, USA).

[7] "Aryabhata the Elder". http://www-history.mcs.st-andrews.ac.uk.

[8] Brittanica Educational Publishing (15 August 2010) *The Brittanica Guide to Numbers and Measurement*, The Rosen Publishing Group, pp. 97, ISBN 978-1-61530-218-5.

[9] B. S. Yadav (28 October 2010) Ancient Indian Leaps into Mathematics, Springer, pp. 88.

[10] Heidi Roupp (1997) *Teaching World History: A Resource Book*, M.E. Sharpe, pp. 112.

[11] K. V. Sharma (2001) "Aryabhata: His name, time and provenance", *Indian Journal of the History of Science*, **36(4)**: 105-115.

[12] S. M. R. Ansari (1977) "Aryabhata I, His Life and His Contributions", *Bulletin of the Astronomical Society of India*, **5(1)**:10-18.

[13] Bibhutibhusan Dutta and Avadesh Narayan Sing (1962) *History of Hindu Mathematics*, Asia Publishing House, Bombay.

[14] Sri Aurovindo (1971) *The Foundation of Indian Culture* (Sri Aurovindo Ashram, Pondicherry, India).