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EFFECTS OF VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY ON FREE CONVECTIVE MHD FLUID FLOW OVER A STRETCHING SHEET

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Abstract

A numerical investigation has been made to discuss the free convective heat and mass transfer over a stretching vertical surface in a steady two dimensional hydromagnetic fluid flow in porous medium. The non-linear governing partial differential equations are transformed into ordinary differential equations using suitable similarity transformations and they are solved numerically by fourth order Runge-Kutta shooting method. The effects of different parameters such as magnetic field parameter, Prandtl number, Schimdt number, radiation parameter, viscosity parameter, thermal conductivity parameter etc. on velocity, temperature and species concentration are shown in graphs and discussed.

Keywords: MHD, variable viscosity, variable thermal conductivity, free convection, heat and mass transfer.

2010 AMS classification: 76M25

1. Introduction

The analysis of the nature of flow in the boundary layer near a stretching sheet has utmost importance in fluid dynamics. Heat transfer and mass transfer also occurs in a number of engineering processes such as metallurgy, glass blowing, polymer processing, paper production, drying of papers and textiles, extrusion of plastic and rubber sheets etc. Magnetohydrodynamics has vast applications in the area of space and astrophysical plasmas. The application of magnetic field on a moving fluid provides a rich variety of phenomena which are also associated with many engineering processes.

Many researchers have studied various aspects of the problems associated with heat and mass transfer, stretching and shrinking sheet, magnetic field etc. Erickson *et al*. (1966)studied heat and mass transfer over a moving continuous surface considering suction/injection effects. Pal (2009)observed the effects of different parameters on velocity, temperature and species concentration profiles in stagnation-point flow over a stretching surface with buoyancy force and thermal radiation. Chen (2004)studied heat and mass transfer in MHD free convective flow on a vertical surface with Ohmic heating and viscous dissipation. Mansour *et al*. (2008) studied the effects of chemical reaction and thermal stratification on MHD free convective heat and mass transfer over a vertical stretching surface in a porous medium. Rashidi*et al*. (2012)investigated analytically a free convective flow of a visco-elastic fluid over a moving stretching surface. Turkyilmazoglu (2011, 2011) analyzed heat and mass transfer of viscoelastic MHD fluid flow over stretching and shrinking surfaces.

Mabood, Khan and Ismail (2014) studied the MHD flow over exponential radiating stretching sheet. Flow over exponentially stretching sheet through porous medium with source/sink was studied by Swain *et al*. (2015). MHD effects on heat transfer over stretching sheet embedded in porous medium with variable viscosity, viscous dissipation and heat source/sink was studied by Dessie and Kishan (2014). Rashidi*et al*. (2014) studied the free convective heat and mass transfer for MHD fluid flow over a vertical stretching sheet in presence of radiation and buoyancy effects. Chaudhary and Chaudhary (2016) examined the heat and mass transfer of MHD flow near a stagnation point over a stretching or shrinking sheet in porous medium. Hayat *et al*. (2016) studied the unsteady MHD flow over exponentially stretching sheet with slip conditions. Mistikawy (2018) studied heat transfer in MHD flow due to a linearly stretching sheet with induced magnetic field.

The main aim of this paper is to study the effects of the temperature dependent viscosity and thermal conductivity on velocity, temperature and species concentration distributions on a steady magnetohydrodynamic fluid flow over a stretching sheet in the presence of buoyancy forces and radiation. The governing equations are solved by developing suitable programming codes in MATLAB for fourth order Runge-Kutta shooting method.

2. Mathematical Formulation

Let us consider a steady laminar two-dimensional boundary layer flow of an incompressible electrically conducting fluid over a permeable stretching sheet (Fig.1). It is assumed that the stretching velocity is of the form $u_w(x)=c(x)^{1/3}$ (two equal and opposite forces are applied along the x – axis with the fixed origin) where c is a constant. The flow is induced due to stretching. A magnetic field of non-uniform strength $B(x)=B_0(x)^{-1/3}$ is applied in the transverse direction of the flow. All the fluid properties are considered as constant except viscosity and thermal conductivity. The induced magnetic field is neglected in comparison with the applied magnetic field and the viscous dissipation is small.

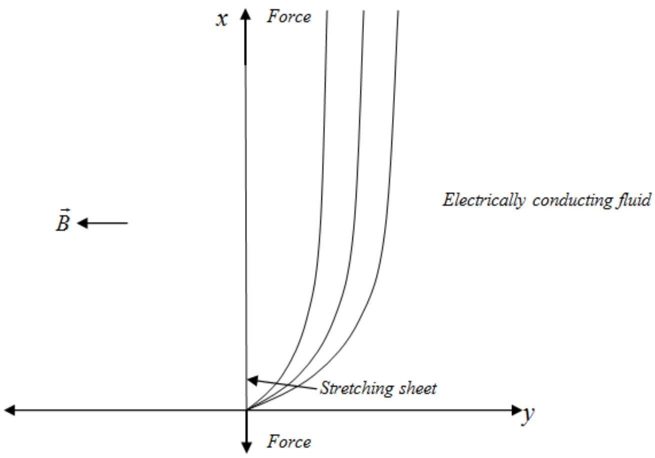

Fig.1: Flow configuration

Using above assumptions together with the Boussinesq and the boundary layer approximations, we get the governing equations as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{1}{\rho}\frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y}-\frac{\sigma B^2(x)u}{\rho}+ g\left\{\beta_T\left(T-T_\infty\right)+\beta_C\left(T-T_\infty\right)\right\} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial \lambda}{\partial y}\frac{\partial T}{\partial y}+\frac{\lambda}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty{}^3}{3\rho C_p \kappa_1}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{\partial D_m}{\partial y}\frac{\partial C}{\partial y} \tag{4}$$

Boundary conditions are:

$$\left.\begin{array}{l} y=0\ :\ u=u_w(x),\ v=v_w,\ -\lambda\dfrac{\partial T}{\partial y}=h_f(x)\left(T_w-T\right),\ C=C_w \\ y\to\infty:\ u\to 0,\ T\to T_\infty,\ C\to C_\infty \end{array}\right\} \tag{5}$$

where u and v are the components of velocity in x and y direction, i.e., along and normal to the surface respectively, υ is the kinematic viscosity, μ is the viscosity of the fluid, σ is the electric conductivity, ρ is the fluid density, g is the acceleration due to gravity, β_T is the coefficient of thermal expansion, β_C is the volumetric coefficient of expansion with species concentration, T is the fluid temperature, C is the fluid concentration, C_p is the specific heat at constant pressure, λ is the thermal conductivity, σ^* is the Stefan-Boltzman constant, κ_1 is the mean absorption coefficient and D_m is the coefficient of mass diffusivity.

The last term of the equation (3) refers to the radiation parameter. Here, Rosseland diffusion model has been considered, where the radiation heat flux q_r is expressed for radiation heat transfer as $q_r=-\frac{4\sigma^*}{3\kappa_1}\frac{\partial T^4}{\partial y}$. Using the Taylor's series, T^4 may be expressed as a linear function of T and expanding it about T_∞ and neglecting the higher order terms, we get $T^4 \cong 4T_\infty{}^3 T-3T_\infty^4$. T_∞ is the free stream temperature.

The viscosity of the fluid is assumed to be an inverse linear function of temperature (Lai and Kulacki (1990)) as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma\left(\overline{T}-\overline{T}_\infty\right)\right] \tag{6}$$

Or,
$$\frac{1}{\mu}=b\left(\overline{T}-\overline{T}_r\right) \tag{7}$$

Here, $b=\frac{\gamma}{\mu_\infty}$ and $\overline{T}_r=\overline{T}_\infty-\frac{1}{\gamma}$, $b<0$ for gas, $b>0$ for liquid. μ_∞ is the free stream viscosity. b and $\overline{T}_\infty$ are constants and their values depend on the reference state and thermal property of the fluid. $\overline{T}_r$ is transformed reference temperature related to viscosity parameter, γ is a constant based on thermal property of the fluid.

The variation of thermal conductivity is assumed as:

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi\left(\overline{T}-\overline{T}_\infty\right)\right] \tag{8}$$

Or,
$$\frac{1}{\lambda}=d\left(\overline{T}-\overline{T}_k\right) \tag{9}$$

Here, $d=\frac{\xi}{\lambda_\infty}$ and $\overline{T}_k=\overline{T}_\infty-\frac{1}{\xi}$.

Introducing a stream function ψ and similarity variable η as given below, the above partial differential equations can be transformed into ordinary form. It is assumed that the temperature and concentration vary in the $x-$ direction, i.e., $T_w=T_\infty+ax$ and $C_w=C_\infty+bx$.

$$\eta=yc^{1/2}x^{-1/3}\upsilon_\infty^{-1/2} \tag{10}$$

$$\psi=x^{2/3}c^{1/2}\upsilon_\infty^{1/2}f\left(\eta\right) \tag{11}$$

$$\therefore u=\frac{\partial\psi}{\partial y}\text{ and }v=-\frac{\partial\psi}{\partial x}. \tag{12}$$

The dimensionless forms of temperature and concentration are introduced as:

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \text{ and } \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}.$$

Using the dimensionless temperature θ, we have

$$\mu=-\frac{\mu_\infty\theta_r}{\theta-\theta_r} \quad (13)$$

$$\lambda=-\frac{\lambda_\infty\theta_k}{\theta-\theta_k} \quad (14)$$

Then theEqs. (2) to (4) becomes:

$$\frac{\theta_r}{\theta-\theta_r}f'''-\frac{2}{3}ff''+\frac{\theta_r}{(\theta-\theta_r)^2}f''\theta'+\frac{1}{3}f'^2+Mf'-(\lambda_T\theta+\lambda_C\varphi)=0 \quad (15)$$

$$\left(Kr-\frac{\theta_k}{\theta-\theta_k}\right)\theta''+\frac{\theta_k}{(\theta-\theta_k)^2}\theta'^2+\frac{Pr}{3}(2f\theta'-3f'\theta)=0 \quad (16)$$

$$\frac{\theta_r}{(\theta-\theta_r)}\varphi''-\frac{\theta_r}{(\theta-\theta_r)^2}\theta'\varphi'-\frac{Sc}{3}(2f\varphi'-3f'\varphi)=0 \quad (17)$$

where superscript (′) denotes the derivative with respect to η, $M=\frac{\sigma B_0{}^2}{\rho C_p}$ is the magnetic field parameter, $\lambda_T=\frac{g\beta_T(T_w-T_\infty)x^{1/3}}{c^2}=\frac{g\beta_T(T_w-T_\infty)x}{c^2x^{2/3}}=\frac{Gr}{Re^2}$ is the buoyancy parameter, where $Gr=\frac{g\beta_T(T_w-T_\infty)x^3}{\upsilon_\infty^2}$ is the Grashof number, $Re=\frac{u_w x}{\upsilon_\infty}$ is the Reynolds number, $\lambda_C=\frac{g\beta_C(C_w-C_\infty)x^{1/3}}{c^2}$ is the concentration buoyancy parameter, $Kr=\frac{16\sigma^* T_\infty^3}{3\lambda_\infty\kappa_1}$ is the radiation parameter, $Pr=\frac{\mu C_p}{\lambda_\infty}$ is the Prandtl number, $Sc=\frac{\upsilon_\infty}{D}$ is the Schmidt number, θ_r and θ_k are viscosity and thermal conductivity parameter, respectively.

The boundary conditions (5) are reduced to:

$$\left.\begin{array}{l}\eta=0\ \ : f(\eta)=f_w,\ f'(\eta)=1,\ \varphi'(\eta)=-Bi\left[1-\theta(0)\right],\ \varphi(\eta)=1\\ \eta\to\infty: f'(\eta)=0,\ \theta(\eta)=0,\ \varphi(\eta)=0\end{array}\right\}\quad(18)$$

3. Results And Discussion

The system of ordinary differential equations (15) to (17) together with the boundary conditions (18) are solved numerically by fourth order Runge-Kuttashooting method. This study has been done to study the effects of various parameters such as $\theta_r,\ \theta_k,\ M,\ \Pr,\ Kr,\ Sc$ on velocity, temperature and species concentration profiles. A representative set of numerical results are shown graphically in Figs. (2) to (14). In the following discussion, the values of the parameters are taken as $\theta_r=5,\ \theta_k=5,\ M=0.5,\ \Pr=0.71,\ Kr=0.05,\ Sc=0.78$ and $Bi=0.5$, unless otherwise stated.

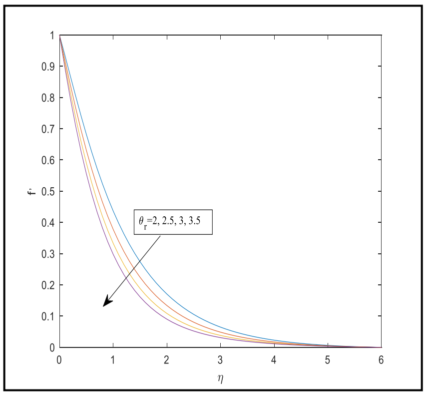

Fig.2: Effects of θ_r on velocity distribution

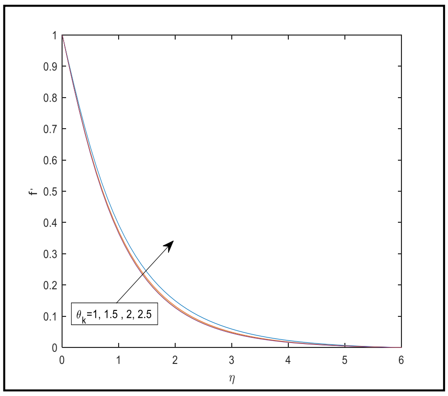

Fig.3: Effects of θ_k on velocity distribution

The variations of velocity profile with the variation of different parameters are plotted in graph (Figs. 2 to 6). Fig.2 displays dimensionless velocity distribution $f'(\eta)$ with the variation of θ_r and it is observed that the velocity decreases with the increases of θ_r. This is due to the fact that with the increase of the value of the viscosity parameter the velocity boundary layer thickness decreases. Physically, this is because of that a larger θ_r implies higher temperature difference between the surface and the fluid. Fig.3 shows the distribution of velocity with the variation of θ_k. It is seen that velocity increases with the increasing value of θ_k. This is due to the fact that temperature decreases with the increasing value of θ_k which implies decreasing of viscosity and so velocity increases.

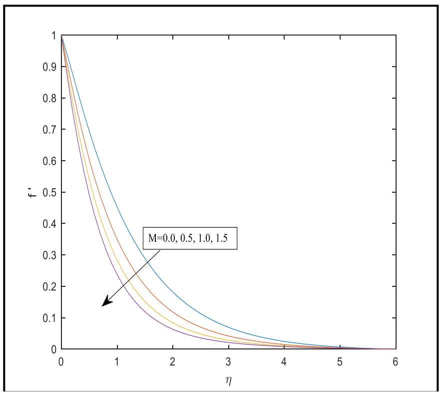

Fig.4: Effects of *M* on velocity distribution

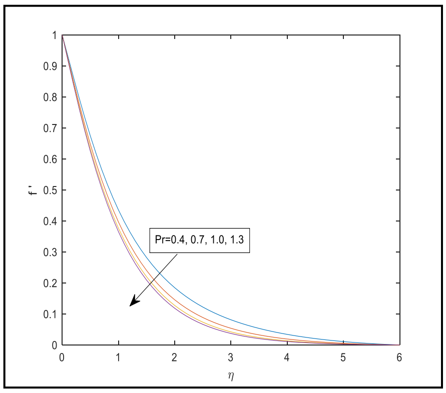

Fig.5: Effects of *Pr*on velocity distribution

In Fig.4, it is observed that with the increase of Hartmann number M, velocity decreases. This is because of the reason that presence of magnetic field normal to the flow in an electrically conducting fluid produces Lorentz force which opposes the flow. Velocity decreases with increasing value of Prandtl number Pr (Fig.5). This is because of that with the increase of Pr, viscosity increases, so velocity decreases. Increasing the value of radiation parameter Kr increases the velocity (Fig.6).

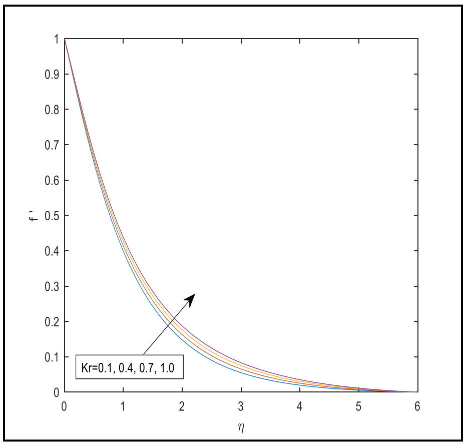

Fig.6: Effects of *Kr* on velocity distribution

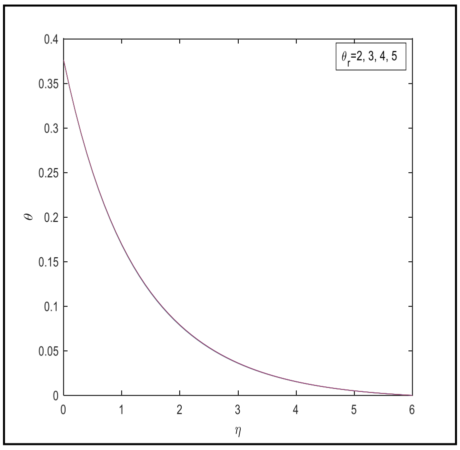

Fig.7: Effects of θ_r on temperature distribution

From Fig.7, a very negligible change has been noticed in temperature profile $\theta(\eta)$ with the increase in θ_r. With the increasing value of θ_k temperature increases (Fig.8).

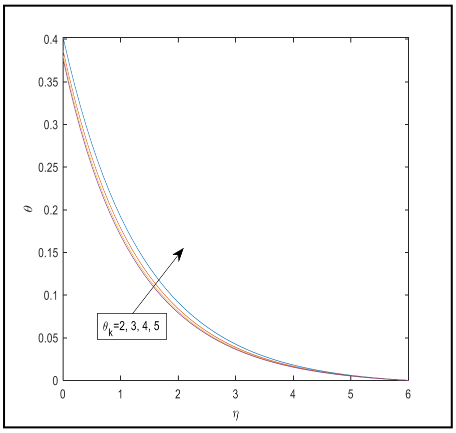

Fig.8: Effects of θ_k on temperature distribution

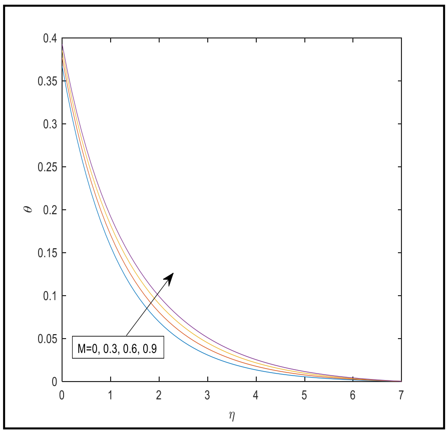

Fig.9: Effects of M on temperature distribution

Fig.9 indicates that temperature increases with the increase of magnetic parameter due to the Lorentz force. Fig.10 displays the effect of Prandtl number on temperature distribution. It is noticed that with the increase of Pr temperature of the fluid decreases. For higher Prandtl number the fluid has a relatively high thermal conductivity which decreases the temperature.

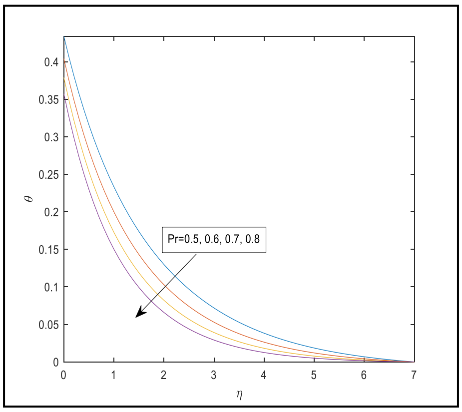

Fig.10: Effects of Pron temperature

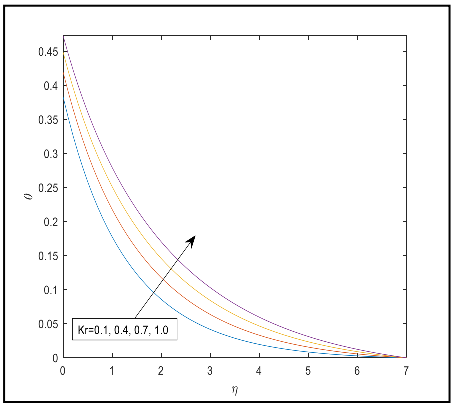

Fig.11: Effects of Kr on temperature

In Fig.11, it is observed that with the increasing value of Kr temperature increases due to the fact that the thermal boundary layer thickness increases with the increase of Kr and hence temperature.

The species concentration $\varphi(\eta)$ decreases for increasing value of θ_r (Fig.12). With the increasing value of *M* species concentration increases (Fig.13), while decreases with the increasing value of *Sc* (Fig.14).

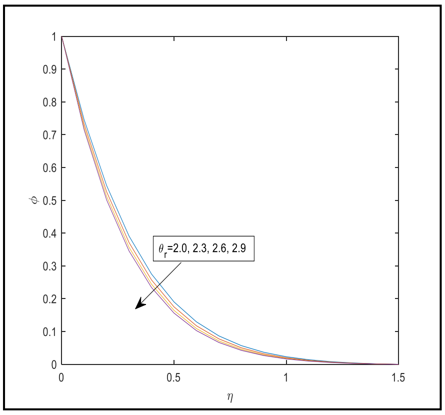

Fig.12: Effects of θ_r on species concentration distribution

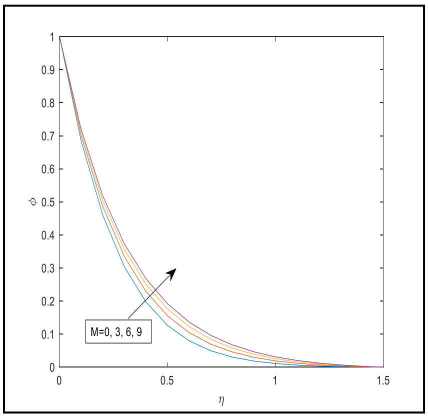

Fig.13: Effects of *M* on species concentration distribution

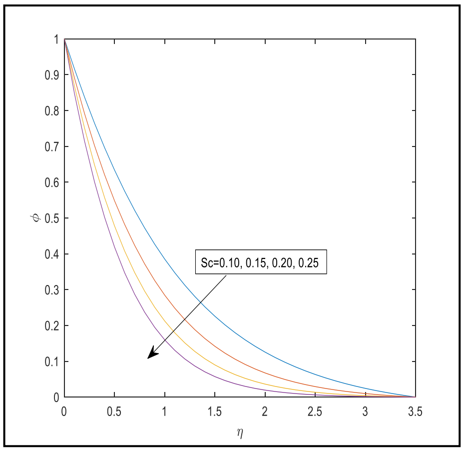

Fig.14: Effects of *Sc*on species concentration distribution

4. Conclusion

From the above analysis following conclusions may be derived:

(i) With the increase of viscosity parameter velocity and species concentration decreases while temperature increases.

(ii) With the increase of thermal conductivity parameter velocity and temperature increases.

(iii) Velocity decreases while temperature increases with the increase of magnetic field parameter.

(iv) Both velocity and temperature decreases with the increase of the value of Prandtl number.

(v) With the increase of the radiation parameter velocity and temperature increases.

(vi) Species concentration increases with the increase of the magnetic field parameter while decreases with the increasing value of Schmidt number.

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