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HEAT AND PARTIAL SLIP IMPACT ON ELASTICO-VISCOUS FLUID FLOW PAST A FLAT PERMEABLE PLATE

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Abstract

An investigation is initiated to examine the heat transport and slip effects on steady elastico-viscous boundary layer fluid motion past a flat permeable plate. The non-Newtonian fluid model Walters Liquid (Model B') is taken for elastico-viscous fluid. The special forms of slip factors involving local Reynolds number are considered. Using similarity variables, the governing equations of fluid motion along with boundary conditions are reduced to self-similar form to solve by inbuilt MATLAB solver 'bvp4c'. The computed results of velocity, temperature, temperature gradient at the plate and skin-friction coefficients are plotted for discussion to find the impact of involved flow parameters. The study reveals that elastico-viscosity plays a significant role to enhance the fluid velocity, heat transport rate and friction at the surface.

Keywords: Boundary layer, elastic-viscous fluid, heat transfer, similarity variables, temperature gradient, velocity slip.

2010 AMS classification: 76A05, 76A10

1. Introduction

The elastico-viscous fluid has little temperature and salinity sensitivity, and is easily regulated by rheology. The elastico-viscous fluid technology generates highly efficient fractures with low damage to conductivity providing excellent control of fluid loss and high properties of proppant transport to generate geometry of design fractures. The rheology of elastico-viscous fluid are used to enhance biomodeling. Biofilms are also elastico-viscous materials that are capable of dissipating energy from external forces and overcoming external mechanical stresses.

The elastico-viscous boundary layer fluid motion through permeable surface along with heat transport in presence of partial slip has influenced researchers a lot these days. The application of such flow is often noticed in engineering and industrial processes, especially in the petroleum and chemical industry. The fluid flow over the porous surface has lots of applications in filtration and purification processes, metal processing, heat exchangers, catalytic reactors, insulation, etc. The physical concept of temperature variation between the plate and the surrounding fluid has many geothermal and engineering applications.

Blasius (1908) firstly studied the progress of velocity in the boundary layer when fluid flows over a flat surface. Pohlhausen (1921) investigated the heat transport phenomenon of Blasius result. The numerical investigation of Blasius findings of flow problems was further carried out by Howarth (1939). Cheng and Minkowycz (1977) presented the fluid flow in a porous media due to natural convection. Rabadi and Hamdan (2000) demonstrated the natural convection fluid flow and heat transport over the inclined plate ingrained in a porous medium with the variation of permeability. Mukhopadhyay and Layek (2009) studied numerically the convective heat transport mechanism of boundary layer fluid motion past a permeable plate and analyzed the impact of radiation over temperature field.

Ishak (2010) investigated the fluid motion and heat transport through a permeable plate considering convection heating at the boundary surface. Bhattacharyya *et al.*(2012) examined numerically using the shooting method the fluid flow through boundary layer past a flat moving surface taking slip effects at the boundary. Bhattacharyya and Layek (2012) also presented the solute diffusion due to the chemical reaction for boundary layer Newtonian fluid past a permeable plate. Abbas *et al.*(2009) studied the heat transfer phenomenon taking slip condition at the boundary of viscous fluid past an infinite oscillating sheet. Khan *et al.*(2014) demonstrated the heat transport and slip effects over a flat surface for carbon nanotubes. Ambreen *et al.*(2016) discussed the heat transport and slip impact on hydromagnetic peristaltic non-Newtonian fluid flow. Sarojamma *et al.* (2018) studied the stratified casson fluid motion and the heat transition taking radiation effect into account. Izadi *et al.*(2019) presented the numerical solution of nanofluid containing microorganisms past a stretching sheet imposing slip condition at the sheet and studied the heat transfer mechanism.

Inspired by the above mentioned studies, this paper aims to analyze the elastico-viscous boundary layer fluid flow and heat transport with partial slip effects over a porous flat plate. The non-Newtonian fluid model Walters Liquid (Model B')(1960, 1962) is considered for viscous and elastic characteristics. The self-similar reduced governing equations together with boundary conditions are evaluated employing MATLAB inbuilt solver 'bvp4c'. The numerically computed values are presented with graphs to analyze the impact of different involved flow feature parameters for discussion.

2. Mathematical Formulation

The incompressible steady two-dimensional elastico-viscous boundary layer fluid motion past a permeable flat plate with slip effects is considered. The geometrical fluid flow model is displayed in Fig.1. The fluid flow governed by equations of continuity, momentum, and energy (2011, 1968) taking boundary layer approximations into account are given by:

$$u_x + v_y = 0 \tag{1}$$

$$u u_x + v u_y = \nu u_{yy} - \frac{\mathrm{k_0}}{\rho}\left(u u_{xyy} + v u_{yyy} - u_y u_{xy} - v_y u_{yy}\right) - \frac{\nu}{\mathrm{k}}(u - U_\infty) \tag{2}$$

$$u T_x + v T_y = \left(\frac{\mathrm{K}}{\rho C_p}\right) T_{yy} \tag{3}$$

where $\nu = \frac{\mu}{\rho}$.

The relevant boundary conditions taking slip effects are as follows:

$$u = G_1 u_y,\ v = v_w \ \text{ at } \ \mathrm{y} = 0; \quad u \to U_\infty \text{ as } y \to \infty \tag{4}$$

$T = T_W + H_1 T_y$ at $y = 0$; $T \to T_\infty$ as $y \to \infty$ (5)

where, $G_1 = G_0 (Re_x)^{\frac{1}{2}}$, $H_1 = H_0 (Re_x)^{\frac{1}{2}}$, $Re_x = \frac{U_\infty x}{\nu} v_W = \frac{v_0}{(x)^{\frac{1}{2}}}$.

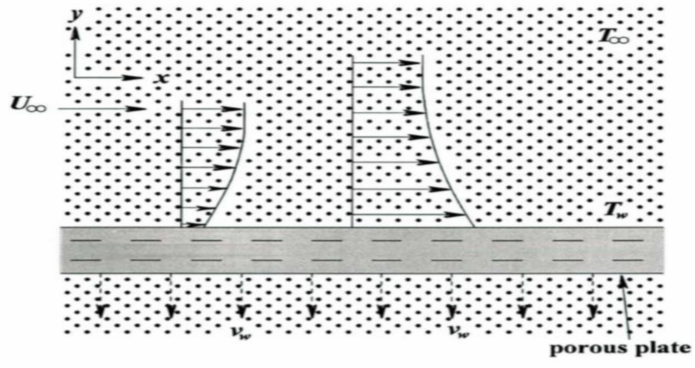

Figure 1 Geometrical model of the flow problem

A careful inspection gives the following set of similarity transformations:

$$\Psi = \sqrt{U_\infty \nu x}\, f(\eta), T = T_\infty + (T_w - T_\infty)\theta(\eta), \text{ and} \eta = \frac{y}{x}\sqrt{Re_x}\,, \quad (6)$$

where Ψ denotes the stream function satisfying $u = \frac{\partial \Psi}{\partial y}$and $v = -\frac{\partial \Psi}{\partial x}$.

Using similarity transformations, equations (6), (2) and (3) finally reduced to self-similar forms as follows:

$$f''' + \frac{1}{2} f f'' + k_1[2f'f''' + f f^{\text{iv}} - (f'')^2] + k^*(1 - f') = 0 \quad (7)$$

$$\theta'' + \frac{1}{2} Pr f \theta' = 0 \quad (8)$$

where dashes denote differentiation with respect to η and$k_1 = \frac{k_0 U_\infty}{2\mu x}$, $k^* = \frac{1}{Da_x Re_x}$, $Da_x = \frac{k}{x^2}$, $Pr = \frac{\mu C_p}{K}$.

The reduced form of boundary conditions (4) and (5) are:

$$f(\eta) = S,\ f'(\eta) = \delta f''(\eta)\ at\ \eta = 0\,; \quad f'(\eta) = 1,\ f''(\eta) = 0\ as\ \eta \to \infty \quad (9)$$

$$\theta(\eta) = 1 + \beta\theta'(\eta) at\ \eta = 0\,;\ \theta(\eta) = 0\ as\, \eta \to \infty \quad (10)$$

where, $S=\left(-\frac{2v_w}{U_\infty}\right)(Re_x)^{\frac{1}{2}}=-\frac{2,v_0}{(U_\infty \nu)^{\frac{1}{2}}}$, $S>0$ (for$v_0<0$) represents suction and $S<0$ (for$v_0>0$) represents blowing, δ=$\frac{G_0 U_\infty}{\nu}$ and β=$\frac{H_0 U_\infty}{\nu}$.

3. Method of Solution

The inbuilt numerical method 'bvp4c' of Matlab(1995, 2001) is a collocation method used to solve differential equations of the form $\frac{dy}{dx}=\mathrm{g}(\mathrm{x},\mathrm{y},\mathrm{q})$, $\mathrm{x}\in[a,b]$ subject to non-linear boundary conditions $\mathrm{h}(\mathrm{y}(\mathrm{a}),\mathrm{y}(\mathrm{b}),\mathrm{q})=0$, where q is an unknown parameter. This method is an effective solver different from the shooting method and it is based on an algorithm. It can compute inexpensively the approximate value of $y(x)$ for any x in $[a,b]$ taking boundary conditions at every step. In this method, infinity conditions at the boundary are replaced with some finite point which reasonably satisfies the given problem.

The self-similar governing equations (7) and (8) are transformed to differential equations of the first order as follows:

$$f=f_1, f'=f_2,\ f''=f_3, f'''=f_4\ ,\theta=f_5, \theta'=f_6 \quad (11)$$

From (11), we can write

$$f_1'=f_2,\qquad f_2'=f_3,\ f_3'=f_4,\qquad f_5'=f_6 \quad (12)$$

In view of equations (11) and (12), reduced governing equations (7) and (8) and boundary conditions (9) and (10) can be written as:

$$f_4'=\frac{1}{f_1}\left[(f_3)^2-2f_2f_4-\left(\frac{1}{k_1}\right)\left\{f_4+\frac{1}{2}f_1f_3+k^*(1-f_2)\right\}\right] \quad (13)$$

$$f_6'=-\frac{1}{2}Prf_1f_6 \quad (14)$$

$$f_1(0)=S, f_2(0)=\delta f_3(0)\ and\ \ f_2(\infty)=1, f_3(\infty)=0 \quad (15)$$

$$f_5(0)=1+\beta f_6(0)\ and\ \ f_5(\infty)=0 \quad (16)$$

The MATLAB inbuilt solver 'bvp4c' is employed to compute the above equations together with different involved flow feature parameters.

4. Results And Discussion

The expression for friction at the surface for the above flow problem is obtained from

$$\tau = f''(0) + k_1 f(0) f'''(0) \qquad (17)$$

The expressions for velocity $(f'(\eta))$, temperature $(\theta(\eta))$, temperature gradient $(-\theta'(0))$ and skin friction coefficient (τ) are obtained numerically using MATLAB built-in 'bvp4c' solver for involved dimensionless flow parameters viz., elastico-viscous parameter (k_1), permeability parameter (k^*), velocity slip parameter (δ), suction/blowing parameter (S), thermal slip parameter (β) and Prandtl parameter (Pr). The computed results are graphically presented to observe the effects of involved flow parameters to explain the physical reasons.

To judge the accuracy of the numerically obtained results by 'bvp4c' and to validate the present work, the skin friction coefficient is evaluated without considering elastico-viscous and permeability parameters and is obtained as $f''(0) = 0.3321$ which is well accord with the standard results obtained by Howarth (1939) as $f''(0) = 0.33206$ and Bhattacharyya and Layek (2012) as $f''(0) = 0.332058$.

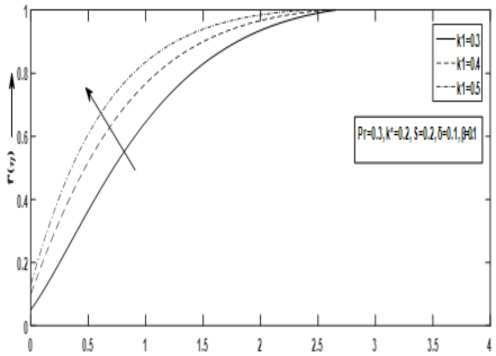

Fig.2. Effects of k_1 on velocity curve $f'(\eta)$ against η

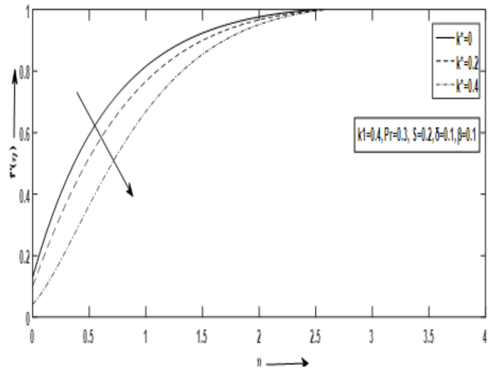

Fig.3. Effects of k * on velocity curve $f'(\eta)$ against η

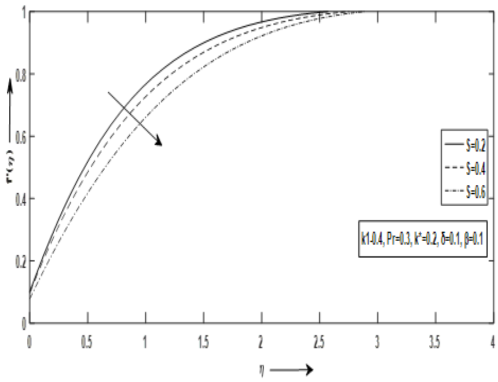

Fig.4. Effects of S on velocity curve $f'(\eta)$ against η

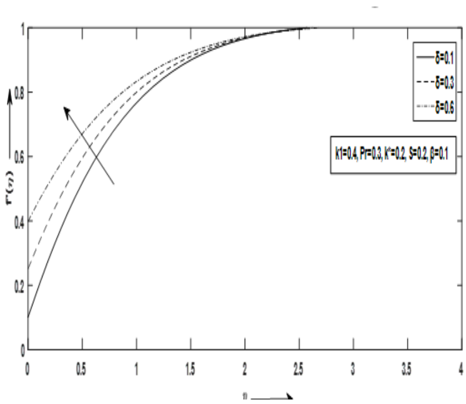

Fig.5. Effects of δ on velocity curve $f'(\eta)$ against η

Figures 2-5 illustrate the velocity distribution for variation of involved flow parameters. The velocity curves $f'(\eta)$ tends to 1 very rapidly as η →3 and thus considering 3 as infinity appears to justify the boundary conditions at infinity. Figure 2 depicts the velocity curves for different incremental values of k_1 and it's clear that velocity $f'(\eta)$ enhances with the growth of $\mathrm{k}_1$and consequently diminishes the momentum boundary layer thickness. The thermal motion deforms the polymers of visco-elastic material and hence increase the fluid motion. Figure 3 shows the effect of the permeability parameter k^*on the velocity curves. It is noticed that the velocity $f'(\eta)$ on the plate decelerates with rising values of k^*.With the growth of the permeability parameter, Darcian body force diminishes and thus reduces the fluid motion.As the fluid is elastico-viscous so considerable high drag is accomplished and thus the velocity retarded. Figure 4 displays the velocity curves for suction parameters S and it is noticed that the flow rate decelerates with increasing values of S.As the particles of the fluid absorbed through the permeable plate, the fluid motion diminishes and causes a reduction in the thickness of velocity boundary layer.The velocity profile $f'(\eta)$ for variation of slip factor δ is plotted in Figure 5.The rate of fluid transport enhances as the slip factor δ rises due to frictional resistance between the fluid and the boundary surface. The positive values of velocity close to the plate is noticed due to slip and thus the thickness reduces for momentum boundary layer.

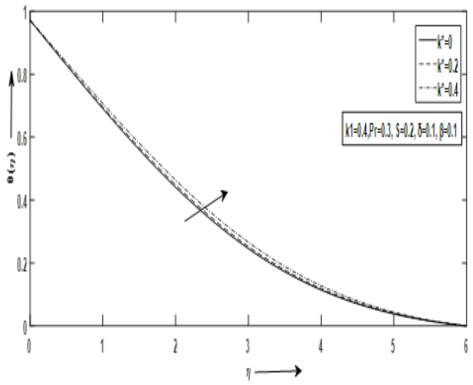

Fig.6. Effects of k_1 on temperature curve $\theta(\eta)$ against η

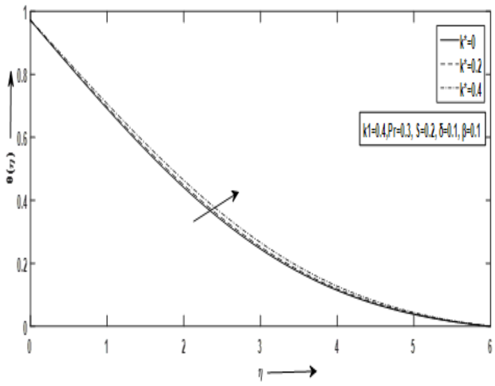

Fig.7. Effects of k* on temperature curve $\theta(\eta)$ against η

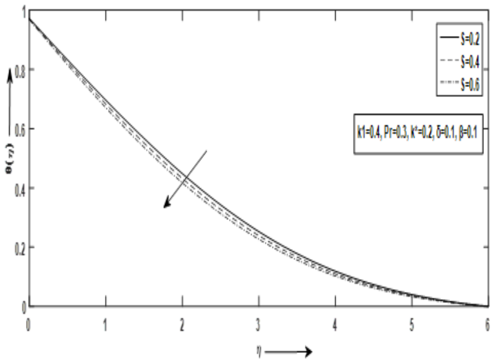

Fig.8. Effects of S on temperature curve $\theta(\eta)$ against η

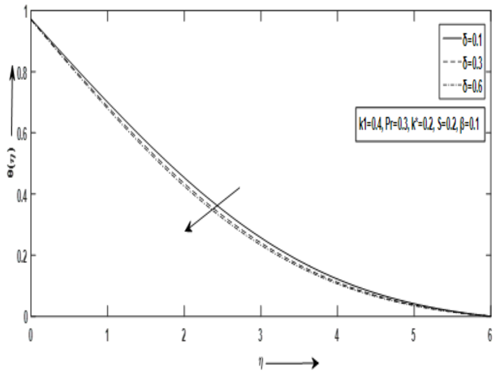

Fig.9. Effects of δ on temperature curve $\theta(\eta)$against η

The temperature curves are graphically presented in Figures 6-9 for various values of involved flow feature parameters. Figure 6 shows that the fluid temperature reduces with the growth ofk_1. As fluid becomes more viscous it hinders the transition of thermal energy to the fluid easily and thus temperature curves decrease. It signifies that the thermal boundary layer thickness diminishes with the growth of k_1. Figure 7 demonstrates that the fluid temperature accelerates with the rise of permeability factor and thus we can conclude that the thickness of the thermal boundary enhances with the rising permeability.Growing permeability reduces the momentum boundary layer thickness and helps to enhance the fluid temperature. Figure 8 depicts the temperature profile for various values of S. It is noticed that fluid temperature diminishes with growing values of suction. The thickness of thermal boundary layer decreases and thus heat transport increases. The increasing values of suction bring fluid near to the plate and diminish the fluid temperature. Figure 9 indicates that the fluid temperature diminishes as slip factor δ enhances and thus less amount of heat is deported from the porous plate to the fluid. A high rate of fluid motion is observed near the plate due to the slip factor which enhances the heat transfer.

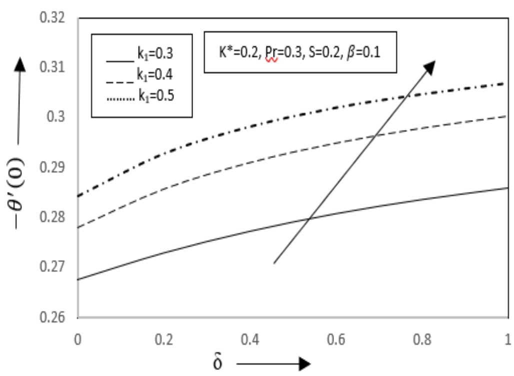

Fig.10. Effects of k_1 on temperature gradient curve $-\theta'(0)$ against δ

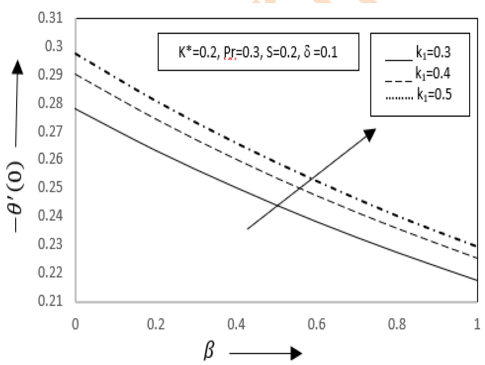

Fig.11. Effects of k_1 on temperature gradient curve $-\theta'(0)$ against β

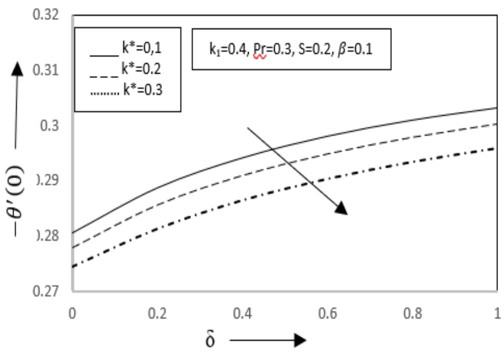

Fig.12. Effects of k* on temperature gradient curve $-\theta'(0)$ against δ

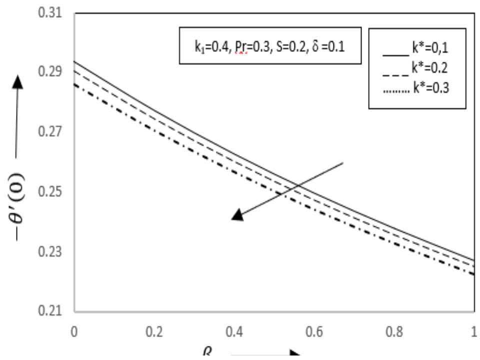

Fig.13. Effects of k* on temperature gradient curve $-\theta'(0)$ against β

The temperature gradient curves $(-\theta'(0))$ against the velocity and thermal slip factors δ and β for variation of involved flow parameters are plotted in Figures 10-

13.It is noticed from Figure 10 that as slip and elastico-viscous parameters enhance, the plate temperature rises. But, Figure 11 shows that the heat transport rate enhances from the plate with growing elastico-viscous parameter but diminishes with increasing thermal slip parameter. Figure 12 indicates that the heat transition rate reduces with increasing permeability factor but enhances with the growth of slip flow parameter whereas, from Figure 13, it is evident that the heat transition rate reduces with the growth of permeability and thermal slip parameters.

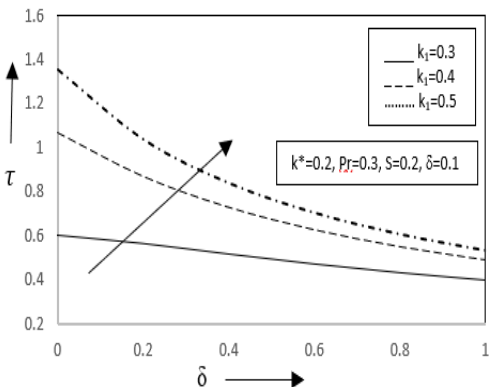

Fig.14. Effects of k_1 on skin friction coefficient τ against δ

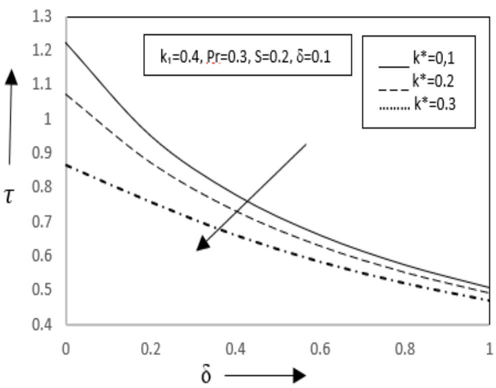

Fig.15. Effects of k* on skin friction coefficient τ against δ

The determination of skin friction is very much important in engineering to estimate the total frictional drag exerted on the object and to evaluate the heat transition rate on its surface. The friction (τ) on the surface of the plate is depicted in Figures 14-15 against δ for several values of elastico-viscous and permeability parameters for fixed values of involved flow parameters. From Figure 14, it is apparent that viscous drag on the surface enhances with the rising values of the elastico-viscous parameter and gradually diminishes with the growing slip parameters. The result is justified as increasing viscosity together with elasticity impede the forward movement of the fluid at the plate due to strong frictional drag. From Figure 15, it is noticed that the friction at the surface reduces rapidly with the growth of permeability and thus more fluid passes through the plate and approaches to zero as slip parameters are enhanced.

5. Conclusion

The study shows that elastico-viscosity plays an important role by reducing the drag on the flow to enhance the fluid velocity, heat transport rate and friction at the surface in combination of other flow parameters involved in the solution. The slip factor parameter role is also noticed to reduce the fluid temperature, heat transport rate and friction at the surface. The permeability parameter plays significant role to controlled the heat transition rate and the friction at the plate. The applied suction parameter brings fluid close to the plate and diminish the flow velocity and fluid temperature. A future study investigating the flow simulation and stability analysis of the problem would be very interesting. A comparative study of the solution of the same problem using different numerical and analytical methods may also possible. It is expected that the physics of flow across a flat plate can be used from our study as the basis of many scientific and engineering applications. The findings of this study will serve as a source of inspiration for future experimental work, which appears to be missing at the moment.

References

[1] Abbas, Z., Wang, Y., Hayat, T., Oberlack, M. (2009). Slip effects and heat transfer analysis in a viscous fluid over an oscillatory stretching surface. Int. J. Numer. Method H., 59, 443–458.

[2] Ambreen, A.K., Hafsa, U., Vafai, K., R Ellahi, R. (2016). Study of peristaltic flow of magnetohydrodynamics Walter's B fluid with slip and heat transfer. Int. J, Sci. Tech., 23, 2650-2662.

[3] Ascher, U., Mattheij, R., & Russell, R. (1995). Numerical solution of boundary valueproblems for ordinary differential equations. SIAM, Philadelphia, PA.

[4] Bhattacharyya, K., Mukhopadhyay, S., Layek, G.C. (2011). Steady boundary layer slip flow and heat transfer over a flat porous plate embedded in a porous media. J. Petrol. Sci. Eng.,78 304–309.

[5] Bhattacharyya, K., Layek, G.C., Reddy, R.S. (2012). Slip Effect on Boundary Layer Flow on a Moving Flat Plate in a Parallel Free Stream. International Journal of Fluid Mechanics Research, 39, 438-447.

[6] Bhattacharyya, K., Layek, G.C. (2012). Similarity solution of MHD boundary layer flow with diffusion and chemical reaction over a porous flat plate with suction/blowing. Meccanica, 47, 1043–1048. DOI 10.1007/s11012-011-9461-x

[7] Blasius, H. (1908). Grenzschichten in FliissigkeitenmitkleinerReibung. Z. Math. U. Phys., 56, 1-37.

[8] Cheng, P., Minkowycz, W.J. (1977). Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a disk. J. Geophys. Res., 88, 2040-2044.

[9] Howarth, L. (1939). On the solution of the laminar boundary layer equations. Proc. Roy. Soc. London A., 164, 547-579.

[10] Ishak, A. (2010). Similarity solutions for flow and heat transfer over a permeable surface with convective boundary conditions. Appl. Math. Comput., 217, 837-846.

[11] Izadi, M., Mehryan, S.A.M., Chamkha, A.J., Lorenzini, G. (2019). The impacts of heat generation/absorption and partial slip on boundary layer flow and heat transfer of a nonofluid comprising of self-impelled motile microorganisms passing a stretching sheet. Mathematical Modelling of Engineering Problems, 6(1), 10-20. https://doi.org/10.18280/mmep.060102

[12] Khan, W.A., Khan, Z.H., Rahi, M. (2014). Fluid flow and heat transfer of carbon nanotubes along a flat plate with navier slip boundary. Applied Nanoscience, 4, 633–641.

[13] Kierzenka, J. and Shampine, L.F. (2001). A BVP Solver Based on Residual Control and the MATLAB PSE. ACM Transaction on Mathematical Software, 27(3), 299-316. http://doi.org/10.1145/502800.502801

[14] Mukhopadhyay, S., Layek, G.C. (2009). Radiation effect on forced convective flow and heat transfer over a porous plate in a porous medium. Meccanica, 44, 587-597.

[15] Pohlhausen, E. (1921). OerWarmeaustauschzwischenfestenKorpem und FliissigkeitenmitkleinerReibung und kleinerWarmeleitung. Z. Angew. Math. Mech., 1,151-121.

[16] Rabadi, N.J., Hamdan, E.M. (2000). Free convection from inclined permeable walls embedded in variable permeability porous media with lateral mass flux. J. Petrol. Sci. Eng., 26, 241-251.

[17] Sarojamma, G., Sreelakshmi, K., Vajravelu, K. (2018). Effects of dual stratification on non-orthogonal non-Newtonian fluid flow and heat transfer. International Journal of Heat and Technology, 36(1), 207-214. https://doi.org/10.18280/ijht.360

[18] Schlichting, H. (1968), Boundary Layer theory, 6th edition, McGraw-Hill, New York.

[19] Walters, K. (1960). The motion of an elastico-viscous liquid contained between co-axial cylinders. Q. J. Mech. Appl. Math., 13, 444-456.

[20] Walters, K. (1962). The solutions of flow problems in the case of materials with memories, J. Mecanique, 1, 473-478.

Nomenclature

u	fluid velocity in x-direction
v	fluid velocity in y-direction
k_0	elastico-viscous parameter
k	permeability parameter
k_1	modified elastico-viscous parameter
k*	modified permeability parameter
K	thermal conductivity
C_p	specific heat
T	temperature
G_1	velocity slip factor
G_0	initial velocity slip flow factor
S	suction parameter
H_1	thermal slip factor
H_0	initial thermal flow slip factor
U_∞	free stream velocity
T_∞	free stream temperature
v_w	suction or blowing parameter
T_w	constant plate temperature
Re_x	local Reynolds number
Da_x	local darcy number
Pr	prandtl number

Greek symbols

ν	kinematic viscosity
ψ	dimensionless stream function
μ	coefficient of viscosity
ρ	Density of the fluid
η	similarity variable
θ	temperature function
δ	velocity slip factor
β	thermal slip factor