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DISPERSION RELATION OF UPPER HYBRID WAVE THROUGH NONLINEAR WAVE PARTICLE INTERACTION IN AN INHOMOGENEOUS PLASMA

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Abstract

The dispersion relation of the upper hybrid wave propagating perpendicular to a uniform magnetic field in a spatially inhomogeneous plasma is derived. A uniform force is considered along the direction opposite to the density gradient. Also the back ground plasma is considered to be excited by a low frequency lower hybrid wave turbulence field. The interaction of waves is governed by Vlasov-Poisson's system of equations for the perturbed potential field in inhomogeneous plasma. We have derived nonlinear dispersion relation and seperated the imaginary part which is involved in estimation of growth rate.

Keywords: Non-linear dispersion relation, density gradient, lower hybrid drift wave turbulence, plasma maser- effect, weak turbulence theory.

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1. Introduction

Upper hybrid wave is a high frequency quasi-electrostatics wave that are

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propagating through magnetized plasma perpendicularly to the ambient magnetic field. The high frequency quasi-electrostatics fluctuations in the upper hybrid wave frequency range are a constant and pervasive feature in the Earth's radiation belt environment[1]. These upper hybrid wave had been observed in various region of the earth or the planetary magnetosphere such as I_0 torus [2,3]. The upper hybrid waves have been studied both experimentally[4] as well as theoretically [5-8] in a wide range of situation. Idehara et al.[4] for the first time in 1974 experimentally observed the upper hybrid waves in a laboratory plasma. Different characteristics and properties of upper hybrid waves are studied [9 - 14]. Some of them study on decay instabilities of upper hybrid wave in magnetized plasma [9], creation of upper hybrid wave from by conversion of ordinary and extraordinary waves [10], role of upper hybrid wave in magnetic reconnection[11], Terahertz wave generation by the upper hybrid wave[12], structure of upper hybrid waves[13], dynamics of nonlinearly coupled upper-hybrid waves[14]. Recently,there has been a great deal of interest in studying electrostatic upper hybrid waves in dusty plasmas [15 - 18].

The lower hybrid drift wave turbulence is a common occurrence in the laboratory plasmas and also they appear in earth's magnetosphere in a limit of whistler modes [19-21]. Recently in several observations of the lower hybrid drift wave has been made in the magnetosphere [22,23], as well as in laboratory plasmas [24,25]. Since earth's magnetosphere are in general very dynamical and active regions, often containing different sorts of waves. Among them the lower hybrid drift waves are supported by the free energy stored in density gradients.

In this present paper, we have derived the dispersion relation of upper hybrid wave through nonlinear wave particle interaction in presence of lower hybrid drift wave turbulence in an inhomogeneous plasmas. In the case the density gradient varies in a direction perpendicular to the magnetic field and a uniform force is considered along the direction opposite to the density gradient. Since upper hybrid wave is high frequency electrostatics wave propagated in a magnetized plasma so we have used the Vlasov-Poisson's equation for inhomogeneous plasma to derive the dispersion relation. As the lower hybrid drift wave turbulence is considered in the magnetized plasma through nonlinear wave particle interaction process so we have also used the Fourier transformation and then integrating along unperturbed orbit in the evaluation for the dispersion relation. We have organised this paper mainly in four sections for discussion. In section 2, we have formulated the problem where the geometry of the problem was described. In section 3, we have described the required governing equation i.e. Vlasov-Poisson's equation which is used for perturb and unperturb fields and distribution function to obtain necessary differential equations. And then in section

4, we have used fourier transformation and integrating along unperturbed orbit for differential equations. In the section 5, we have derived the detail dispersion relation of upper hybrid wave in the considered system. At the last section 6, we have finally evaluated the dispersion relation of upper hybrid wave considering only the imaginary terms applying plasma maser effect.

2. Formulation for the problem

We consider electrostatic lower hybrid drift wave turbulence to be present in the system with propagation vector $k = (k_\perp, 0, k_\parallel)$. The gradients in density $\nabla n_0(y)$ and we consider a magnetized plasma where the external magnetic field $\mathbf{B} = B_0(y)$ is taken in the z-direction and the system has spatial gradient in y-direction. The Geometry of the model is shown in the figure 1. For such an inhomogeneous plasma, the electron distribution function is considered [26].

$$f_e(y, \mathbf{v}) = \left(\frac{m}{2\pi T}\right)^{\frac{3}{2}} \left[1 + \epsilon' \left(y + \frac{v_x}{\Omega_e}\right)\right] exp\left[-\left(\frac{m\mathrm{v}^2}{2T} - \frac{Fy}{T}\right)\right] (1)$$

where $\Omega_e = \frac{eB_0}{mc}$ is the cyclotron frequency of the electron and $\mathbf{F} = F\hat{y}$ is a uniform force field applied in the y-direction and $\epsilon' = \frac{1}{f_e}\frac{df_e}{dy}]_{y=0} - \frac{F}{T}$ is the small density gradient.

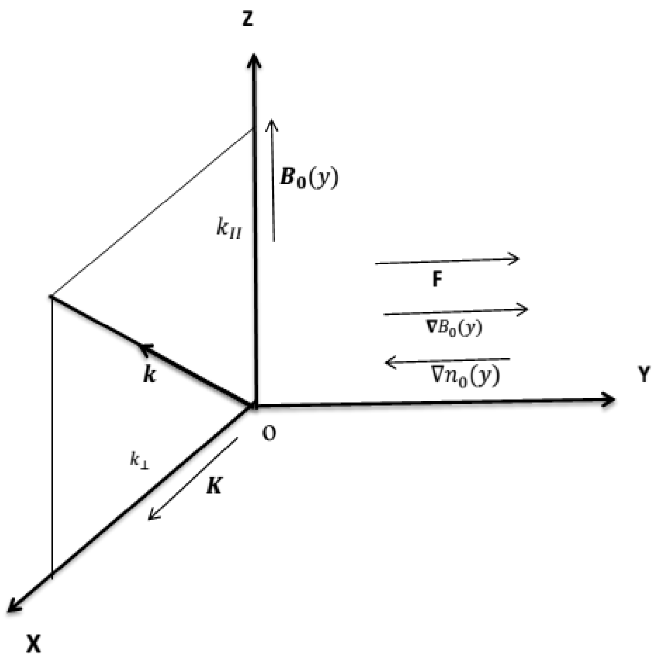

Figure 1: Geometry of the Model: Where $\mathbf{k} = (\mathrm{k}_\perp, 0, \mathrm{k}_\parallel)$ is the propagating vector of

lower hybrid drift wave, $\mathbf{K} = (K_\perp, 0,0)$ is the propagating vector of upper hybrid wave.

3. Governing Equations

The basic governing equations describing the interaction of the high frequency upper hybrid wave with low frequency lower hybrid drift wave turbulence are Vlasov Poisson's equations

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \left\{\frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) - \frac{\mathbf{F}}{m}\right\}.\frac{\partial}{\partial \mathbf{v}}\right] F_{0e}(\mathbf{r}, \mathbf{v}, t) = 0 \quad (2)$$

$$\nabla.\mathbf{E} = -4\pi e n_e \int f_e(\mathbf{r}, \mathbf{v}, t) d\mathbf{v} \quad (3)$$

And the Fourier transformation

$$A(\mathbf{r}, \mathbf{v}, t) = \sum_{\mathbf{k},\omega} A(\mathbf{k}, \omega, \mathbf{v})\, exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)] \quad (4)$$

According to the linear response theory of a plasma [27], the unperturbed electron distribution function and fields are

$$F_{0e} = f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e} \quad (5)$$

and

$$\mathbf{E}_{oe} = \epsilon \mathbf{E}_l + \epsilon^2 \mathbf{E}_2 \quad (6)$$

where ϵ is a small parameter associated with lower hybrid drift wave turbulence field $\mathbf{E}_l = (E_{l\perp}, 0, E_{l\parallel}), f_{0e}$ is space and time average parts, f_{1e} and f_{2e} are fluctuating parts of the distribution function, $\mathbf{E}_2$ is the second order electric field.

On putting these in Eq.(2)

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \left\{\frac{e}{m}\left(\epsilon \mathbf{E}_l + \epsilon^2 \mathbf{E}_2 + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) - \frac{\mathbf{F}}{m}\right\}.\frac{\partial}{\partial \mathbf{v}}\right] (f_{0e} + \epsilon f_{1e} + \epsilon^2 f_{2e}) = 0$$

We have to the order of ϵ

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{r}} - \left\{\frac{e}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right) - \frac{\mathbf{F}}{m}\right\}.\frac{\partial}{\partial \mathbf{v}}\right] f_{1e} = \frac{e}{m}\left(\mathbf{E}_l.\frac{\partial}{\partial \mathbf{v}} f_{oe}\right) \quad (7)$$

To this quasi-steady state we apply perturbation $\mu\delta\mathbf{E}_h$ of test high frequency electrostatic upper hybrid wave with propagating vector $\mathbf{K} = (K_\perp, 0,0)$ with electric field $\delta\mathbf{E} = (\delta E_h, 0,0)$ and a frequency Ω. Thus the total perturbed electric field and the distribution function are given

$$\delta\mathbf{E} = \mu\delta\mathbf{E}_h + \mu\epsilon\delta\mathbf{E}_{lh} + \mu\epsilon^2 \triangle \mathbf{E}$$

$$\delta\mathbf{B} = 0$$

and

$$\delta f = \mu\delta f_h + \mu\epsilon\delta f_{lh} + \mu\epsilon^2 \triangle f$$

where $\delta\mathbf{E}_{lh}$, $\triangle\mathbf{E}$ are the modulation fields , δf_{lh}, $\triangle f$ are fluctuating parts of electron distribution function corresponding to modulation field. We put these in Vlasov Eq.(2).

$$\left[\frac{\partial}{\partial t}+\mathbf{v}.\frac{\partial}{\partial\mathbf{r}}-\left\{\frac{e}{m}\left(\mathbf{E}_{0e}+\delta\mathbf{E}_h)+\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right)-\frac{\mathbf{F}}{m}\right\}.\frac{\partial}{\partial\mathbf{v}}\right](F_{0e}+\delta f)(\mathbf{r},\mathbf{v},t)=0$$

$$\Rightarrow\left[\frac{\partial}{\partial t}+\mathbf{v}.\frac{\partial}{\partial\boldsymbol{r}}-\left\{\frac{e}{m}\left(\epsilon\mathbf{E}_{lh}+\epsilon^2\mathbf{E}_2+\mu\delta\mathbf{E}_h+\mu\epsilon\delta\mathbf{E}_{lh}+\mu\epsilon^2\triangle\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right)-\frac{\mathbf{F}}{m}\right\}.\frac{\partial}{\partial\mathbf{v}}\right]$$
$$(f_{0e}+\epsilon f_{1e}+\epsilon^2 f_{2e}+\mu\delta f_h+\mu\epsilon\delta f_{lh}+\mu\epsilon^2\triangle f)(\mathbf{r},\mathbf{v},t)=0(8)$$

Then to the order of $\mu,\mu\epsilon,\mu\epsilon^2$ we get

$$P\delta f_h=\frac{e}{m}\delta\mathbf{E}_h.\frac{\partial}{\partial\mathbf{v}}f_{0e}(9)$$

$$P\delta f_{lh}=\frac{e}{m}\delta\mathbf{E}_{lh}.\frac{\partial}{\partial\mathbf{v}}\delta f_h+\frac{e}{m}\delta\mathbf{E}_h.\frac{\partial}{\partial\mathbf{v}}f_{1e}+\frac{e}{m}\delta\mathbf{E}_{lh}.\frac{\partial}{\partial\mathbf{v}}f_{0e}(10)$$

$$P\triangle f=\frac{e}{m}\delta\mathbf{E}_l.\frac{\partial}{\partial\mathbf{v}}\delta f_{lh}+\frac{e}{m}\delta\mathbf{E}_{lh}.\frac{\partial}{\partial\mathbf{v}}f_{1e}(11)$$

where

$$P=\left[\frac{\partial}{\partial t}+\mathbf{v}.\frac{\partial}{\partial\mathbf{r}}-\left\{\frac{e}{m}\left(\mathbf{E}+\frac{\mathbf{v}\times\mathbf{B}_0}{c}\right)-\frac{\mathbf{F}}{m}\right\}.\frac{\partial}{\partial\mathbf{v}}\right]$$

4. Integration along unperturbed orbit

Using the Fourier transform of Eq.(3) and we solve these differential Eqs.(7,9-11) for fluctuating parts of the distribution functions $f_{1e}, f_h, f_{lh}, \triangle f$ over the electron trajectories whose equations are given by

$$\frac{d\mathbf{r}\prime}{dt\prime}=\mathbf{v}',\ \frac{d\mathbf{v}'}{dt\prime}=\frac{e}{m}\left(\frac{\mathbf{v}'\times\mathbf{B}_0}{c}\right)+\frac{F}{m}\hat{y}$$

$$r'(t'=t)=\mathbf{r},\mathbf{v}'(t'=t)=\mathbf{v}$$

The unperturbed orbits are found by integrating these differential equations which are given by

$$v'_x=v_\perp cos(\theta-\Omega_e\tau)-\frac{F}{m\Omega_e},v'_y=v_\perp sin(\theta-\Omega_e\tau),v'_z=v_\parallel$$

where $\tau=t'-t$

And then by integrating,we have

$$x' = x - \frac{v_\perp}{\Omega_e} sin(\theta - \Omega_e \tau) + \frac{v_\perp}{\Omega_e} sin\theta - \frac{F}{m\Omega_e}\tau$$

$$\Rightarrow x' = x - \frac{v_\perp}{\Omega_e} sin(\theta - \Omega_e \tau) + \frac{v_\perp}{\Omega_e} sin\theta + v_F \tau \text{ , where } v_F = -\frac{F}{m\Omega_e}$$

$$y' = y + \frac{v_\perp}{\Omega_e} cos(\theta - \Omega_e \tau) + \frac{v_\perp}{\Omega_e} cos\theta$$

$$z' = z + v_\parallel \tau$$

Using Fourier transform Eq.(3) and integrating along unperturbed orbit; the fluctuating part f_{1e} of the low frequency turbulence field is obtained from Eq.(7)

$$f_{1e} = \frac{e}{m}\int_{-\infty}^{0}\left(\mathbf{E}_l.\frac{\partial}{\partial \mathbf{v}} f_{0e}\right) exp[i\{\mathbf{k}.(\mathbf{r}' - \mathbf{r}) - \omega\tau\}]d\tau$$

$$= \frac{e}{m}\int_{-\infty}^{0}\left[E_{l\perp}\frac{\partial}{\partial v_x} f_{0e} + E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}\right] exp[i\{\mathbf{k}.(\mathbf{r}' - \mathbf{r}) - \omega\tau\}]d\tau$$

$$= i\frac{e}{m}\left[\frac{m}{T_e k_\perp} E_{l\perp}\left\{1 + \left(\omega - k_\parallel v_\parallel - \frac{\epsilon T_e k_\perp}{m\Omega_e}\right) S_{a,b}\right\} f_{0e} - E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel} S_{a,b}\right] (12)$$

where

$$S_{a,b} = \sum_{a,b}\frac{J_a(\alpha)J_b(\alpha)exp[i(b-a)\theta]}{a\Omega_e + k_\perp v_F - \omega + k_\parallel v_\parallel + i0}, \alpha = \frac{k_\perp v_\perp}{\Omega_e}$$

and $v_F = -\frac{F}{m\Omega_e}$ is represent the $\mathbf{F} \times \mathbf{B}$ drift velocity. Here i0 imaginary parts, so involving term $S_{a,b}$ is always become imaginary

We have from Eq.(9)

$$\delta f_h(\boldsymbol{K}, \Omega) = \frac{e}{m}\int_{-\infty}^{0}\left(\delta\mathbf{E}_h.\frac{\partial}{\partial \mathbf{v}} f_{0e}\right) exp[i\{\mathbf{K}.(\mathbf{r}' - \mathbf{r}) - \Omega\tau\}]d\tau$$

$$= \frac{e}{m}\int_{-\infty}^{0}\left(\delta E_h.\frac{\partial}{\partial \mathbf{v}'_x} f_{0e}\right) exp[i\{K_\perp(x' - x) - \Omega\tau\}]d\tau$$

$$= \frac{ie}{m}\delta E_h \frac{m}{K_\perp T_e}\left[1 + \left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right)\sum_{s,t}\frac{J_s(\alpha')J_t(\alpha')exp[i(t-s)\theta]}{s\Omega_e - \Omega + K_\perp v_F}\right] f_{0e}$$

$$= \frac{ie}{m}\delta E_h \frac{m}{K_\perp T_e}\left[1 + \left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right) Q_{s,t}\right] f_{0e} (13)$$

where

$$Q_{s,t} = \sum_{s,t}\frac{J_s(\alpha')J_t(\alpha')exp[i(t-s)\theta]}{s\Omega_e - \Omega + K_\perp v_F}, \alpha' = \frac{K_\perp v_\perp}{\Omega_e}$$

From Eq.(10), we have

$$\delta f_{lh}(\mathbf{K} - \mathbf{k}, \Omega - \omega) = \frac{e}{m}\int_{-\infty}^{0}[\delta\mathbf{E}_l.\frac{\partial}{\partial \mathbf{v}}\delta f_h + \delta\mathbf{E}_h.\frac{\partial}{\partial \mathbf{v}} f_{1e} + \delta\mathbf{E}_{lh}.\frac{\partial}{\partial \mathbf{v}} f_{0e}]$$

$$\times exp[i\{(\mathbf{K} - \mathbf{k})(\mathbf{r}' - \mathbf{r}) - (\Omega - \omega)\tau\}]d\tau$$

$$= I_{lh}^1 + I_{lh}^2 + I_{lh}^3 (14)$$

Where

$$I_{lh}^{1} = \frac{e}{m}\int_{-\infty}^{0}\left[\delta\mathbf{E}_{l}.\frac{\partial}{\partial\mathbf{v}}\delta f_{h}\right]exp[i\{(\mathbf{K}-\mathbf{k})(\mathbf{r}'-\mathbf{r})-(\Omega-\omega)\tau\}]d\tau$$

$$= \frac{e}{m}\int_{-\infty}^{0}\left[E_{l\perp}\frac{\partial}{\partial v'_{x}}+E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}\right]\delta f_{h}exp[i\{(\mathbf{K}-\mathbf{k})(\mathbf{r}'-\mathbf{r})-(\Omega-\omega)\tau\}]d\tau$$

$$= \frac{e}{m}\left[E_{l\perp}\frac{im}{(K_{\perp}-k_{\perp})T_{e}}\{1+\left(\Omega-\omega+k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}(K_{\perp}-k_{\perp})}{m\Omega_{e}}\right)T_{p,q}\}-iE_{l\parallel}\frac{\partial}{\partial v_{\parallel}}T_{p,q}\right]\times$$

$$\left[\frac{ie}{m}\frac{m}{K_{\perp}T_{e}}\delta E_{h}\left\{1-(\Omega-\frac{\epsilon T_{e}K_{\perp}}{m\Omega_{e}})Q_{s,t}\right\}\right]f_{0e}$$

$$= -(\frac{e}{m})^{2}\delta E_{h}\frac{m}{T_{e}K_{\perp}}$$

$$\times\left[E_{l\perp}\frac{m}{(K_{\perp}-k_{\perp})T_{e}}\{1+\left(\Omega-\omega+k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}(K_{\perp}-k_{\perp})}{m\Omega_{e}}\right)T_{p,q}\}\right.$$

$$\left.-E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}S_{p,q}\right]$$

$$\times\left\{1+\left(\Omega-\frac{\epsilon T_{e}K_{\perp}}{m\Omega_{e}}\right)S_{s,t}\right\}f_{0e} \quad (15)$$

Where

$$T_{p,q} = \sum_{p,q}\frac{J_{p}(\xi)J_{q}(\xi)exp[i(q-p)\theta]}{p\Omega_{e}-\Omega+(K_{\perp}-k_{\perp})v_{F}+\omega-k_{\parallel}v_{\parallel}}, \xi = \frac{(K_{\perp}-k_{\perp})v_{\perp}}{\Omega_{e}}$$

$$I_{lh}^{2} = \frac{e}{m}\int_{-\infty}^{0}\left[\delta\mathbf{E}_{h}.\frac{\partial}{\partial\mathbf{v}}f_{1e}\right]exp[i\{(\mathbf{K}-\mathbf{k})(\mathbf{r}'-\mathbf{r})-(\Omega-\omega)\tau\}]d\tau$$

$$= \frac{e}{m}\int_{-\infty}^{0}\left[\delta E_{h}.\frac{\partial}{\partial v'_{x}}f_{1e}\right]exp[i\{(K_{\perp}-k_{\perp})(x'-x)-k_{\parallel}(z'-z)-(\Omega-\omega)\tau\}]d\tau$$

$$= \frac{e}{m}\frac{im\delta E_{h}}{(K_{\perp}-k_{\perp})T_{e}}\left[1+\left(\Omega-\omega+k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}(K_{\perp}-k_{\perp})}{m\Omega_{e}}\right)T_{p,q}\right]$$

$$\times\frac{ie}{m}\left[\frac{m}{T_{e}k_{\perp}}E_{l\perp}\left\{1+(\omega-k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}k_{\perp}}{m\Omega_{e}})S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_{\parallel}}S_{a,b}\right]$$

$$= -(\frac{e}{m})^{2}\frac{\delta E_{h}m}{(K_{\perp}-k_{\perp})T_{e}}\left[1+\left(\Omega-\omega+k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}(K_{\perp}-k_{\perp})}{m\Omega_{e}}\right)T_{p,q}\right]$$

$$\times\left[\frac{m}{T_{e}k_{\perp}}E_{l\perp}\left\{1+\left(\omega-k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}k_{\perp}}{m\Omega_{e}}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_{\parallel}}S_{a,b}\right] \quad (16)$$

$$I_{lh}^{3} = \frac{e}{m}\int_{-\infty}^{0}\left[\delta\mathbf{E}_{lh}.\frac{\partial}{\partial\mathbf{v}}f_{0e}\right]exp[i\{(\mathbf{K}-\mathbf{k})(\mathbf{r}'-\mathbf{r})-(\Omega-\omega)\tau\}]d\tau$$

$$= \frac{e}{m}\int_{-\infty}^{0}\left[\frac{\delta E_{lh}}{|\mathbf{K}-\mathbf{k}|}(K_{\perp}-k_{\perp})\frac{\partial}{\partial v'_{x}}-\frac{\delta E_{lh}k_{\parallel}}{|\mathbf{K}-\mathbf{k}|}\right]f_{0e}$$

$$\times exp[i\{(K_{\perp}-k_{\perp})(x'-x)-k_{\parallel}(z'-z)-(\Omega-\omega)\tau\}]d\tau$$

$$= \frac{ie}{m}\frac{\delta E_{lh}}{|\mathbf{K}-\mathbf{k}|}\left[\frac{m}{T_{e}}\left\{1+\left(\Omega-\omega+k_{\parallel}v_{\parallel}-\frac{\epsilon T_{e}(K_{\perp}-k_{\perp})}{m\Omega_{e}}\right)T_{p,q}\right\}+k_{\parallel}\frac{\partial}{\partial v_{\parallel}}T_{p,q}\right]f_{0e} \quad (17)$$

From Eq.(11), we have

$$\triangle f(\mathbf{K},\Omega) = \frac{e}{m}\int_{-\infty}^{0}\left[\mathbf{E}_l.\frac{\partial}{\partial \mathbf{v}}\delta f_{lh} + \delta\mathbf{E}_{lh}.\frac{\partial}{\partial \mathbf{v}} f_{1e}\right] exp[i\{\mathbf{K}.(\mathbf{r}'-\mathbf{r}) - \Omega\tau\}]d\tau$$
$$= I_{\triangle}^1 + I_{\triangle}^2 \quad (18)$$

$$I_{\triangle}^1 = \frac{e}{m}\int_{-\infty}^{0}\left[\mathbf{E}_l.\frac{\partial}{\partial \mathbf{v}}\delta f_{lh}\right] exp[i\{\mathbf{K}.(\mathbf{r}'-\mathbf{r}) - \Omega\tau\}]d\tau$$
$$= \frac{e}{m}\int_{-\infty}^{0}\left[E_{l\perp}\frac{\partial}{\partial v\prime} + E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}\right]\delta f_{lh} \times exp[i\{\mathbf{K}.(\mathbf{r}'-\mathbf{r}) - \Omega\tau\}]d\tau$$
$$= \frac{e}{m}\left[E_{l\perp}\frac{im}{K_{\perp}T_e}\left\{1 + \left(\Omega - \frac{\epsilon T_e K_{\perp}}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e} - iE_{l\parallel}\frac{\partial}{\partial v_{\parallel}}Q_{s,t}\right] \times \delta f_{lh} \quad (19)$$

$$I_{\triangle}^2 = \frac{e}{m}\int_{-\infty}^{0}\left[\delta\mathbf{E}_{lh}.\frac{\partial}{\partial \mathbf{v}} f_{1e}\right] exp[i\{\mathbf{K}.(\mathbf{r}'-\mathbf{r}) - \Omega\tau\}]d\tau$$
$$= \frac{e}{m}\int_{-\infty}^{0}\left[\frac{\delta E_{lh}}{|\mathbf{K}-\mathbf{k}|}\left\{(K_{\perp}-k_{\perp})\frac{\partial}{\partial v\prime} - k_{\parallel}\frac{\partial}{\partial v_{\parallel}}\right\}f_{1e}\right] exp[i\{\mathbf{K}.(\mathbf{r}'-\mathbf{r}) - \Omega\tau\}]d\tau$$
$$= -(\frac{e}{m})^2\frac{\delta E_{lh}}{|\mathbf{K}-\mathbf{k}|}\left[\frac{m}{T_e}\left\{1 + \left(\Omega - \omega + k_{\parallel}v_{\parallel} - \frac{\epsilon T_e(K_{\perp}-k_{\perp}}{m\Omega_e}\right)T_{p,q}\right\} + k_{\parallel}\frac{\partial}{\partial v_{\parallel}}T_{p,q}\right]$$
$$\times\left[\frac{m}{T_e k_{\perp}}E_{l\perp}\left\{1 + \left(\omega - k_{\parallel}v_{\parallel} - \frac{\epsilon T_e k_{\perp}}{m\Omega_e}\right)S_{a,b}\right\}f_{0e} - E_{l\parallel}\frac{\partial f_{0e}}{\partial v_{\parallel}}S_{a,b}\right] \quad (20)$$

Now using Eqs.(15-17) in Poisson's equation we have the modulating electric field δE_{lh} as

$$\mathbf{\nabla}.\delta\mathbf{E}_{lh} = -4\pi n_e e\int \delta f_{lh}(\mathbf{K}-\mathbf{k})d\mathbf{v}$$
$$\Rightarrow \delta E_{lh} = -\frac{4\pi n_e e}{i|\mathbf{K}-\mathbf{k}|}\int \left[I_{lh}^1 + I_{lh}^2 + I_{lh}^3\right]d\mathbf{v}$$
$$= -\frac{4\pi n_e e}{i|\mathbf{K}-\mathbf{k}|}\int\left[\left[-(\frac{e}{m})^2\delta E_h\frac{m}{T_e K_{\perp}}\left[E_{l\perp}\frac{m}{(K_{\perp}-k_{\perp})T_e}\{1 + (\Omega - \omega + k_{\parallel}v_{\parallel}\right.\right.\right.$$
$$\left.\left. - \frac{\alpha T_e(K_{\perp}-k_{\perp})}{m\Omega_e}\right)T_{p,q}\right\} - E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}T_{p,q}\right] \times \left\{1 + \left(\Omega - \frac{\epsilon T_e K_{\perp}}{m\Omega_e}\right)Q_{s,t}\right\}\right] f_{0e}$$
$$-(\frac{e}{m})^2\frac{\delta E_h m}{(K_{\perp}-k_{\perp})T_e}\left[1 - \left(\Omega - \omega + k_{\parallel}v_{\parallel} - \frac{\epsilon T_e k_{\perp}}{m\Omega_e}\right)T_{p,q}\right]$$
$$\times\left[\frac{m}{T_e k_{\perp}}E_{l\perp}\left\{1 + \left(\omega - k_{\parallel}v_{\parallel} - \frac{\epsilon T_e k_{\perp}}{m\Omega_e}\right)S_{a,b}\right\}f_{0e} - E_{l\parallel}\frac{\partial f_{0e}}{\partial v_{\parallel}}S_{a,b}\right]$$
$$+\frac{ie}{m}\frac{\delta E_{lh}}{|\mathbf{K}-\mathbf{k}|}[\frac{m}{T_e}\left\{1 + \left(\Omega - \omega + k_{\parallel}v_{\parallel} - \frac{\epsilon T_e(K_{\perp}-k_{\perp})}{m\Omega_e}\right)T_{p,q}\right\}$$
$$+k_{\parallel}\frac{\partial}{\partial v_{\parallel}}T_{p,q}]f_{0e}] \quad d\mathbf{v}$$
$$= \frac{4\pi n_e e}{R|\mathbf{K}-\mathbf{k}|}(\frac{e}{m})^2\delta E_h\int\left[\frac{m}{T_e K_{\perp}}\left[E_{l\perp}\frac{m}{(K_{\perp}-k_{\perp})T_e}\{1 + (\Omega - \omega + k_{\parallel}v_{\parallel}\right.\right.$$
$$\left. - \frac{\alpha T_e(K_{\perp}-k_{\perp})}{m\Omega_e}\right)T_{p,q}\right\} - E_{l\parallel}\frac{\partial}{\partial v_{\parallel}}T_{p,q}\right] \times \left\{1 + \left(\Omega - \frac{\epsilon T_e K_{\perp}}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}$$
$$+\frac{m}{(K_{\perp}-k_{\perp})T_e}\left[1 + \left(\Omega - \omega + k_{\parallel}v_{\parallel} - \frac{\epsilon T_e k_{\perp}}{m\Omega_e}\right)T_{p,q}\right]$$

$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\Bigg]d\mathbf{v} \quad (21)$$

Where

$$R=1+\frac{4\pi e^2 n}{m(|\mathbf{K}-\mathbf{k}|)^2}\int\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]f_{0e}d\mathbf{v}$$

$$=1+\frac{w_{pe}^2}{(|\mathbf{K}-\mathbf{k}|)^2}\int\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]f_{0e}d\mathbf{v} \quad (22)$$

5. Dispersion relation of Upper Hybrid wave

We obtain the dielectric response function for upper hybrid wave in presence of lower hybrid drift wave turbulence in inhomogeneous plasma. For this we have used the Poisson's equation for the perturbed electric field and fluctuating densities function, which is given by

$$\nabla.\delta\mathbf{E}_h=-4\pi e n_e\int[\delta f_h+\triangle f]d\mathbf{v}$$

Now putting the values for δf_h from Eq.(12) and for $\triangle f$ from Eqs.(19) and (20)in above equation, we have as

$$\nabla.\delta\mathbf{E}_h=-4\pi e n_e\int[\delta f_h+\triangle f]dv$$

$$\Rightarrow\delta E_h\left[1+\frac{4\pi e^2 n_e}{mK_\perp}\int\left[\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{s\Omega_e}\right)Q_{s,t}\right\}f_{0e}\right]dv\right]$$

$$\delta E_h[-\frac{4\pi e n_e}{iK_\perp}(\frac{e}{m})^2\int[\frac{ie}{m}[E_{l\perp}\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}$$

$$-E_{l\parallel}\frac{\partial}{\partial v_\parallel}Q_{s,t}]\times\frac{m}{T_e K_\perp}[E_{l\perp}\frac{m}{(K_\perp-k_\perp)T_e}|\{1+(\Omega-\omega+k_\parallel v_\parallel$$

$$-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}|)T_{p,q}\}-E_{l\parallel}\frac{\partial}{\partial v_\parallel}T_{p,q}]\times\left[1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right]f_{0e}$$

$$+\frac{m}{(K_\perp-k_\perp)T_e}\left[1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)T_{p,q}\right]$$

$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]]d\mathbf{v}]$$

$$\delta E_{lh}[\frac{\omega_{pe}^2}{K_\perp|\mathbf{K}-\mathbf{k}|^2}(\frac{e}{m})\int[[E_{l\perp}\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}-E_{l\parallel}\frac{\partial}{\partial v_\parallel}Q_{s,t}]\times$$

$$\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]f_{0e}$$
$$+\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]$$
$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]]d\mathbf{v}]=0(23)$$

Now putting the value of δE_{lh} from Eq.(21) in above equation can be written as

$$D(\mathbf{K},\Omega)\times\delta E_h=0$$

where $D(\mathbf{K},\Omega)$ is the dispersion relation of upper hybrid wave in presence of lower hybrid drift wave turbulence through nonlinear wave particle interaction in an inhomogeneous magnetized plasma. The dispersion relation can be divided in three parts as

$$D(\mathbf{K},\Omega)=D_0(\mathbf{K},\Omega)+D_d(\mathbf{K},\Omega)+D_p(\mathbf{K},\Omega)(24)$$

Where D_0 is linear part, D_d is direct coupling part and D_p is polarization coupling part of the dispersion relation and these values are given by

$$D_0(\mathbf{K},\Omega)=1+\frac{4\pi e^2 n_e}{mK_\perp}\int\left[\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{s\Omega_e}\right)Q_{s,t}\right\}f_{0e}\right]d\mathbf{v}(25)$$

$$D_d(\mathbf{K},\Omega)=-\frac{4\pi e n_e}{iK_\perp}\left(\frac{e}{m}\right)^2\int\left[\frac{ie}{m}\left[E_{l\perp}\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}-E_{l\parallel}\frac{\partial}{\partial v_\parallel}Q_{s,t}\right]\times\right.$$
$$\frac{m}{T_e K_\perp}[E_{l\perp}\frac{m}{(K_\perp-k_\perp)T_e}|\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}|\right)T_{p,q}\right\}-E_{l\parallel}\frac{\partial}{\partial v_\parallel}T_{p,q}]\times$$
$$\left[1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right]f_{0e}+\frac{m}{(K_\perp-k_\perp)T_e}\left[1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)T_{p,q}\right]$$
$$\left.\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\right]d\mathbf{v}(26)$$

$$D_p(\mathbf{K},\Omega)=\frac{\omega_{pe}^4}{R|\mathbf{K}-\mathbf{k}|^2}\left(\frac{e}{m}\right)^2$$
$$\int\left[\left[E_{l\perp}\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}-E_{l\parallel}\frac{\partial}{\partial v_\parallel}Q_{s,t}\right]\times\right.$$
$$\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]f_{0e}$$
$$+\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]$$
$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]$$
$$\times\left[\frac{m}{T_e K_\perp}\left[E_{l\perp}\frac{m}{(K_\perp-k_\perp)T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}-\right.\right.$$

$$E_{l\parallel}\frac{\partial}{\partial v_\parallel}T_{p,q}\Big]$$

$$\times\left\{1+\left(\Omega-\frac{\epsilon T_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}+\frac{m}{(K_\perp-k_\perp)T_e}\left[1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)T_{p,q}\right]$$

$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]\Bigg]\Bigg]d\mathbf{v}$$

$$\Rightarrow D_p(\mathbf{K},\Omega)=\frac{\omega_{pe}^4}{R|\mathbf{K}-\mathbf{k}|^2}\left(\frac{e}{m}\right)^2(E+F)\times(G+H)\quad(27)$$

where

$$E=\int\left[\left[E_{l\perp}\frac{m}{K_\perp T_e}\left\{1+\left(\Omega-\frac{\epsilon T\prime_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}f_{0e}-E_{l\parallel}\frac{\partial}{\partial v_\parallel}Q_{s,t}\right]\times\right.$$
$$\left.\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]f_{0e}\right]d\mathbf{v}\quad(28)$$

$$F=\int\left[\frac{m}{T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}+k_\parallel\frac{\partial}{\partial v_\parallel}T_{p,q}\right]$$
$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]d\mathbf{v}\quad(29)$$

$$G=\int\left[\frac{m}{T_e K_\perp}\left[E_{l\perp}\frac{m}{(K_\perp-k_\perp)T_e}\left\{1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right\}\right.\right.$$
$$\left.\left.-E_{l\parallel}\frac{\partial}{\partial v_\parallel}T_{p,q}\right]\times\left\{1+\left(\Omega-\frac{\epsilon T\prime_e K_\perp}{m\Omega_e}\right)Q_{s,t}\right\}\right]f_{0e}d\mathbf{v}\quad(30)$$

$$H=\int\frac{m}{(K_\perp-k_\perp)T_e}\left[1+\left(\Omega-\omega+k_\parallel v_\parallel-\frac{\epsilon T_e(K_\perp-k_\perp)}{m\Omega_e}\right)T_{p,q}\right]$$
$$\times\left[\frac{m}{T_e k_\perp}E_{l\perp}\left\{1+\left(\omega-k_\parallel v_\parallel-\frac{\epsilon T_e k_\perp}{m\Omega_e}\right)S_{a,b}\right\}f_{0e}-E_{l\parallel}\frac{\partial f_{0e}}{\partial v_\parallel}S_{a,b}\right]d\mathbf{v}\quad(31)$$

6 Discussion and conclusion

Previous studies in refs.[1–18] are not concerned in non-linear dispersion relation of upper hybrid wave through lower hybrid wave turbulence field in inhomogeneous plasma. In this paper, we have tried to derive the non-linear dispersion relation of upper hybrid wave in the considered system. For detail the dispersion relation of Upper Hybrid wave given by Eq.(24) after using the Eq.(25) to Eq.(31). This dispersion relation is more complex to evaluate in details due to nonlinearity as well as inhomogeneity of the plasma density. So a large number of studies[28–44] related in plasma waves in astrophysics and laboratory plasmas have been developed plasma maser effect[28–41] as well as weak turbulence theory[41–44] to overcome the complexity for evaluation of the dispersion relation through nonlinear wave particle interaction in an inhomogeneous plasma. The Plasma maser effect occurs when both resonant and nonresonant waves are present in the system. The resonant waves are those which are Cherenkov resonance condition $\omega - \mathbf{k}.\mathbf{v} \doteq 0$ is satisfied on the other hand the nonresonant waves are those for which both the linear and nonlinear conditions are not satisfied, i.e. $\Omega - \mathbf{K}.\mathbf{v} \neq 0$ and $(\Omega - \omega) - (\mathbf{K} - \mathbf{k}).\mathbf{v} \neq 0$. Here, ω and Ω are frequencies of the resonant and the non-resonant waves, respectively and $\mathbf{k}$ and $\mathbf{K}$ are the corresponding wave numbers. Also by weak turbulence theory it meant as the perturbative nonlinear theory based upon Vlasov equation, truncated at the second (or upto third) order nonlinearity and ensemble averaged.

We consider the plasma-maser interaction between the upper hybrid wave and lower hybrid drift wave turbulence. The condition for the plasma-maser is $w = k_{\parallel} v_{\parallel}$ and assuming $\Omega < K v_{\parallel}$, we first estimate the linear part of the dielectric function of the upper hybrid wave from Eq.(25). Considering the fact that for the upper hybrid wave the most dominant contribution to Bessels sums comes from the term $a = b = 0, s = t = p = q = 1$

$$D_0(\mathbf{K}, \Omega) = 1 + \left(\frac{w_{pe}}{K_\perp}\right)^2 \frac{m}{T_e} + \frac{w_{pe}^2}{2\Omega^2}\left[\frac{\Omega}{(\Omega_e - \Omega + K_\perp v_F)} - \frac{\epsilon T_e K_\perp}{m\Omega_e(\Omega_e - \Omega + K_\perp v_F)}\right] \quad (32)$$

We now calculate the imaginary part of the direct coupling term using Eq.(26).

$$Im\, D_d(\mathbf{K}, \Omega) = -\frac{w_{pe}^2}{K_\perp(K_\perp - k_\perp)} \frac{e}{m} \frac{m}{T_e} \frac{\sqrt{\pi}}{v_e |k_\parallel} \Big[E_{l\perp} \frac{\epsilon\prime}{\Omega_e} + 2E_{l\parallel} \frac{v_d}{v_e^2}$$

$$1 + \frac{(K_\perp - k_\perp)^2 v_e^2}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp) v_F)} \left(\Omega - \frac{\epsilon T_e k_\perp}{m\Omega_e}\right)\Big] e^{-\left(\frac{v_d}{v_e}\right)^2} \quad (33)$$

where $v_d = \frac{\omega - k_\perp v_F}{k_\parallel}$ drift velocity of the lower hybrid drift wave turbulence.

Now for considering only the imaginary terms, we have from Eq.(27) as

$$Im\, D_p(\mathbf{K}, \Omega) = -\frac{\omega_{pe}^4}{R|\mathbf{K} - \mathbf{k}|^2} \left(\frac{E_{l\parallel}}{K_\perp}\right)^2 \left(\frac{e}{m}\right)^2 \int\, [E \times ImH + G \times ImF] d\mathbf{v} (34)$$

Now evaluating Eqs.(28-31),we have obtained as

$$E = \frac{E_{l\parallel}}{K_\perp}\left(\frac{m}{T_e}\right)^2 \left[1 + \frac{K_\perp^2 v_e^2}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)}\left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right) + \frac{(K_\perp - k_\perp)^2 v_e^2}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)}\left(\Omega - \frac{\epsilon T_e (K_\perp - k_\perp)}{m\Omega_e}\right) + \frac{K_\perp^2 (K_\perp - k_\perp)^2 v_e^4}{8\Omega_e^4(\Omega_e - \Omega + K_\perp v_F)(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)} \times \left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right)\left(\Omega - \frac{\epsilon T_e (K_\perp - k_\perp)}{m\Omega_e}\right)\right] \quad (35)$$

$$ImF = \frac{\sqrt{\pi}}{v_e|k_\parallel} \left[\left(E_{l\perp}\frac{\epsilon\prime}{\Omega_e} - 2E_{l\parallel}\frac{v_d}{v_e^2}\right)\frac{2}{v_e^2} + \frac{2(K_\perp - k_\perp)^2 v_e^2}{4v_e^2\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)}\left(\Omega - \frac{\epsilon T_e (K_\perp - k_\perp)}{m\Omega_e}\right) + \frac{(K_\perp - k_\perp)^2 v_e^2 k_\parallel^2}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)^2}\right] e^{-\left(\frac{v_d}{v_e}\right)^2} \quad (36)$$

$$G = \frac{m}{T_e K_\perp} E_{l\perp} \frac{m}{(K_\perp - k_\perp)T_e}\left[1 + \frac{K_\perp^2 v_e^2}{4\Omega_e^2(\Omega_e - \Omega + K_\perp v_F)}\left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right) + \frac{(K_\perp - k_\perp)^2 v_e^2}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)}\left(\Omega - \frac{\epsilon T_e (K_\perp - k_\perp)}{m\Omega_e}\right) + \frac{K_\perp^2 (K_\perp - k_\perp)^2 v_e^4}{8\Omega_e^4(\Omega_e - \Omega + K_\perp v_F)(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)}\left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right)\left(\Omega - \frac{\epsilon T_e (K_\perp - k_\perp)}{m\Omega_e}\right)\right] - \frac{m}{T_e K_\perp} E_{l\parallel}\left[\frac{(K_\perp - k_\perp)^2 v_e^2 k_\parallel}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)^2} + \frac{K_\perp^2 (K_\perp - k_\perp)^2 v_e^4 k_\parallel}{8\Omega_e^4(\Omega_e - \Omega + K_\perp v_F)(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)^2}\left(\Omega - \frac{\epsilon T_e K_\perp}{m\Omega_e}\right)\right] \quad (37)$$

$$ImH = \frac{m}{(K_\perp - k_\perp)T_e}\frac{\sqrt{\pi}}{v_e|k_\parallel}\left[\left(E_{l\perp}\frac{\epsilon\prime}{\Omega_e} + 2E_{l\parallel}\frac{v_d}{v_e^2}\right)\left\{1 + \frac{(K_\perp - k_\perp)^2 v_e^2}{4\Omega_e^2(\Omega_e - \Omega + (K_\perp - k_\perp)v_F)}\left(\Omega - \frac{\epsilon T_e (K_\perp - k_\perp)}{m\Omega_e}\right)\right\}\right] e^{-\left(\frac{v_d}{v_e}\right)^2} \quad (38)$$

Here,in Eq.(22) R is expanded about the small argument $\mathbf{k}$ and ω using the relation $D_0(\mathbf{K},\Omega) = 0$. We obtain $R|\mathbf{K} - \mathbf{k}|^2 \sim k_\parallel^2$ at the lowest order approximation.

Now from Eq.(34), we have the dispersion relation of upper hybrid wave by using Eqs.(35-38) considering only the imaginary terms. Upper hybrid wave instability in presence of lower hybrid wave can be investigated in inhomogeneous plasma using these imaginary parts.

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