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MEDICAL DECISION MAKING UNDER PICTURE FUZZY ENVIRONMENT

[1]Palash Dutta[*], [2]Tabendra N. Das, [3]Satyaranjan Dash

[1]Dept of Mathematics, Dibrugarh University, Dibrugarh-786004, India.
[2]Dept. of Mathematics, Dhakuakhana College, Dhakuakhana-787055,India
[3]KIIT University, Bhubeneswar-24, India

Email: [1]Palash.dtt@gmail.com, [2]tabendra2@gmail.com,
[3]satyaranjan.dash@gmail.com

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Abstract:

The vague, partial and imprecise character of medical documents and uncertain data/information gathered for decision making in medical diagnosis requires the use of picture fuzzy set.

This paper aims to perform medical decision making using picture fuzzy set incorporating the refusal degree concept. For this purpose, hybridization between Hausdorff and Hamming distance measures as well as Hausdorff and Euclidean distance measures on picture fuzzy set have been studied. To achieve this objective an algorithm has been proposed and to obtain the patient-disease relationship the hybridized distance measures are used in the proposed algorithm.

Fuzzy set theory plays an important role in medical diagnosis. Various studies have been done so far in medical diagnosis using fuzzy sets, interval valued fuzzy sets, Intuitionistic fuzzy sets, interval valued Intuitionistic fuzzy sets etc. However, in all the theory, the important concept of degree of neutrality is missing which is more often occurred in medical decision making. A medical database has been assembled in the form of PFS. To formulate the data base, for patient A, B, C and D;

five symptoms Temperature, Headache, Stomach pain, Cough and Chest pain and five diseases, Viral Fever, Malaria, Typhoid, Stomach problem and Chest problem are considered. The study revealed that this approach makes it possible to introduce weights of all symptoms and consequently patient can be diagnosed directly.

The performance of the proposed technique is effectively established as it has the ability to initiate weights of all signs of diseases properly and accordingly patients can be diagnosed directly. It is observed that it gives efficient, flexible, simple, logical, technically sound solution for medical decision making problem.

This paper provides an opposite algorithm which enables to successfully carry out medical diagnosis. The major contribution of this paper is to develop a medical diagnostic system based on distance measures on PFSs. The reason for choosing PFS is its ability to address imprecision, uncertainty and vagueness in a proper manner incorporating the concept of degree of neutrality.

Keywords: Medical investigation, Uncertainty, Fuzzy set, Picture Fuzzy Set, Distance Measure.

2010 AMS Classification: 03E72

1. Introduction:

Uncertainty is an integral part of any real world problems and nobody can escape from it. *Fuzzy set theory* (FST) is a well-established mathematical tool to handle uncertainty arises due to lack of precision, deficiency in data, diminutive sample sizes or data acquire from specialist opinion of existing data/information (Zadeh, 1965). After commencement of FST, numerous direct/indirect extensions of have been made and successfully applied in most of the problems of real world situations including medical diagnosis. After that the concept of interval valued fuzzy sets (IVFS) has been developed and named as $\phi-$fuzzy set (Sambuc, 1975).

An important generalization of fuzzy set theory is the theory of intuitionistic fuzzy set (IFS), ascribing a membership degree and a non-membership degree separately in such a way that sum of the two degrees must not exceed one (Atanassov, 1986). Distance measure between IFSs is an important concept in fuzzy mathematics because of its wide applications in real world. Later, another direct extension of IFS is Picture fuzzy set (PFS) incorporating the concept of positive, negative and neutral membership degree of an element (Cuong and Kreinovich, 2013). Nevertheless, different theories have been applied in medical diagnosis, but yet no enough applications have been seen in medical decision making using PFS. It is observed that concept of PFS has more and more importance in medical diagnosis and can effectively be used. For example, PFS based model may be adequate in situations when we face human opinions involving more answers of type: yes, abstain, no, refusal. For example, in a democratic election station, the council issues 500 voting papers for a candidate. The voting results are divided into four groups accompanied with the number of papers namely “vote for” (300), “abstain” (64), “vote against” (115) and “refusal of voting” (21). Group “abstain” means that the voting paper is a whitepaper rejecting both “agree” and “disagree” for the candidate but still takes the vote. Group “refusal of voting” is either invalid voting papers or bypassing the vote (Son, 2016). On the other hand, in case of medical decision making, if someone is suffering from the diseases *stomach* and *chest problems* then it is clear that the symptoms *temperature* and *headache* have no effect on those diseases. That is, in such circumstances, these symptoms may have neutral effect on those diseases. Similarly, symptoms *stomach pain* and *chest pain* have neutral effect on the diseases *viral fever*, *malaria*, *typhoid* etc. On the other hand, symptom *temperature* has direct effect on malaria and in this case, degree of neutrality can be considered as around zero. So, concept of *neutral membership degree* can be considered as an integral part of medical decision making process which is not seen in the other theories yet.

The main objectives of this present study are to present hybridization of Hausdorff and Hamming distance measures as well as Hausdorff and Euclidean distance measures incorporating the refusal degree concept of PFS. Then, to carry out medical diagnosis based on the hybridized distance measures between PFSs.

1.1 Related Works:

Some authors investigated and suggested various methods for measuring distance on IVFSs Bustince and Burillo (1996) established that vague sets are intuitionistic fuzzy sets, Grzegorzewski (2004) proposed distance measures between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric, Zeng and Guon (2008) studies normalized distance, similarity measure, inclusion measure and entropy of interval-valued fuzzy sets and their relationship, Park et al., (2008) studied distances between interval-valued intuitionistic fuzzy sets, Li (2009) performed distance measures between interval-valued fuzzy sets.

Later many distance measures on IFSs have been proposed. Szmidt and Kacprzyk, (1997, 2000, 2006 & 2008) discussed and presented various distance measures on IFSs; Wang and Xin (2005) also studied distance measure between intuitionistic fuzzy sets, Chen (2007) further proposed distance measures between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric, Tcvetkov et al., (2009) presented some issues related to the distances between the Atanassov intuitionistic fuzzy sets, Szmidt and Janusz (2011) studied further on intuitionistic fuzzy sets in the context of a Hausdorff distance, Papakostas et al., (2013) studied distance and similarity measures between intuitionistic fuzzy sets, Szmidt (2014) discussed in details on distances and similarities in intuitionistic fuzzy sets. Some study on PFSs can be seen in literature. Cuong (2014) further discussed picture fuzzy sets, Phong et al., (2014) presented some compositions of picture fuzzy relations, Cuong and Hai (2015) proposed some fuzzy logic operators for picture fuzzy sets, Cuong et al., (2015) studied negator and some De Morgan triples on picture fuzzy sets, Son (2016) presented generalized picture distance measure, Cuong et al., (2016) discussed classification of representable t-norm operators for picture fuzzy sets. FST has been successfully and extensively applied in medical diagnosis. Sanchez (1976) introduced the application of FST in medical diagnosis. After that Sanchez (1979) further discussed medical diagnosis via fuzzy relation equations, Yao and Yao (2001) studied fuzzy decision making for medical diagnosis based on fuzzy number and compositional rule of inference, Elizabeth and Sujatha (2013) applied fuzzy membership matrix in medical diagnosis. IVFNs have been also successfully applied in medical decision making by a few researchers. Chetia and Das (2010) presented an application of interval valued fuzzy soft set in medical diagnosis, Ahn et al., (2011) also discussed an application of interval-valued intuitionistic fuzzy sets in medical diagnosis of headache, Meenakshi and Kaliraja (2011) further applied interval valued fuzzy matrices in medical diagnosis, Choi et al., (2012) presented a medical diagnosis study based on interval valued fuzzy set, Elizabeth and Sujatha (2014) studied medical diagnosis based on interval valued fuzzy number matrices, Dutta (2017) further studied decision making in medical diagnosis via distance measures on interval valued fuzzy. Similarl way, IFSs have also been applied in medical diagnosis. De et al., (2001) applied intuitionistic fuzzy sets in medical diagnosis, Szmidt and Kacprzyk (2001) further discussed application of intuitionistic fuzzy sets in some medical applications, Vlachos and Sergiadis (2007) studied intuitionistic fuzzy information and applied in medical diagnosis, Ye (2009) proposed cosine similarity measures for intuitionistic fuzzy sets and same applied in medical analysis, Own (2009) presented relationship between type-2 fuzzy sets and intuitionistic fuzzy sets and applied in medical diagnosis, Samuel and Balamurugan (2012a,b, 2013) carried out deep study on IFS in medical diagnosis using rank function, correlation coefficient, Boran and Akay (2014) proposed a biparametric similarity measure on intuitionistic fuzzy sets and applied medical decision making, Song et al., (2014) presented a novel similarity measure on intuitionistic fuzzy sets and applied in medical investigation, Davvaz and Hassani (2016) also applied intuitionistic fuzzy sets in medicine. Some extended fuzzy set application can also be obtained in literature. Comas et al., (2011) carried out a survey of medical images using Type-2 Fuzzy Logic, Srivastava an Maheshwari (2016) decision making in medical investigations using new divergence measures for intuitionistic fuzzy sets, Memmedova, (2017) presented a fuzzy logic modelling to evaluate anxiety and aggression levels of students , Mishraand and Prakash, (2018) studied of fuzzy logic in medical data analytics, Samuel, and Narmadhagnanam, (2018) presented an Intuitionistic fuzzy sets approach in medical diagnosis, Dutta and Dash, (2018) studied medical decision making via the arithmetic of generalized triangular fuzzy numbers, Mahmood et al., (2018) discussed an approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets.

1.2 Motivations:

It is obvious that when human beings suffer from diseases they often approach to a medical expert and for diagnosis, medical expert enquiry patients regarding their situation or condition to construct a list of possible symptoms. It is also observed that patients often use the linguistic expression to explain their conditions/situations which are usually vague/partial. Medical expert requires preparing a list of possible symptoms for the respective diseases based on their vague linguistic expressions. On the other hand, the relationships between symptoms and their corresponding diseases are not often one-to-one. Also, knowledge base associating the symptom-disease relationship comprises of imprecision, vagueness and uncertainty in medical decision making process. Therefore, PFS can be used to represent vagueness, imprecision, uncertainty incorporating the important concept i.e., *degree of neutrality* in medical decision making process, which are not been as of now in other theories. Thus, it can be opined that PFS has more importance in medical decision making process and can effectively be used.

1.3 Major Contributions:

The applicability and validation of the proposed approach has been shown by solving a medical decision making problem. The present study made it feasible to initiate weights of all signs of diseases properly and accordingly patients can be diagnosed directly. It is observed that it gives efficient, flexible, simple, logical, technically sound solution for medical decision making problem. It is also obtained that the neutrality membership functions have utmost important role in medical decision making problem as there is always a reasonable possibility of existence of non-zero refusal degree at each point of assessment of every unfamiliar objects. The benefits of this study can be pointed out as follows: it presents an effort to carry out medical diagnosis by considering medical expert's medical knowledge in terms of PFSs; it has more usefulness because of its capability to include neutrality affect of symptoms and diseases together with membership and non-membership values; it provides assistance to medical experts to carry out medical diagnosis providing better efficiency to the output and people living in the rural areas will be mostly benefited.

The guide for rest of the chapter has been condensed as follows. Section 2 explains some basic relevant definitions on FS, IFS, PFS and picture distance measure. Section 3 discusses hybridized distance measures on PFSs. Section 4 presents medical decision making. Section 5 gives details on results and discussions, section 7 presents concrete conclusion of the chapter. \

2. Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory (Zadeh, 1965; Atanassov, 1986; Cuong and Kreinovich, 2013; Dutta*et al.*, 2011) are reviewed.

Fuzzy Set:

Fuzzy set is a set in which every element has degree of membership of belonging in it. Mathematically, let *X*be a universal set. Then the fuzzy subset *A* of *X* is defined by its membership function

$$\mu_A : X \to [0,1]$$

Which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in A$, where the value of $\mu_A(x)$ at *x* shows the grade of membership of *x* in *A.*

Intuitionistic Fuzzy Set:

A Intuitionistic fuzzy set *A* on a universe of discourse X is of the form

$A = \{(x, \mu_A(x), \nu_A(x) : x \in X\},$

Where $\mu_A(x) \in [0,1]$ is called the "degree of membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of non-membership of x in A", and where $\mu_A(x)$ and $\nu_A(x)$ satisfy the following condition:

$$0 \le \mu_A(x) + \nu_A(x) \le 1.$$

The amount $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called hesitancy of x which is reflection of lack of commitment or uncertainty associated with the membership or non-membership or both in A.

Picture Fuzzy Set:

A picture fuzzy set *A* on a universe discourse *X* is an object of the form

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x) : x \in X\},$$

Where $\mu_A(x) \in [0,1]$ is called the "degree of positive membership of x in A", $\eta_A(x) \in [0,1]$ is called the "degree of neutral membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of negative membership of x in A", and where $\mu_A(x)$, $\eta_A(x)$ and $\nu_A(x)$ satisfy the following condition:

$$0 \le \mu_A(x) + \eta_A(x) + \nu_A(x) \le 1.$$

The amount $\rho_A(x) = 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$ is called the degree of refusal of x in A which is reflection of lack of commitment or uncertainty associated with the positive membership, neutral membership, negative membership or all in A.

Picture Distance Measure:

A function $d(A,B)$ with $A, B \in PFS(X)$ is called picture distance measure if it satisfies (Son, 2016):

(i) $0 \le d(A,B) \le 1$,

(ii) $d(A,B) = d(B,A)$,

(iii) $d(A,B) = 0 \Leftrightarrow A = B$

(iv) $\mu_{AB} \times d(A,B) + \mu_{AC} \times d(A,C) \geq \mu_{BC} \times d(B,C) \forall A,B,C \in PFS(X)$

where the symbol $"\times"$ *is the arithmetic product* μ_{AB} , μ_{BC} *and* μ_{AC} *are composition operations of* $A,B,C \in PFS(X)$. *As an example, the following min-max composition formulae are used to calculate the triple* $(\mu_{AB}, \mu_{BC}, \mu_{AC})$, *from the membership functions of* $A,B,C \in PFS(X)$

$$\mu_{AB} = \min_i \left\{ \max \left\{ \mu_A(x_i), \mu_B(x_i) \right\} \right\}$$

$$\mu_{BC} = \min_i \left\{ \max \left\{ \mu_B(x_i), \mu_C(x_i) \right\} \right\}$$

$$\mu_{AC} = \min_i \left\{ \max \left\{ \mu_A(x_i), \mu_C(x_i) \right\} \right\}$$

3. Proposed Hybridized Distance Measures On PFSs:

In this section, considering the refusal degree of membership of PFS hybrid distance measures have been proposed inspired by (Son, 2016). Medical judgment is often tough, because many signs and symptoms are nonspecific. For example, redness of the skin (erythema), by itself, is a sign of many disorders and thus doesn't tell the medical expert what is wrong. Thus differential diagnosis, in which several possible explanations are compared and contrasted, must be performed. This involves the correlation of various pieces of information followed by the recognition and differentiation of patterns. Thus, it is really difficult to find out the relationship between patient's and diseases. Distance measures play important role in medical diagnosis by evaluation the required relationship from the patients-symptoms profile and symptoms-diseases profile. It should be noted that smaller distance gives strong relationship between patient and disease, while huge distance indicates weak relationship. Thus, it can be concluded that shortest distance between patient and disease will indicate that the patient is likely to have the disease.

I) Hybridization of Hausdorff and Hamming distance measures

For $A, B \in PFS(X)$,

$$d_1(A,B) = \frac{X}{X+Y+1} \quad (1)$$

Where,

$$X = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\Delta\mu_i + \Delta\eta_i + \Delta\gamma_i + \Delta\rho_i}{4} + \max\left(\Delta\mu_i, \Delta\eta_i, \Delta\gamma_i, \Delta\rho_i\right)\right)$$

$$Y = \frac{1}{N}\sum_{i=1}^{N}\left(\max\{\varphi_i^A, \varphi_i^B\} + \left|\varphi_i^A - \varphi_i^B\right|\right)$$

II) Hybridization of Hausdorff and Euclidean distance measures

$$d_2(A,B) = \frac{X'}{X'+Y'+1}$$

Where

$$X' = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\Delta\mu_i^2 + \Delta\eta_i^2 + \Delta\gamma_i^2 + \Delta\rho^2{}_i}{4} + \max\left(\Delta\mu_i^2, \Delta_i^2, \Delta\gamma_i^2, \Delta\rho^2{}_i\right)\right)^{1/2}$$

$$Y' = \frac{1}{N}\sum_{i=1}^{N}\left(\max\{\varphi_i^A, \varphi_i^B\} + \left|\varphi_i^A - \varphi_i^B\right|^2\right)^{1/2}$$

$$\Delta\mu_i = \left|\mu_A(x_i) - \mu_B(x_i)\right|, (i = 1, \ldots, N)$$

$$\Delta\eta_i = \left|\eta_A(x_i) - \eta_B(x_i)\right|, (i = 1, \ldots, N)$$

$$\Delta\gamma_i = \left|\gamma_A(x_i) - \gamma_B(x_i)\right|, (i = 1, \ldots, N)$$

$$\Delta\rho_i = |\rho_A(x_i) - \rho_B(x_i)|, (i = 1, 2, ..., N)$$

$$\phi_i^A = \left|\mu_A(x_i) + \eta_A(x_i) + \lambda_A(x_i)\right|, (i = 1, ..., N)$$
$$\phi_i^B = \left|\mu_B(x_i) + \eta_B(x_i) + \lambda_B(x_i)\right|, (i = 1, ..., N)$$

Proof: We shall prove the distance measure-I here. II is left for readers.

Conditions (i)-(iii) are obvious. We need to prove the condition (iv). Son (2016) idea has been adopted to prove the property.

In the extent of this proof, we will show that exists discrete values for the triple $(\mu_{AB}, \mu_{BC}, \mu_{AC})$

Consider p=1. For $A,B \in PFS(X)$, let us denote:

$$AB_1 = \sum_{i=1}^{N} \frac{\Delta\mu_i + \Delta\eta_i + \Delta\gamma_i + \Delta\rho_i}{4},$$

$$AB_2 = \max\{\Delta\mu_i, \Delta\eta_i, \Delta\gamma_i, \Delta\rho_i\},$$

$$AB_3 = \max_i\{\phi_i^A, \phi_i^B\},$$

$$AB_4 = \sum_{i=1}^{N}\left|\phi_i^A - \phi_i^B\right|$$

The following inequality is needed to prove:

$$\frac{AB1 + AB2}{AB1 + AB2 + AB3 + AB4 + 1} + \frac{AC1 + AC2}{AC1 + AC2 + AC3 + AC4 + 1} \geq \frac{BC1 + BC2}{3(BC_1 + BC2 + BC3 + BC4 + 1)}$$

The facts below come from the definition of PFS.

$$\left|\mu_A(x_i)-\mu_B(x_i)\right|+\left|\mu_A(x_i)-\mu_C(x_i)\right|\geq\left|\mu_B(x_i)-\mu_B(x_i)\right|,$$
$$\left|\eta_A(x_i)-\eta_B(x_i)\right|+\left|\eta_A(x_i)-\eta_C(x_i)\right|\geq\left|\eta_B(x_i)-\eta_B(x_i)\right|,$$
$$\left|\gamma_A(x_i)-\gamma_B(x_i)\right|+\left|\gamma_A(x_i)-\gamma_C(x_i)\right|\geq\left|\gamma_B(x_i)-\gamma_B(x_i)\right|.$$

It follows that,

$$AB1+AC1\geq BC1$$
$$AB2+AC2\geq BC2$$

Assume:

$$\max\{\left|\mu_B(x)-\mu_C(x)\right|,\left|\eta_B(x)-\eta_C(x)\right|,\left|\gamma_B(x)-\gamma_C(x)\right|\}$$
$$=\left|\mu_B(x)-\mu_C(x)\right|$$

Then,

$$\left|\mu_B(x)-\mu_C(x)\right|\leq\left|\mu_A(x)-\mu_B(x)\right|+\left|\mu_A(x)-\mu_C(x)\right|$$
$$\leq\max\{\left|\mu_A(x)-\mu_B(x)\right|,\left|\eta_A(x)-\eta_B(x)\right|,\left|\gamma_A(x)-\gamma_B(x)\right|\}$$
$$+\max\{\left|\mu_A(x)-\mu_C(x)\right|,\left|\eta_A(x)-\eta_C(x)\right|,\left|\gamma_A(x)-\gamma_C(x)\right|\}$$

$$\frac{BC1+BC2}{BC1+BC2+BC3+BC4+1}\leq\frac{AB1+AB2}{AB1+AB2+AC1+AC2+BC3+BC4+1} \tag{3}$$

If one of the facts below happen,

$$\max_i\{\phi_i^A,\phi_i^B,\phi_i^C\}=\phi_j^B,$$
$$\max_i\{\phi_i^A,\phi_i^B,\phi_i^C\}=\phi_j^B,$$
$$Then\ BC3\geq AB3$$

Again if

$$\max_i \{\phi_i^A, \phi_i^B, \phi_i^C\} = \phi_j^A,$$

Then

$$\max_i \{\phi_i^A, \phi_i^B\} - \max_i \{\phi_i^B, \phi_i^C\} \le \max_i \{\phi_i^A - \phi_i^B, \phi_i^B - \phi_i^C\} \le \max_i \{\phi_i^A - \phi_i^B + \phi_i^B - \phi_i^C\}$$

$$= \max_i \{\phi_i^A - \phi_i^C\} \le 3AC_2$$

Analogously, we get

$$BC_2 \ge AB_4$$

$$or\ 3AC_2 + BC_4 \ge AB_4$$

Thus,

$$3(AB_1 + AB_2 + AC_1 + AC_2 + BC_3 + BC_4 + 1) \ge AB_1 + AB_2 + AB_3 + AB_4 + 1 \quad (4)$$

$$3(AB_1 + AB_2 + AC_1 + AC_2 + BC_3 + BC_4 + 1) \ge AC_1 + AC_2 + AC_3 + AC_4 + 1 \quad (5)$$

Combining (3), (4) and (5), the inequality is proven. Thus, *d* is a picture distance measure. In a similar way, equation (2) can be shown as a distance measure which is straightforward and hence left for the readers.

4. Medical Decision Making

Medical diagnosis is the most productive and attractive field of applications of fuzzy set theory. In real world problems, due to the imprecise nature of medical documents and uncertain information gathered for decision making requires the use of fuzzy. In medical decision making it is observed that the relationship between symptoms and their corresponding diseases are not often straightforward and distinct. The exhibition of the same disease may not be identical with different patients and even at different disease stages. Moreover, a particular symptom may signify various diseases and again in some situations in a particular patient may disarray the presumed structure of symptoms. However, knowledge base associating the symptom-disease relationship comprises of imprecision, vagueness and uncertainty in medical diagnosis process. Hence, fuzzy sets can be used as the state as well as symptoms of diseases of the patient which can be only identified by medical expert with a very limited degree of accuracy. Therefore, PFSs should be taken into consideration under study as the PFN has the ability to deal with uncertain information in a more flexible manner because refusal degree and degree of neutrality can especially represent the uncertainty more specific way.

The tactic for medical decision making can be recapitulated as: suppose *P*, *S* and *D* are the sets of patients, symptoms and diseases respectively. To identify the patients and their corresponding disease that they are suffering from, it is utmost important to determine the symptoms and accordingly need to formulate the medical knowledge via picture fuzzy relations say, symptoms-diseases fuzzy relation (***Q***) and patient-symptoms fuzzy relation (***R***) with the help of medical experts. To determine the patients-diseases relation (***T)*** the hybrid distance measures (I) and (II) can be employed as a composition operation between *Q* and *R*

Then, the entries of the patients-diseases relation (***T)*** will be real numbers between 0 and 1. It should be noted that the least/smallest value in each row will indicate the suffering of disease of the respective patient. If medical expert does not satisfy with the result, then modification will be done on symptoms-diseases (***Q***) and compute from the beginning. A flowchart has been depicted in figure-1 in for easy understanding of the readers.

5. Case Study

In this part, a hypothetical case study has been carried out to perform medical decision making using the concept of picture fuzzy sets based on the proposed distance measures. Here, it is proposed to take into account the four parameters characterization of picture fuzzy sets: the positive membership degree (μ), neutral membership degree (η), the negative membership degree (ν).

Let *P* = {*A*, *B*, *C*, *D*} be the set of patients admitted in Lakhimpur Medical Hospital hospital in Assam, India, *S* = {*temperature, headache, stomach pain, cough, chest pain*} be the set of symptoms, *D* = {*viral fever, Malaria, typhoid, stomach problem, chest problem*} be the set of diseases. Our intention is to carry out the right decision for each patient $p_i,\ i = 1,2,3,4$ from the set of symptoms $s_j,\ j = 1,2,\ldots,5$ for each disease $d_k,\ d = 1,2,\ldots,5.$

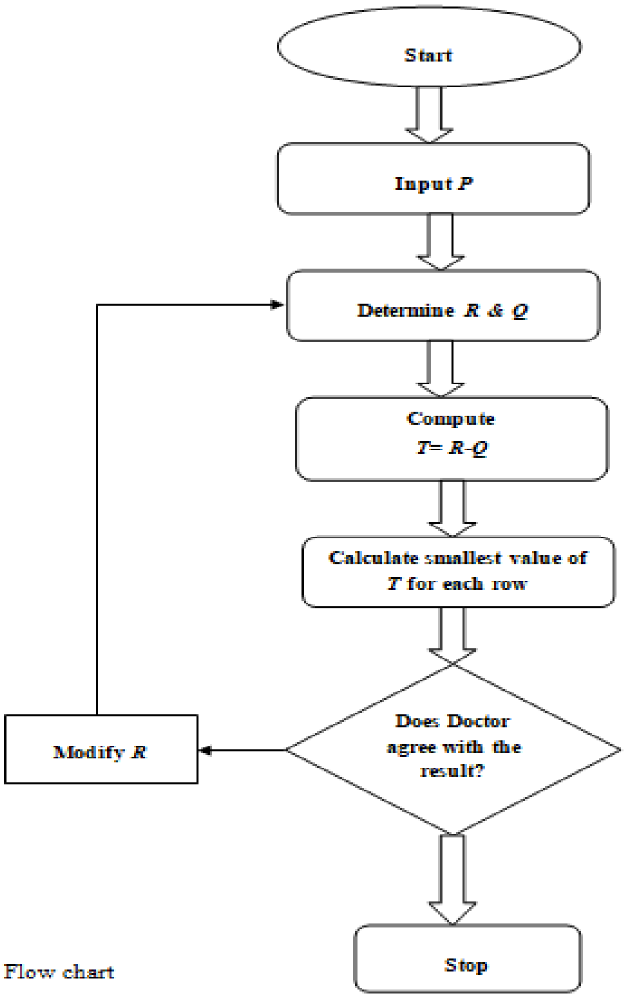

Fig. 1: Flow chart

Fig. 1: Flow chart

The symptom-disease picture fuzzy relation (***R***) and patient-symptom picture fuzzy relation (Q) are given in table-1 and table-2 respectively.

Table-1: symptom-disease picture fuzzy relation

R	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4,0,0,0)	(0.7,0,0)	(0.3,0.4,0.3)	(0.1,0.3,0.5)	(0.1,0.3,0.5)
Headache	(0.3,0.2,0.4)	(0.2,0.4,0.35)	(0.6,0.2,0.1)	(0.2,0.4,0.3)	(0,0.5,0.35)
Stomach Pain	(0.10.35,0.5)	(0,0.4,0.5)	(0.2,0.3,0.4)	(0.8,0,0)	(0.2,0.3,0.5)
Cough	(0.4,0.3,0.2)	(0.7,0.1,0)	(0.2,0.35,0.3)	(0.2,0.4,0.3)	(0.2,0.35,0.4)
Chest Pain	(0.1,0.25,0.5)	(0.1,0.3,0.5)	(0.1,0.2,0.6)	(0.2,0.35, 0.3)	(0.8,0,0.1)

Table-2: patients-symptoms picture fuzzy relation

Q	Temperature	Headache	Stomach Pain	Cough	Chest Pain
A	(0.8,0,0.1)	(0.6,0.3,0.1)	(0.2,0.4,0.4)	(0.6,0.15,0.1)	(0.1,0.4,0.4)
B	(0,0.5,0.4)	(0.4,0.25,0.3)	(0.6,0.2,0.1)	(0.1,0.3,0.6)	(0.1,0.35,0.4)
C	(0.8,0,0.1)	(0.8,0,0.1)	(0,0.4,0.5)	(0.2,0.3,0.4)	(0,0.4,0.4)
D	(0.6,0.2,0.1)	(0.5,0.25,0.25)	(0.3,0.3,0.2)	(0.7,0,0.25)	(0.3,0.4,0.2)

Now, evaluating refusal degree in the data set (i.e., in table-1 & table-2) we have table-3 & table-4.

Table-3: symptom-disease picture fuzzy relation incorporating refusal degree.

R	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4,0,0,0,0.6)	(0.7,0,0,0.3)	(0.3,0.4,0.3,0)	(0.1,0.3,0.5,0.1)	(0.1,0.3,0.5,0.1)

Headache	(0.3,0.2,0.4,0.1)	(0.2,0.4,0.35,0.05)	(0.6,0.2,0.1,0.1)	(0.2,0.4,0.3,0.1)	(0,0.5,0.35,0.15)
Stomach Pain	(0.1,0.35,0.5,0.05)	(0,0.4,0.5,0.1)	(0.2,0.3,0.4,0.1)	(0.8,0,0,0.2)	(0.2,0.3,0.5,0)
Cough	(0.4,0.3,0.2,0.1)	(0.7,0.1,0,0.2)	(0.2,0.35,0.3,0.15)	(0.2,0.4,0.3,0.1)	(0.2,0.35,0.4,0.05)
Chest Pain	(0.1,0.25,0.5,0.15)	(0.1,0.3,0.5,0.1)	(0.1,0.2,0.6,0.1)	(0.2,0.35, 0.3,0.15)	(0.8,0,0.1,0.1)

Table-4: patients-symptomss picture fuzzy relation incorporating refusal degree.

Q	Temperature	Headache	Stomach Pain	Cough	Chest Pain
A	(0.8,0,0.1,0.1)	(0.6,0.3,0.1,0)	(0.2,0.4,0.4,0)	(0.6,0.15,0.1,0.15)	(0.1,0.4,0.4,0.1)
B	(0,0.5,0.4,0.1)	(0.4,0.25,0.3,0.05)	(0.6,0.2,0.1,0.1)	(0.1,0.3,0.6,0)	(0.1,0.35,0.4,0.15)
C	(0.8,0,0.1,0.1)	(0.8,0,0.1,0.1)	(0,0.4,0.5,0.1)	(0.2,0.3,0.4,0.1)	(0,0.4,0.4,0.2)
D	(0.6,0.2,0.1,0.1)	(0.5,0.25,0.25,0)	(0.3,0.3,0.2,0.2)	(0.7,0,0.25,0.05)	(0.3,0.4,0.2,0.1)

The relation between picture fuzzy sets for all the symptoms of the i^{th} –patient from the k^{th} –diagnosis can be calculated by the hybrid distance measures and depicted in table-5 & 6.

Table-5: Patient-disease relation

T	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.0773	**0.0621**	0.0790	0.1294	0.1404
B	0.1021	0.1318	0.0834	**0.0640**	0.1106
C	0.0887	0.0894	**0.0773**	0.1425	0.1431
D	0.0885	**0.0801**	0.0992	0.1163	0.1309

Table-6: Patient-disease relation

T	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
A	0.0610	**0.0494**	0.0657	0.1000	0.1099
B	0.0805	0.1051	0.0613	**0.0468**	0.0871
C	0.0724	0.0789	**0.0617**	0.1174	0.1207
D	0.0627	**0.0592**	0.0745	0.0888	0.0952

6. Results and Discussion:

FST plays a significant role in dealing with uncertainty in medical decision making. Numerous experiments have been done so far in medical decision making using FSs, IVFSs, IFSs, etc. Nevertheless, IFS gained more popularity in medical diagnosis because of its interesting concept of membership and non-membership degree. De and co-workers first presented medical diagnosis with notion of IFS (De *et al.*, 2001)). Then, another study in medical diagnosis has been done using the same IFS medical data and criticised the earlier study having with different results (Szmidt and Kacprzyk 2001). Further studies have been made on medical diagnosis problem for the same IFSs data and obtained different results from the existing studies (Valchos and Sergiadis, 2007; Own, 2009; Ye, 2009; Boran and Akay, 2014; Song et al., 2014). Later, a comparative study has been made via the existing distance measures for the same medical diagnosis problem and obtained different results except with the second study in this sequence (Davvaz and Sadrabadi, 2016). It is seen that for the same diagnosis problem addressed by various researchers using different approaches lead to different chaotic results and this is because of the non inclusion of important concept i.e., *neutrality functions* in the existing studies. The present studyeffectively resolves the drawbacks of existing studies by incorporating the *neutral membership functions* and *refusal degree* in the distance measures on PFSs.

We try to incorporate all the information into analysis as well as PFSs are used to represent uncertainty involved in medical decision making process. In this regard, two hybrid distance measures have proposed and proved in which the refusal degree of membership of PFS has been taken into consideration. Then, to structure the data base, for the patients *A*, *B*, *C and D*; five symptoms *temperature, headache, stomach pain, cough, chest pain;* the five diseases *viral fever, Malaria, typhoid, stomach problem, chest problem* are considered. Finally, patients-diseases relation has been computed using hybrid distance measures (I) and (II). As shortest distance (smallest value) between patient and disease indicates that the patient is likely to have the disease. Therefore, from the table 5-6, it can be observed that smallest values in the first row are 0.0621and 0.0494 which indicates that *A* suffers from Malaria; similarly, in the second rows of table 5 & 6, the smallest values are 0.0640 and 0.0468 which also indicates that *B* suffers from stomach problem, Smallest values in the third and fourth rows are 0.0773 & 0.0617 and 0.0801 & 0.0592 respectively that indicates *C* suffers from typhoid while *D* suffers from malaria.

From this analysis, it is observed that our study provides a realistic result and a rigorous analysis of the IFS medical diagnosis data also corroborates the same fact.

7. Conclusion:

The most important contribution of this article is to build up a medical decision making system based on proposed distance measures on PFSs. The motive for prefer PFS is its ability to address imprecision, uncertainty and vagueness properly incorporating the concept of degree of neutrality in its own way. Most appropriate medical diagnosis has been obtained. Both the distance measures give similar results. These measures made it possible to introduce weights of all symptoms and accordingly patients have been diagnosed directly.

Conflict Of Interest:

The authors declare no conflict of interest.

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