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INVERSE EDGE DOMINATION IN VAGUE GRAPHS

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Abstract

This paper uses the strong arcs for introducing various definitions of the inverse edge dominating sets, inverse edge domination vague set and also the minimum inverse edge domination number pertaining to a vague graph. This paper also investigates some of the properties that are related to above concepts by giving suitable illustrations.

Keywords: Vague graph; Inverse edge dominating set; Inverse edge domination number; Minimum inverse edge domination number; Maximum inverse edge domination number.

2010 AMS classification: 03B60, 06B10, 06B20**.**

1. Introdction

Originally, the inverse domination concept on graphs was proposed by V.R. Kulli and L. S. C. Sigarkanti [5] in the year 1991. After inverse edge domination in graph some new parameters were given by V. R. Kulli and R. Nirmala Nandargi [6] in 1997. A. Nagoor Gani and S. Vasantha Gowri [7] introduced inverse domination using strong arc on the fuzzy type of graphs. Further A. Nagoor Gani and K. Prasanna Devi [8] introduced the inverse edge dominating sets and the domination number which are based on fuzzy type of graphs. J. Johnstephen and N. Vinothkumar [3]

introduced the complementary domination set and complementary kind of domination number to be used in fuzzy kind of graphs which are intuitionistic in nature.
R.A. Borzooeiy, E. Darabianz, and H. Rashmanlou [1, 2] introduced the notion of domination in vague graphs, and obtained the strong domination numbers with applications. Yahya Talebi and Hossein Rashmanlou [10] introduced the concept of application of dominating sets in vague graphs. Recently, the authors [4] introduced the notion of edge domination vague graph, independent edge domination and edge domination numbers of vague graphs and investigated some related properties with illustrations.

In this manuscript, the definitions for the inverse edge dominating set is introduced along with inverse edge domination vague set and inverse edge domination number for vague graphs using strong arc. Among the various usage of inverse domination theory in vague graphs regularly discussed is a wireless communication. This paper also investigates some of the properties that are related to above concepts by giving suitable illustrations.

2. Preliminaries

Definition 2.1 [1] Given a vague graph represented by $G = (P, Q)$ is considered as strong if cardinality of the edge is $t_Q(v_i v_j) = \min\{t_P(v_i), f_P(v_j)\}$ and $f_Q(v_i v_j) = \max\{t_P(v_i), f_P(v_j)\}$ for all $v_i v_j \in Q$.

Definition 2.2 [2] Consider $G = (P, Q)$ as a vague graph, where in $P = (t_P,\ f_P)$ and $Q = (t_Q,\ f_Q)$ are vague sets on V and $Q \subseteq P \times P$ respectively. It defined as $t_Q(uv) \leq \min\{t_P(u), f_P(v)\}$ and $f_Q(uv) \geq \max\{t_P(u), f_P(v)\}$ for all $uv \in Q$.

Definition 2.3[10] let u denote a vertex of a vague graphs represented by $G = (P, Q)$, the neighborhood of u is represented as

$$N(u) = \left\{v \in \frac{V}{u,v} \text{is considered as a strong arc}\right\}.$$

Definition 2.4[2] Let e_i be an edge in vague graph the $G = (P, Q)$. Then, the strong neighborhood is $N_s(e_i) = \{e_i, e_i \in Q(G) \text{and } (e_i, e_j) \text{is a strong edge in } G\}$.

Definition 2.5[1] Consider $G = (P, Q)$ as a vague graph, the edges (e_i, e_j) are considered to be strong. If $t_Q(uv) \geq (t_Q)^{\infty}(uv)$ and $f_Q(uv) \leq (f_Q)^{\infty}(uv)$,

Where $(t_Q)^{\infty}(uv) = \max\{(t_Q)^k(uv): k = 1,2,\ldots,n\}$ and

$$(f_Q)^{\infty}(uv) = \min\{(f_Q)^k(uv): k = 1,2,\dots,n\}.$$

Definition 2.6[4] Let $G = (P,Q)$ be a vague graph. Then e_i and e_j be a two edges of vague graph of G.We say that e_i edge dominates e_j, if e_i is a strong arc in G and adjacent to e_i.

Definition 2.7[4] The minimum vague cardinality taken all the minimal edges which are in dominating set are called as lower edge dominating number of G and it is represented as $\gamma(G)$.

Definition 2.8 [1] The total count of edge, that is, the cardinality value of Q is termed as the order size in a vague graph and it is represented as

$$O(S) = \sum_{v_i v_j \in Q} \left(\frac{1 + t_Q(v_i v_j) - f_Q(v_i v_j)}{2} \right) \text{ for all } v_i v_j \in Q.$$

Definition 2.9[4] The maximum vague cardinality is taken by considering all the minimal edges of the dominating set and it is termed as the upper edge dominating number pertaining to G and are denoted by $\gamma(G)$.

Definition 2.10[4] Consider $G = (P,Q)$ as a vague graph, which is considered as an isolated edge if the graph is not present adjacent to any of the strong edges in G.

3. Inverse edge domination

In the following section, the inverse edge dominating set and the inverse edge dominating number for a vague graph are introduced. Also, few results on the same is discussed.

Definition 3.1 Consider $G = (P,Q)$ as a vague graph, which has a minimal edge dominating set inside D. In this case, $Q(G) - D$ also has the edge dominating set D^{-1} of G, where D^{-1} is termed as the inverse edge dominating set of G.

Example 3.2 Let $G = (P,Q)$ be a vague graph as shown in the Fig. 1 from the edge set $Q = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. Then, we have $e_1, e_4, e_5, e_6,$ and e_7 are strong arcs of the vague graph. Here, $\{e_2, e_4\}$, $\{e_1, e_4, e_7\}$,$\{e_1, e_4\}$ and $\{e_2, e_5\}$ are edge dominating set of G, for $D = \{e_1, e_4\}$ are minimum edge dominating set of G. Then the sets $\{e_2, e_4\}$, $\{e_1, e_4, e_7\}$ and $\{e_2, e_5\}$ are inverse edge dominating sets of vague graph G.

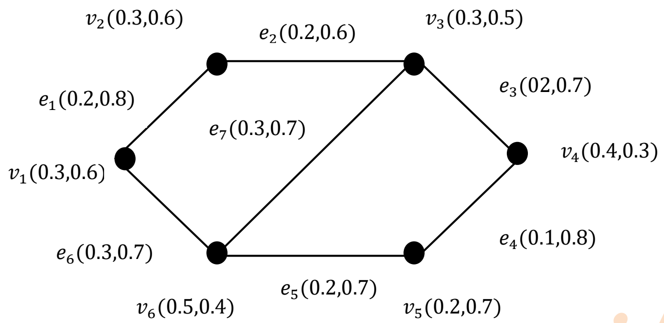

Fig. 1: Inverse edge dominating sets of vague graph

Definition 3.3 Consider $G = (P, Q)$ as a vague graph, which has a minimal inverse edge dominating set of a given vague graph is termed as the inverse edge dominating number of the graph, and it is represented as $\gamma^{-1}(D)$ in G.

Example 3.4 Consider that the Fig.1 is represented as $\{e_{2,}e_4\}$ which has a minimal inverse edge dominating set belongs to G. In this case, the minimum inverse dominating number will be $\gamma^{-1}(D) = 0.45$.

Proposition 3.5 If $G = (P, Q)$ is a vague graph, then $2\gamma^{-1}(D) \leq O(S)$.

Proof: Consider $G = (P, Q)$ as a vague graph, then the minimum inverse edge dominating set is represented as D^{-1} in correspondence to D. It is obtained from definition 3.1 that,

$$D^{-1} \subseteq Q(G) - D \Longrightarrow |D^{-1}| \leq |Q(G)| - |D|.$$

Therefore, $\gamma^{-1}(D) \leq O(S) - \gamma(G)$.

From the definition 3.1, we have $\gamma^{-1}(D) \leq O(S) - \frac{O(S)}{2}$

$$\gamma^{-1}(D) \leq \frac{O(S)}{2}$$

Therefore, $2\,\gamma^{-1}(D) \leq O(S)$.

Example 3.6: Consider $G = (P, Q)$ as a vague graph as depicted in Fig. 2. Given the edge set $Q = \{e_{1,}e_{2,}e_{3,}e_{4,}\}$, we see that $\{e_1\}$ and $\{e_4\}$ are strong arcs in the graph G.Then $\{e_1\}$ is an edge dominating set and $\{e_4\}$ is an inverse edge dominating set.

So edge dominating number is $\gamma(G) = 0.20$ and the inverse edge dominating number is $\gamma^{-1}(D) = 0.35$. Then, we have order size of the vague graph is $O(S) = 0.55$.

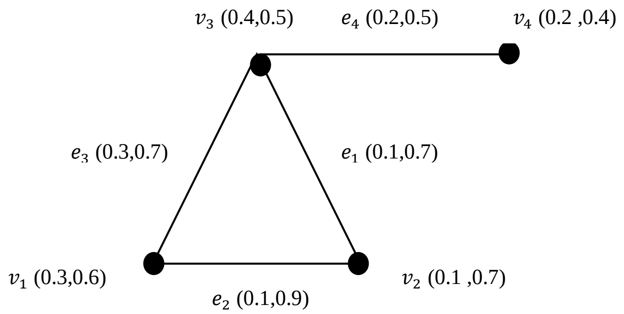

Fig. 2: Inverse edge dominating number

Proposition 3.7 Consider $G = (P, Q)$ as a vague graph, it has two is joined edge dominating set if and only if G has an inverse edge dominating set..

Proof: Consider D as a large edge dominating set which has at least two disjointed edge dominating set vague graph of G. Then D have be any minimum edge dominating set of vague graph G. We consider D_1 and D_2 two disjoint edges dominating set vague graph of G.
To prove: G has an edge dominating set $D \subseteq D_1 \cup D_2$
Consider G and D as the vague graph and its edge dominating set. Then, it has an at least two disjoint edge dominating set. Now, $D \subseteq D_1$, then D and D_2 are disjoint sets. Here, other edge dominating set $D_2 \subseteq S - D$. Therefore, G has also inverse edge dominating set.

Consider G and D as the vague graph and its edge dominating set. Then, it has an at least two disjoint edge dominating set. Now, $D \subseteq D_2$. Then, D and D_1 are disjoint sets. Here, other edge dominating set $D_1 \subseteq S - D$. Therefore, the vague graph G has also inverse edge dominating set. Thus, G has an edge dominating set $D \subseteq D_1 \cup D_2$.

Conversely, let $|D| = |D_1|$ considered having a minimum edge domination set in G and so the D_2 is taken as the inverse domination set. Here, the vague graph G is considered to have an inverse edge dominating set.

Let $|D| = |D_2|$ considered to have a minimum edge domination set in G and so the D_2 is taken as the inverse domination set. Here, the vague graph G is considered to have an inverse edge dominating set.

Example 3.8 Consider $G = (P, Q)$ as a vague graph as depicted in Fig.3 and the set $Q = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}$. Then, we have $\{e_5\}$, $\{e_3, e_4\}$ are disjoined edge dominating set of in G. Here, $\{e_5\}$ is a minimum edge dominating set and $\{e_3, e_4\}$ is an inverse edge dominating set of G.

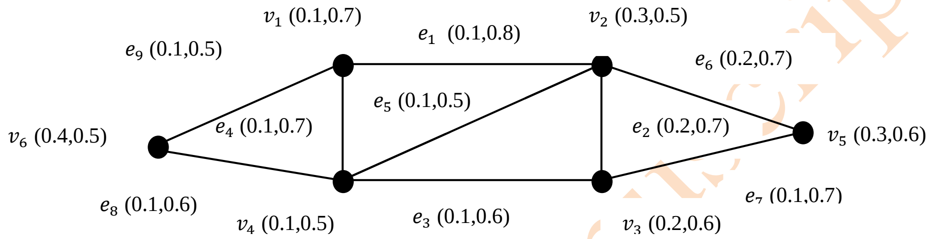

Fig 3: Minimum edge dominating set

Proposition 3.9 Consider $G = (P, Q)$ as a vague graph, and in case of G having only a single strong edge, then $\gamma^{-1}(D)$= 0

Proof: Consider $G = (P, Q)$ as a vague graph and D is the minimal edge dominating set in G which has only one strong arc. Then there exists $D^{-1} \in Q(G) - D$ is an inverse edge dominating set in G.Then, we have the edge dominating number which is $|Q(G)| = |D| = O(S)$

The definition of 3.1 inverse edge domination set is denoted by $D^{-1} \in Q(G) - D$.

Then, we have $|D^{-1}| = |Q(G) - D|$

By the triangle inequality $|D^{-1}| \leq |Q(G)| - |D|$

From the definition of 3.3, we have $|\gamma^{-1}(D)| \leq |Q(G)| - |D|$

$$|\gamma^{-1}(D)| = 0$$

$$\gamma^{-1}(D) = 0.$$

Example 3.10 Consider $G = (P, Q)$ as a vague graph as depicted in Fig. 4. As in the edge set $Q = \{e_1, e_2, e_3, e_4, e_5\}$. Also, we have only one strong arc e_3 and e_4 $\{e_3, e_4\}$ is an edge domination set in the vague graph G no any other strong arc, then, it has no inverse edge dominating set in G.

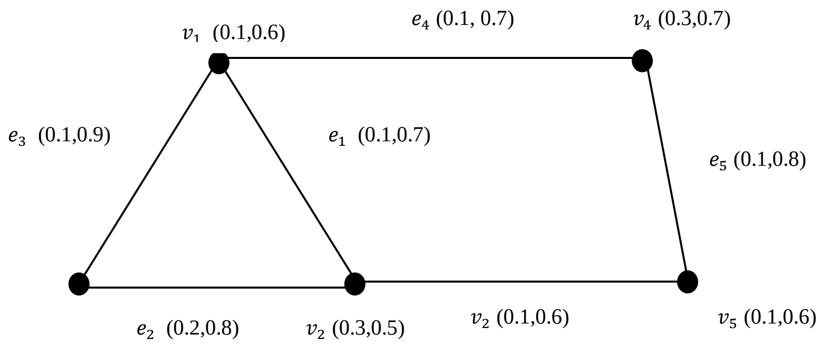

Fig. 4: No inverse edge dominating set

Proposition 3.11 Let $G = (P, Q)$ be a vague graph, then D has both minimal and maximum edge dominating set from G and do not have a inverse edge domination when compared to D.

Proof: Consider $G = (P, Q)$ is a vague graph and D is an edge dominating set of G having only one strong arc.

Here, minimal edge domination set of minimal cardinality $d_e(G)$ and minimal dominating set of maximal cardinality $D_e(G)$ are same. Here, every minimum edge dominating set of G is a maximal edge dominating set of G, then G does not have the inverse edge set corresponding to any of the minimal edges of G. Hence, G do not have even a single inverse edge dominating set.

Example 3.12 Consider $G = (P, Q)$ as a vague graph as presented in Fig. 5. From the edge set $Q = \{e_1, e_2, e_3\}$, from Fig. 5, $\{e_2\}$ have the only one strong arc. Then, we have $\{e_2\}$ is a minimum and maximum of edge dominating set of G. Hence there are no inverse edge dominating set of G.

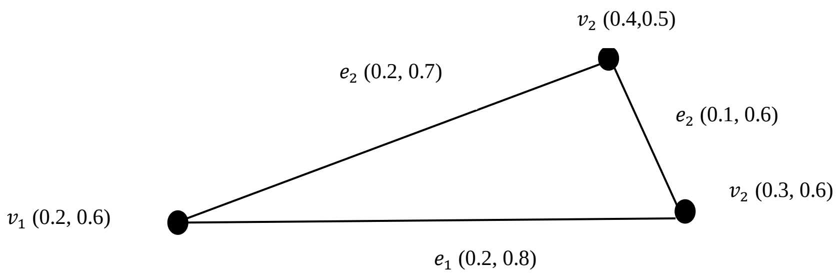

Fig. 5: Minimum and maximum of edge dominating set

Proposition 3.13 Consider $G = (P, Q)$ as a vague graph, it has at least one inverse edge dominating set, then $\gamma(G) \leq \gamma^{-1}(D)$.

Proof: Consider $G = (P, Q)$ as a vague graph and D be the edge dominating set. Then, G having at least one inverse edge dominating set..

From the definition 3.1, any inverse edge dominating set of vague graph of G, that is D^{-1} also a edge dominating set.

Here, $\gamma(G)$ is a minimum cardinality of edge dominating set and $\gamma^{-1}(D)$is a minimum cardinality of inverse edge dominating set of G.

Example 3.14 Let $G = (P, Q)$ be a vague graph as shown in the Fig. 6. From the edge set $Q = \{e_{1,}e_{2,}\}$. Then, we have $\{e_1\}$ and $\{e_2\}$ are edge dominating set of G. Here, $\{e_1\}$ is an edge dominating set and $\{e_2\}$ is a inverse edge dominating set of a vague graph.
Also, $\{e_1\}$ is minimal edge dominating number is $\gamma(G)$ = 0.25, and minimal inverse edge dominating number is $\gamma^{-1}(D)$ = 0.40.
Therefore, $\gamma(G) \leq \gamma^{-1}(D)$.

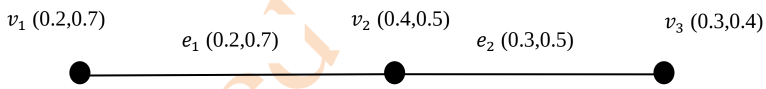

Fig. 6: Minimal inverse edge dominating number

Proposition 3.15 Let $G = (P, Q)$ be a vague graph, it has edge dominating set D without isolated edge. Then it has at least one inverse edge dominating set as in G.

Proof: Consider $G = (P, Q)$ as a vague graph that has a minimum edge with a dominating set represented by D and such as $Q(G) - D$ is not a edge dominating set. Here, there exist edges such as $e_i \in D$.Such that e_j is not a dominated by any edge $Q(G) - D$. Since G is not an isolated edge. Here e_i is considered as the strongest neighbor of one of the edge $Q(G) - D$. Hence, $Q(G) - D$ is determined to be the dominating set that is opposite to the minimality of the $Q(G)$.

Example 3.16 Consider $G = (P, Q)$ as a vague graph as presented in Fig. 7 and from the set $Q = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.Then, we have $\{e_1, e_3\}$, $\{e_2\ e_4\}$,$\{e_3, e_5\}$ are edge dominating set of edges in G. So this not an isolated edge dominating set.

Here, $\{e_1, e_5\}$ is the minimum edge dominating set of G and $\{e_1, e_3\}$ or $\{e_2\ e_4\}$ are the inverse edge dominating set of the given vague graph G.

Remarks 3.17: A vague graph have an edge dominating set D without isolated edge. Then it has an at least one inverse edge dominating set in G.

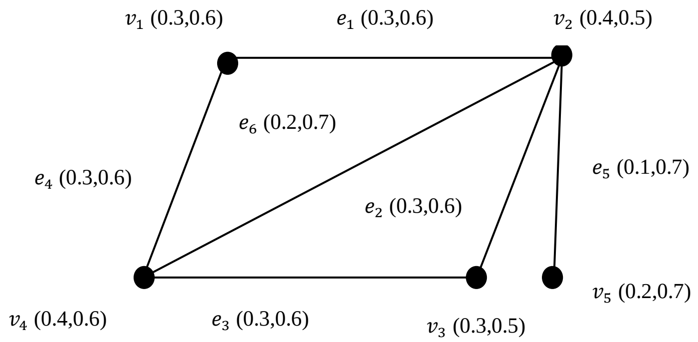

Fig. 7: Inverse edge dominating set

Conclusion

In this manuscript, the definitions of the inverse edge dominating set is introduced along with inverse edge domination vague set and inverse edge domination number for the vague graphs using strong arc. This paper also investigated some of the properties that are related to above concepts by giving suitable illustrations. The expansion of this research work is application of inverse edge domination of vague graph in the area of wireless networks.

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References

[1] Borzooei, R. A. and Rashmanlou, H. (2015), *Domination in Vague Graphs and its Applications, Journal* of Intelligent and Fuzzy Systems 29, 1933-1940.

[2] Borzooei, R. A., Darabian, E. and Rashmanlou, H. (2016), *Strong Domination Numbers of Vague Graphs with Applications*, Applied Mathematics E-Notes 16, 251-267.

[3] Johnstephen, J. and Vinothkumar, N. (2017), *Complementary Domination in Intunionistic Fuzzy Graphs*, International Conference on Mathematical Impacts in Science and Technology, 124-128.

[4] Kaliraja, M., Kanibose, P. and Ibrahim, A. (2020), *Edge Domination in Vague Graph*, International Journal of Advanced Science and Technology Volume 29, Number 1, 1474-1480.

[5] Kulli, V.R. and Sigarkanti, L.S.C. (1991), *Inverse domination in graphs*, National Science Academy Letters 14(12), 473-475.

[6] Kulli, V.R. and Nirmala Nandargi, R. (2012), *Inverse Domination and Some New Parameters*, Advances in Domination Theory-I, 15-24.

[7] Nagoor Gani, A. and Vasantha Gowri, S. (2017), *Inverse Fuzzy Domination Using Strong Arc*, International Journal of Pure and Applied Mathematics Volume 117, Number 11, 9-15.

[8] Nagoor Gani, A. and Prasanna Devi, K. (2014), *Inverse Edge Domination in Fuzzy Graphs*, Advances in Fuzzy Sets and Systems, Volume 18, Number 2, 87-100.

[9] Yahya Talebiy, and Hossein Rashmanlouz, (2017), *Application of Dominating Sets In Vague Graphs*, Applied Mathematics, E-Notes 17, 251-267.

[10] Zadeh, L. A. (1965), *Fuzzy sets*, Information and control 8, 338 -353.