

**EFFECTS OF VARIABLE VISCOSITY AND THERMAL  
CONDUCTIVITY ON UNSTEADY FREE CONVECTION FLOW  
PAST AN IMPULSIVELY STARTED INFINITE VERTICAL PLATE  
WITH NEWTONIAN HEATING IN THE PRESENCE OF THERMAL  
RADIATION AND MASS DIFFUSION**

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**Abstract**

*The influence of variable viscosity and thermal conductivity on unsteady free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion is examined. Both the fluid viscosity and thermal conductivity are considered as an inverse linear function of temperature. The governing boundary layer equations with associated boundary conditions are converted to non-dimensional form. The magnetic Reynold number is assumed to be so small that the induced magnetic field can be neglected. The resulting non-linear partial differential equations are then solved using an iterative method for an implicit finite difference scheme. Effects of various flow governing parameters on the fluid velocity, temperature and concentration fields are presented graphically. Further, the numerical values of skin-friction co-efficient, Nusselt number and Sherwood number are computed and presented in tabular form.*

**Keywords:** Variable viscosity, thermal conductivity, Mass transfer, unsteady free convection flow, thermal radiation, MHD.

**2010 AMS classification:**76M25

## 1. Introduction

The analysis of free convection flow near an infinite vertical plate has been carried out as an important application in many industries. Numerous investigations are performed by using both analytical and numerical methods. Radiation effects on the free convection flow are important in context of space technology, processes in engineering areas occur at high temperature.

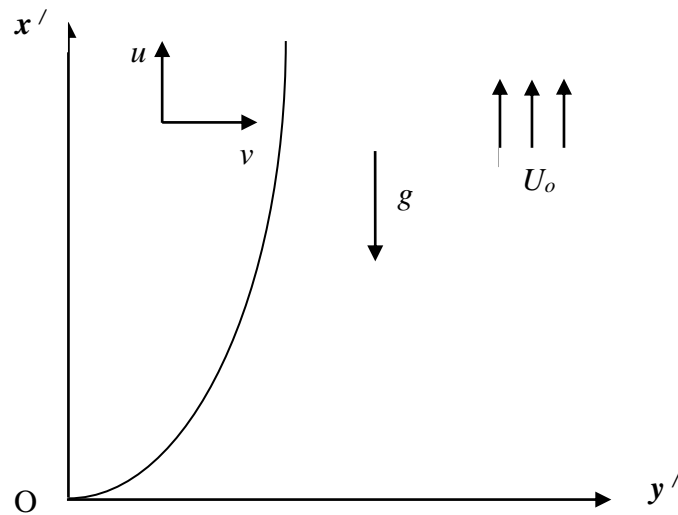
The first exact solution of the Navier-Stokes equation was given by Stokes [13] and explains the motion of a viscous incompressible fluid past an impulsively started infinite horizontal plate in its own plane. This is known as Stoke's first problem in the literature. If the plate is in a vertical direction and gives an impulsive motion in its own plane in a stationary fluid, then the resulting effect of buoyancy force was first studied by Soundalgekar [14] by Laplace transformation technique and the effects of heating or cooling of the plate by free convection currents were discussed. Pohlhausen [10] was among the earlier research workers, who first analyzed the steady free convective flow of a viscous, incompressible fluid past a semi-infinite vertical plate by using integral method. Raptis and Perdikis [12] studied the effects of thermal radiation and free convection flow past a moving vertical plate. Das et al [1] has analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate by the Laplace transform technique. Hossain and Begum [4] observed the effects of mass transfer and free convection past a vertical porous plate. Free convection flow involving coupled heat and mass transfer occurs frequently in nature. For this flow, the driving forces arise due to the temperature and concentration variations in the fluid. For example, in atmospheric flows, thermal convection resulting from heating of the earth by sunlight is affected by differences in water vapour concentration. Elbashbasy [3] studied heat and mass transfer along a vertical plate in the presence of a magnetic field. The transient free convection flow has been investigated by Muthukumaraswamy and Ganesan [7] for an impulsively started vertical plate with heat and mass transfer. Deka and Das [2] studied the radiation effects on free convection flow near a vertical plate with ramped Wall temperature. Narahari and Yunus [9] studied the free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion.

In most of the investigation done earlier on the convective flow, the viscosity and thermal conductivity of the fluid were assumed as constant. However, it is known that these physical properties can change significantly with temperature and when the

variable viscosity and thermal conductivity are taken into account, the flow characteristics are substantially changed compared to the constant cases.

In the present problem, an analysis has been carried out to study the effects of variable viscosity and thermal conductivity on unsteady free convection flow past an impulsively started infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion. The resulting transformed governing equations are then solved by an implicit finite-difference method. Representative results for the velocity, temperature and concentration distributions are presented for various governing parameters.

## 2. Mathematical Formulation



**Figure:** Flow Configuration and Co-ordinate system

Let us consider the unsteady free convection flow of a viscous incompressible fluid past an infinite vertical plate with Newtonian heating in the presence of thermal radiation and mass diffusion. The  $x'$ -axis is taken along the plate in the vertically upward direction and the  $y'$ -axis is taken normal to the plate. Initially, the plate and the fluid are at the same temperature  $T'_{\infty}$  and concentration  $C'_{\infty}$  in a stationary condition. At time  $t' > 0$ , the plate is given an impulsive motion in the vertical direction against the gravitational field, such that it attains uniform velocity  $U_0$  and

the concentration level near the plate is raised to  $C'_w (\neq C'_\infty)$  and it is assumed that the heat transfer from the surface is proportional to the local surface temperature,  $T'$ . As the plate is infinite in the  $x'$ - direction, all physical variables are independent of  $x'$  and are functions of  $y'$  and  $t'$  only.

Under the usual Boussinesq approximation, after neglecting the inertia terms, viscous dissipation heat and Soret and Dufour effects, the flow can be shown to be governed by the following system of equations:

$$\text{Momentum equation: } \frac{\partial u'}{\partial t'} = \frac{1}{\rho_\infty} \frac{\partial}{\partial y'} \left( \mu \frac{\partial u'}{\partial y'} \right) + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) \quad (1)$$

$$\text{Energy equation: } \rho c_p \frac{\partial T'}{\partial t'} = \frac{\partial}{\partial y'} \left( K \frac{\partial T'}{\partial y'} \right) - \frac{\partial q_r}{\partial y'} \quad (2)$$

$$\text{Concentration equation: } \frac{\partial c'}{\partial t'} = \frac{\partial}{\partial y'} \left( D \frac{\partial c'}{\partial y'} \right) \quad (3)$$

With the initial boundary conditions:

$$\left. \begin{array}{l} t' \leq 0: u' = 0, T' = T'_\infty, C' = C'_\infty \quad \text{for } y' \geq 0, \\ t' > 0: \left\{ \begin{array}{l} u' = U_0, \frac{\partial T'}{\partial y'} = -\frac{h}{K} T', C' = C'_w \quad \text{at } y' = 0, \\ u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \end{array} \right. \end{array} \right\} \quad (4)$$

The radiative heat flux term is simplified by making use of the Rosseland approximation (Siegel and Howell, 2002) as:

$$q_r = -\frac{4\sigma}{3K_R} \frac{\partial T'^4}{\partial y'} \quad (5)$$

By using Rosseland approximation, we limit our analysis to optically thick fluids. If temperature differences within the flow are sufficiently small, such that  $T'^4$  may be expressed as a linear function of the temperature, then the Taylor series for  $T'^4$  about  $T'_\infty$  after neglecting higher order terms, is given by:

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (6)$$

In view of equations (5) and (6), equation (2) reduces to:

$$\rho c_p \frac{\partial T'}{\partial t'} = \frac{\partial}{\partial y'} \left( K \frac{\partial T'}{\partial y'} \right) + \frac{16\sigma T_{\infty}'^3}{3K_R} \frac{\partial^2 T'}{\partial y'^2} \quad (7)$$

We introduce the following non-dimensional equalities:

$$\left. \begin{aligned} y &= \frac{y' U_0}{v_{\infty}}, \quad t = \frac{t' U_0^2}{v_{\infty}}, \quad u = \frac{u'}{U_0 Gr}, \quad \theta = \frac{T' - T_{\infty}'}{T_w' - T_{\infty}'}, \quad Pr = \frac{\mu c_p}{k}, \quad Gr = \frac{v_{\infty} g \beta T_{\infty}'}{U_0^3}, \\ R &= \frac{k K_R}{4\sigma T_{\infty}'^3}, \quad C = \frac{c' - c_{\infty}'}{c_w' - c_{\infty}'}, \quad Gm = \frac{v_{\infty} g \beta^* (c_w' - c_{\infty}')}{U_0^3}, \quad Sc = \frac{v_{\infty}}{D}, \quad N = \frac{Gm}{Gr} \end{aligned} \right\} \quad (8)$$

Lai and Kulacki [6] have assumed that the viscosity is an inverse linear function of temperature i.e.

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}} [1 + \gamma(T - T_{\infty})] \quad (9)$$

or 
$$\frac{1}{\mu} = a(T - T_r)$$

where 
$$a = \frac{\gamma}{\mu_{\infty}} \quad \text{and} \quad T_r = T_{\infty} - \frac{1}{\gamma}$$

Here  $\mu$  is the fluid viscosity,  $\mu_{\infty}$  is the viscosity at infinity,  $T$  is the temperature,  $T_{\infty}$  is the temperature of free stream,  $a$ ,  $\gamma$  and  $T_r$  are constants and their values depends on the reference state and thermal property of the fluid and  $a < 0$  for gas,  $a > 0$  for liquid.

Following, Hazarika and Khound [5] have assumed the thermal conductivity as-

$$\frac{1}{k} = \frac{1}{k_{\infty}} [1 + \xi(T - T_{\infty})] \quad (10)$$

or 
$$\frac{1}{k} = b(T - T_c)$$

where 
$$b = \frac{\xi}{k_{\infty}} \quad \text{and} \quad T_c = T_{\infty} - \frac{1}{\xi}$$

Here  $k$  and  $T$  are the thermal conductivity and temperature of the fluid,  $k_{\infty}$  and  $T_{\infty}$  are the thermal conductivity and temperature of free stream;  $b$ ,  $\xi$  and  $T_c$  are constants and their values depend on the reference state and the thermal property of the fluid.

The non-dimensional form of viscosity and thermal conductivity parameters  $\theta_r$  and  $\theta_c$  can be written as-

$$\theta_r = \frac{T_r - T_{\infty}}{T_w - T_{\infty}}, \quad \theta_c = \frac{T_c - T_{\infty}}{T_w - T_{\infty}}$$

Using these two parameters in equation (9) and (10), we can write the coefficient of viscosity and thermal conductivity as follows:

$$\mu = \frac{-\mu_{\infty} \theta_r}{\theta - \theta_r} \quad \text{and} \quad k = \frac{-k_{\infty} \theta_c}{\theta - \theta_c}$$

Equations (1), (7) and (3) are reduced to the following non-dimensional forms:

$$\frac{\partial u}{\partial t} = -\frac{\theta_r}{\theta - \theta_r} \frac{\partial^2 u}{\partial y^2} + \frac{\theta_r}{(\theta - \theta_r)^2} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} + \theta + NC \quad (11)$$

$$3RP_r \frac{\partial \theta}{\partial t} = \left[ 4 - 3R \left( \frac{\theta_c}{\theta - \theta_c} \right) \right] \frac{\partial^2 \theta}{\partial y^2} + 3R \frac{\theta_c}{(\theta - \theta_c)^2} \left( \frac{\partial \theta}{\partial y} \right)^2 \quad (12)$$

$$S_c \frac{\partial c}{\partial t} = -\frac{\theta_r}{\theta - \theta_r} \frac{\partial^2 c}{\partial y^2} + \frac{\theta_r}{(\theta - \theta_r)^2} \frac{\partial c}{\partial y} \frac{\partial \theta}{\partial y} \quad (13)$$

where the dimensionless parameters are defined as follows:

$\Theta$  is the dimensionless temperature,

$N$  is the Buoyancy ratio parameter,

$C$  is the dimensionless concentration,

$R$  is the radiation parameter,

$P_r$  is the Prandtl number,

$S_c$  is the Schmidt number.

The corresponding initial and boundary conditions are:

$$\begin{aligned}
 t \leq 0 : u = 0, \theta = 0, C = 0 & \quad \text{for } y \geq 0, \\
 \left\{ \begin{array}{l} u = \frac{1}{Gr}, \frac{\partial \theta}{\partial y} = -(1 + \theta), C = 1 \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \end{array} \right. & \quad \begin{array}{l} \text{at } y = 0 \\ \text{as } y \rightarrow \infty \end{array}
 \end{aligned} \tag{14}$$

The physical quantities of interest in the present study are the skin friction coefficient  $C_f$ , Nusselt number  $N_u$  and the Sherwood number  $S_h$  which are the rate of shear stress, the rate of heat transfer and the rate of Mass transfer for the plate respectively and they are defined as-

**Coefficient of skin friction:**

The coefficient of skin friction  $C_f$  can be defined as-

$$C_f = \frac{\tau_w}{\rho_\infty U_0^2}, \text{ where } \tau_w = \mu \left( \frac{\partial u'}{\partial y'} \right)_{y'=0} \text{ is the shearing stress.}$$

Using the non-dimensional variables, we finally get the coefficient of skin friction  $C_f$  as-

$$C_f = -\frac{\theta_r}{(\theta - \theta_r)} \left( \frac{\partial u}{\partial y} \right)_{y=0} \tag{15}$$

**Nusselt number:**

The Nusselt number  $N_u$  is defined as-

$$N_u = \frac{x q_w}{k_\infty (T_w - T_\infty)}, \text{ where } q_w = -k \left( \frac{\partial T'}{\partial y'} \right)_{y'=0} \text{ is the heat transfer from the}$$

plate.

Using the non-dimensional variables, we get -

$$N_u = \frac{\theta_c}{(\theta - \theta_c)} \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \tag{16}$$

**Sherwood number:**

The Sherwood number  $S_h$  is defined as-

$Sh = \frac{x h_m}{D(c_w - c_\infty)}$ , where  $h_m = -D \left( \frac{\partial c'}{\partial y} \right)_{y'=0}$  is the mass flux at the surface.

Using the non-dimensional variables, we get-

$$Sh = \frac{\theta_r}{(\theta - \theta_r)} \left( \frac{\partial \varphi}{\partial y} \right)_{y=0} \quad (17)$$

### 3. Method of Solution with Results and Discussions

Partial differential equations are discretize by Crank- Nicolson method and the discretized equations are solved by using an iterative method.

Let us consider a dummy function  $f$ .

Crank- Nicolson formula for the function  $f$  is given below:

$$\frac{\partial f}{\partial t} = \frac{(f_{i+1,j} - f_{i,j})}{\Delta t}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2\Delta y} [f_{i+1,j+1} + f_{i,j+1} - f_{i+1,j} - f_{i,j}]$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{1}{2(\Delta y)^2} [f_{i+1,j+1} + f_{i,j+1} - 2(f_{i+1,j} + f_{i,j}) + 2(f_{i+1,j-1} + f_{i,j-1})]$$

The boundary value problems (11) to (14) are then solved using an iterative method for an implicit finite difference scheme.

The variations of velocity profile, temperature profile and concentration profile are illustrated in figure (1) to figure (9) for the variations of different parameters and from figure (10) to figure (13) illustrate the velocity, temperature and concentration profile for the variations of viscosity parameter  $\theta_r$ , thermal conductivity parameter  $\theta_c$  and Schmidt number  $S_c$  with time.

Table (1) to (4) gives the skin friction coefficient  $C_f$ , Nusselt number  $N_u$  and the Sherwood number  $S_h$  for various values of the flow governing parameters as indicated. Table (5) and (6) demonstrate a comparison of the skin friction coefficient



$C_f$ , Nusselt number  $N_u$  and the Sherwood number  $S_h$  between the work done by Narahari and Yunus [9] and our present work.

The numerical values of different parameters are taken as  $N = 2, P_r = .7, G_r = 13, G_m = 15, \theta_r = 4, \theta_c = 4, R = 1, S_c = .22$  unless otherwise stated.

Velocity profile for various parameters are shown in figure (1) to figure (5) and from these, it is clear that velocity first increases near the vertical plate and then gradually decreases away from the plate with the increase of viscosity parameter  $\theta_r$ , Thermal conductivity parameter  $\theta_c$ , Schmidt number  $S_c$ , Radiation parameter  $R$  and Prandtl number  $P_r$ .

Temperature profiles are illustrated from figure (6) to figure (8). From figure (6), we have found that temperature decreases with the increasing values of thermal conductivity parameter  $\theta_c$ . It is due to the fact that the kinematic viscosity of the fluid increases with the increase of thermal conductivity and as a result temperature decreases. From figure (7), we have found that temperature decreases with the increasing values of Prandtl number  $P_r$ . It is due to the reason that with the increasing values of Prandtl number  $P_r$ , viscosity increases and as a result temperature decreases. From figure (8), we have found that temperature decreases with the increasing values of Radiation parameter  $R$ . Therefore, increases in the value of Radiation parameter  $R$  lead to a decrease in the thermal boundary layer thickness. When radiation effects are present in a fluid, the thermal boundary layer is always found to thicken. The reason for this is that radiation provides an additional means to diffuse energy and as a result temperature decreases.

Concentration profiles are shown in figure (9) and from this, it is observed that concentration decreases with the increasing values of Schmidt number  $S_c$ . Since Schmidt number  $S_c$  is the connecting link between velocity and concentration profiles, therefore with the increasing values of Schmidt number  $S_c$ , molecular mass diffusivity decreases and as a result concentration decreases.

From figure (10), it is observed that velocity decreases with the increasing values of viscosity parameter  $\theta_r$  and velocity increases when time increases. From figure (11), it is seen that velocity decreases with the increasing values of thermal conductivity parameter  $\theta_c$  and velocity increases when time increases. From figure (12), it is observed that temperature decreases with the increasing values of thermal

conductivity parameter  $\theta_c$  and temperature increases when time increases. From figure (13), it is seen that concentration decreases with the increasing values of Schmidt number  $S_c$  and concentration increases when time increases.

From Table (1), (2) and (3), it is observed that with the increasing values of viscosity parameter  $\theta_r$ ; Coefficient of skin friction  $C_f$  and Nusselt Number  $N_u$  are increasing, but Sherwood number  $S_h$  is decreasing. Also from table (1), it is seen that with the increasing values of temperature Grashof number  $G_r$ ; Coefficient of skin friction  $C_f$  is increasing. From table (2), we have found that with the increasing values of prandtl number  $P_r$ ; Coefficient of skin friction  $C_f$  and Nusselt number  $N_u$  are increasing but Sherwood number  $S_h$  is decreasing. From table (3), it is seen that with the increasing values of Radiation parameter  $R$ ; Coefficient of skin friction  $C_f$  and Nusselt number  $N_u$  are increasing, but Sherwood number  $S_h$  is decreasing. Table (4) displays that with the increasing values of thermal conductivity parameter  $\theta_c$ ; Coefficient of skin friction  $C_f$  is increasing, but Sherwood number  $S_h$  and Nusselt number  $N_u$  are decreasing. Also with the increasing values of Radiation parameter  $R$ ; Coefficient of skin friction  $C_f$  and Nusselt number  $N_u$  are increasing, but Sherwood number  $S_h$  is decreasing.

Tables (5) and (6) displays that considering variable viscosity and variable thermal conductivity; the value of Nusselt number  $N_u$  is increased with the increasing values of prandtl number  $P_r$ , Radiation parameter  $R$ , Mass Grashof number  $G_m$  and Schmidt number  $S_c$ . Thus by assuming variable viscosity and variable thermal conductivity we can expect significant results in the flow problems.

#### 4. Conclusion

In this analysis, the problem of affect of variable viscosity and thermal conductivity on unsteady free convection flow past an impulsively started infinite vertical plate is investigated. The resulting non-linear partial differential equations are discretized using Crank Nicolson method and the discretized equations are solved using an iterative method for an implicit finite difference scheme. The results are presented for the major parameters including the viscosity parameter, thermal conductivity parameter, Prandtl number, radiation parameter and Schmidt number and also the numerical values of skin friction coefficient, Nusselt number and Sherwood number are computed and presented in tabular form.

From this analysis, it is observed that

1. Viscosity parameter  $\theta_r$ , Thermal conductivity parameter  $\theta_c$ , Schmidt number  $S_c$ , Prandtl number  $P_r$  and Radiation parameter  $R$  first enhance the fluid velocity near the vertical plate and then retard the fluid velocity away from the plate.
2. Viscosity parameter  $\theta_r$ , thermal conductivity parameter  $\theta_c$ , Schmidt number  $S_c$  and Radiation parameter  $R$  retards the fluid temperature.
3. Mass transfer rate decreases with the increase of Schmidt number  $S_c$ .

**TABLE 1**

Coefficient of skin friction  $C_f$ , Nusselt Number  $N_u$  and Sherwood Number  $S_h$  for various values of Temperature Grashof number  $G_r$  and Viscosity parameter  $\theta_r$ ,  $N = 2, P_r = .7, G_m = 15, \theta_c = 4, R = 1, S_c = .22$

$G_r$	$\theta_r$	$C_f$	$N_u$	$S_h$
12	2	-0.40142	16.77171	10.76068
	4	-0.16357	16.77773	7.296953
	6	-0.11801	16.77985	6.594529
	8	-0.09871	16.78092	6.292304
14	2	-0.30287	16.77171	10.76068
	4	-0.10251	16.77773	7.296953
	6	-0.06438	16.77985	6.594529
	8	-0.04824	16.78092	6.292304
16	2	-0.22896	16.77171	10.76068
	4	-0.05672	16.77773	7.296953
	6	-0.02415	16.77985	6.594529
	8	-0.01039	16.78092	6.292304
18	2	-0.17147	16.77171	10.76068
	4	-0.0211	16.77773	7.296953
	6	0.007143	16.77985	6.594529
	8	0.019048	16.78092	6.292304

**TABLE 2**

Coefficient of skin friction  $C_f$ , Nusselt Number  $N_u$  and Sherwood Number  $S_h$  for various values of Prandtl number  $P_r$  and Viscosity parameter  $\theta_r$

$$N = 2, G_r = 15, G_m = 15, \theta_c = 4, R = 1, S_c = .22$$

$P_r$	$\theta_r$	$C_f$	$N_u$	$S_h$
0.7	2	-0.34836	16.77171	10.76068
	4	-0.1307	16.77773	7.296953
	6	-0.08913	16.77985	6.594529
	8	-0.07153	16.78092	6.292304
0.8	2	-0.34838	16.86577	10.75857
	4	-0.12947	16.8726	7.296585
	6	-0.08769	16.87499	6.594346
	8	-0.07001	16.87621	6.292185

**TABLE 3**

Coefficient of skin friction  $C_f$ , Nusselt Number  $N_u$  and Sherwood Number  $S_h$  for various values of Radiation parameter  $R$  and Viscosity parameter  $\theta_r$ ,

$$N = 2, P_r = .7, G_r = 15, G_m = 15, \theta_c = 4, S_c = .22$$

$R$	$\theta_r$	$C_f$	$N_u$	$S_h$
0.2	2	-0.3488	16.2458	10.7726
	4	-0.13811	16.24612	7.299075
	6	-0.09774	16.24624	6.595587
	8	-0.08063	16.2463	6.292995
0.4	2	-0.34861	16.37176	10.76973
	4	-0.13626	16.3728	7.29856
	6	-0.09561	16.37317	6.59533
	8	-0.07839	16.37336	6.292827
0.6	2	-0.34846	16.50142	10.76678
	4	-0.1344	16.50362	7.298034
	6	-0.09346	16.5044	6.595068
	8	-0.07612	16.50479	6.292656
0.8	2	-0.34838	16.63476	10.76376
	4	-0.13255	16.6386	7.297499
	6	-0.0913	16.63995	6.594801
	8	-0.07383	16.64064	6.292482
1	2	-0.34836	16.77171	10.76068
	4	-0.1307	16.77773	7.296953
	6	-0.08913	16.77985	6.594529
	8	-0.07153	16.78092	6.292304

**TABLE 4**

Coefficient of skin friction  $C_f$ , Nusselt Number  $N_u$  and Sherwood Number  $S_h$  for various values of Radiation parameter  $R$  and Thermal conductivity parameter  $\theta_c$

$$N = 2, P_r = .7, G_r = 15, G_m = 15, \theta_r = 4, S_c = .22$$

$R$	$\theta_c$	$C_f$	$N_u$	$S_h$
0.2	2	-0.08066	28.42826	6.292997
	4	-0.08065	18.95275	6.292996
	6	-0.08064	17.05803	6.292996
	8	-0.08063	16.2463	6.292995
0.4	2	-0.07848	28.64358	6.292835
	4	-0.07845	19.09777	6.292832
	6	-0.07842	17.18995	6.29283
	8	-0.07839	16.37336	6.292827
0.6	2	-0.07633	28.86201	6.292672
	4	-0.07626	19.2458	6.292667
	6	-0.07619	17.32549	6.292662
	8	-0.07612	16.50479	6.292656
0.8	2	-0.07419	29.08333	6.292509
	4	-0.07408	19.39672	6.292501
	6	-0.07396	17.46459	6.292492
	8	-0.07383	16.64064	6.292482
1	2	-0.07208	29.30731	6.292347
	4	-0.07191	19.55042	6.292334
	6	-0.07173	17.60721	6.292319
	8	-0.07153	16.78092	6.292304

**TABLE 5**

Comparison of skin friction coefficient  $C_f$ , Nusselt number  $N_u$  and Sherwood number  $S_h$  with Radiation parameter  $R$  and Prandtl number  $P_r$

$$N = 2, G_r = 15, G_m = 15, \theta_r = 4, \theta_c = 4, S_c = .22$$

$P_r$	$R$	$C_f$		$N_u$		$S_h$	
		Previous	Present	Previous	Present	Previous	Present
0.7	0.2	1.4746	-0.08063	1.689022	16.2463	1.246305	6.292995
	0.4	1.474973	-0.07839	1.700022	16.37336	1.246305	6.292827
	0.6	1.475276	-0.07612	1.70898	16.50479	1.246305	6.292656
	0.8	1.475528	-0.07383	1.716415	16.64064	1.246305	6.292482
	1	1.475739	-0.07153	1.722685	16.78092	1.246305	6.292304
0.8	0.2	1.474668	-0.08032	1.69104	16.26385	1.246305	6.292972
	0.4	1.475098	-0.07776	1.703707	16.40926	1.246305	6.29278
	0.6	1.475448	-0.07519	1.714063	16.55981	1.246305	6.292585
	0.8	1.475739	-0.0726	1.722685	16.71547	1.246305	6.292387
	1	1.475984	-0.07001	1.729974	16.87621	1.246305	6.292185



**TABLE 6**

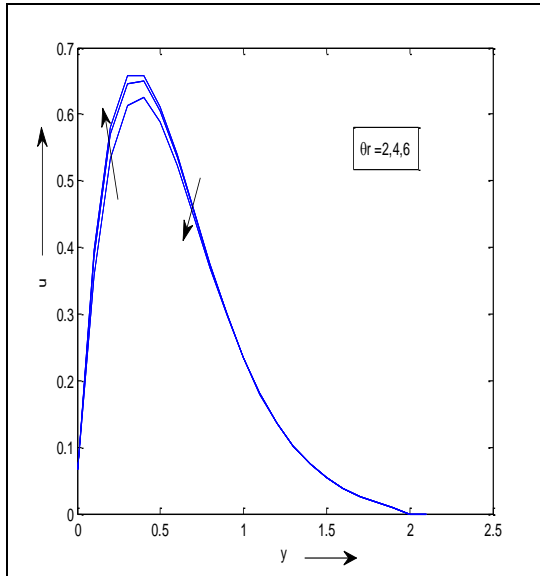
Comparison of skin friction coefficient  $C_f$ , Nusselt number  $N_u$  and Sherwood number  $S_h$  with Mass Grashof number  $G_m$  and Schmidt number  $S_c$

$$N = 2, P_r = .7, \quad G_r = 15, \theta_r = 4, \theta_c = 4, R = 1$$

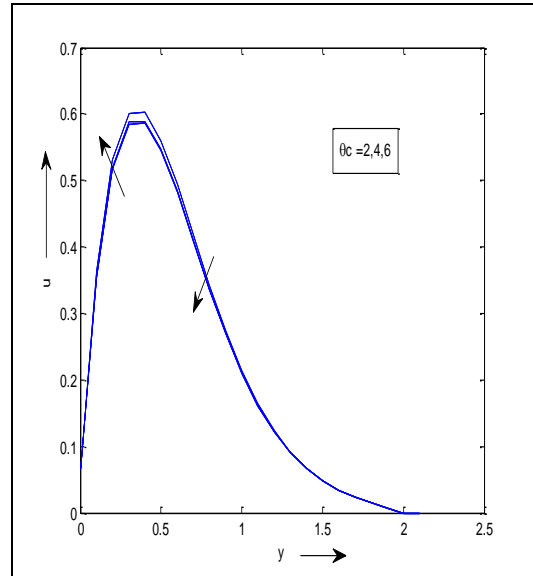
$G_m$	$S_c$	$C_f$		$N_u$		$S_h$	
		Previous	Present	Previous	Present	Previous	Present
10	0.2	1.475631	-0.12288	1.722685	16.78092	1.264936	6.293861
	0.4	1.476741	-0.1225	1.722685	16.78092	1.072841	6.278841
	0.6	1.477918	-0.12211	1.722685	16.78092	0.867027	6.264958
	0.8	1.479168	-0.12171	1.722685	16.78092	0.645987	6.252081
13	0.2	1.475631	-0.0921	1.722685	16.78092	1.264936	6.293861
	0.4	1.476741	-0.09161	1.722685	16.78092	1.072841	6.278841
	0.6	1.477918	-0.0911	1.722685	16.78092	0.867027	6.264958
	0.8	1.479168	-0.09059	1.722685	16.78092	0.645987	6.252081
16	0.2	1.475631	-0.06133	1.722685	16.78092	1.264936	6.293861
	0.4	1.476741	-0.06072	1.722685	16.78092	1.072841	6.278841
	0.6	1.477918	-0.0601	1.722685	16.78092	0.867027	6.264958
	0.8	1.479168	-0.05947	1.722685	16.78092	0.645987	6.252081



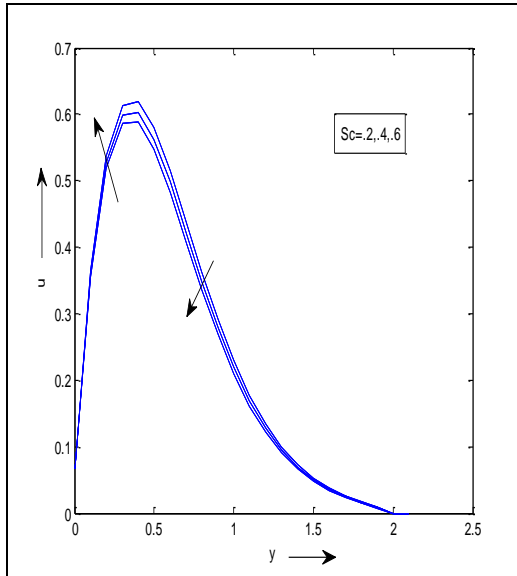
FIGURES



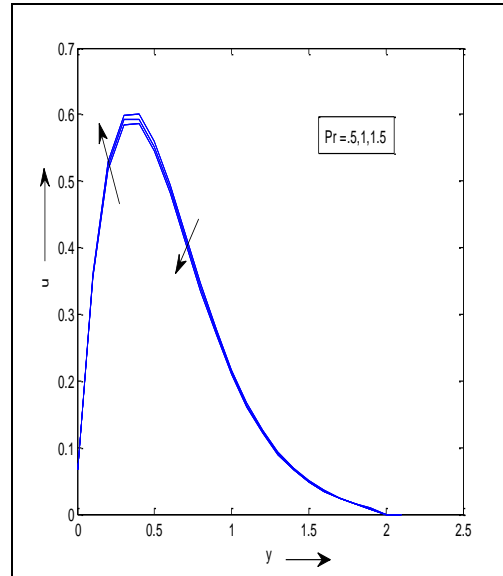
**Fig. 1:** Variation of velocity profile against  $y$  for different values of  $\theta_r$



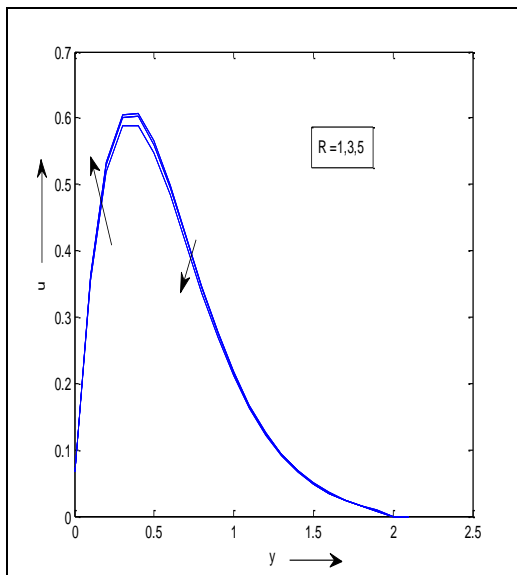
**Fig. 2:** Variation of velocity profile against  $y$  for different values of  $\theta_c$



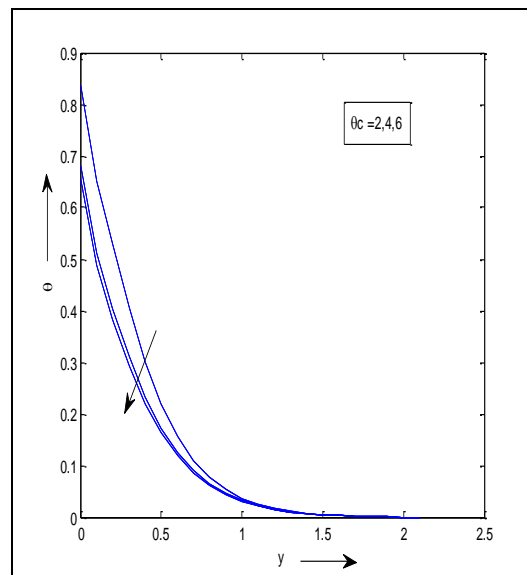
**Fig. 3:** Variation of velocity profile against  $y$  for different values of  $Sc$



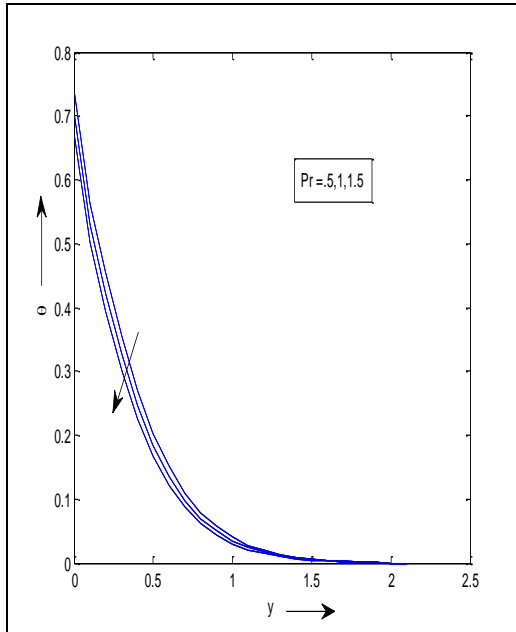
**Fig. 4:** Variation of velocity profile against  $y$  for different values of  $Pr$



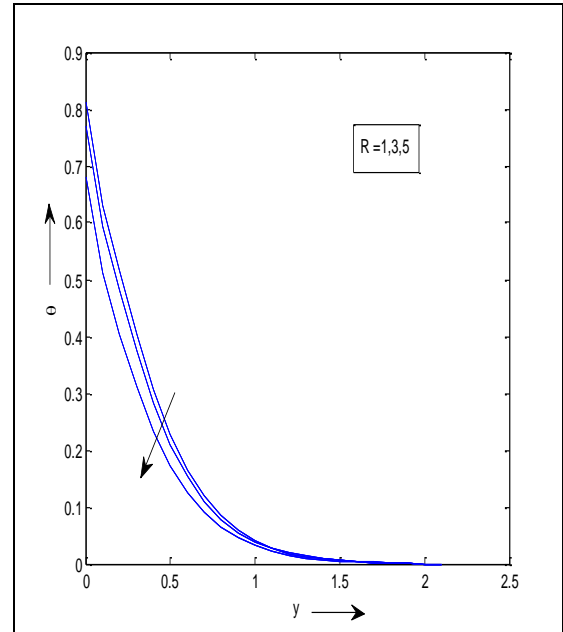
**Fig. 5:** Variation of velocity profile against  $y$  for different values of  $R$



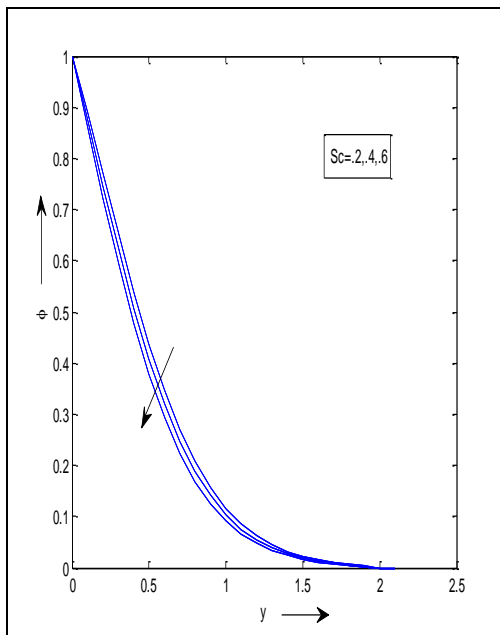
**Fig. 6:** Variation of temperature profile against  $y$  for different values of  $\theta_c$



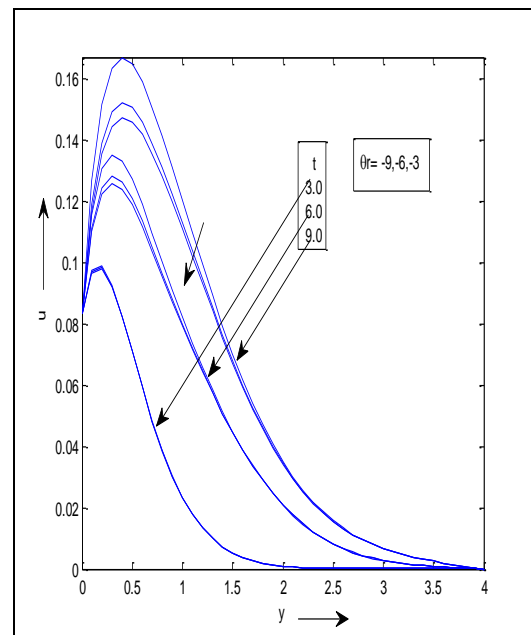
**Fig. 7:** Variation of temperature profile against  $y$  for different values of  $Pr$



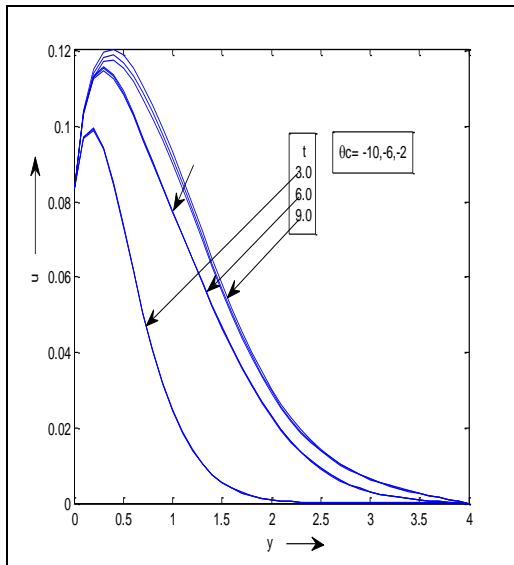
**Fig. 8:** Variation of temperature profile against  $y$  for different values of  $R$



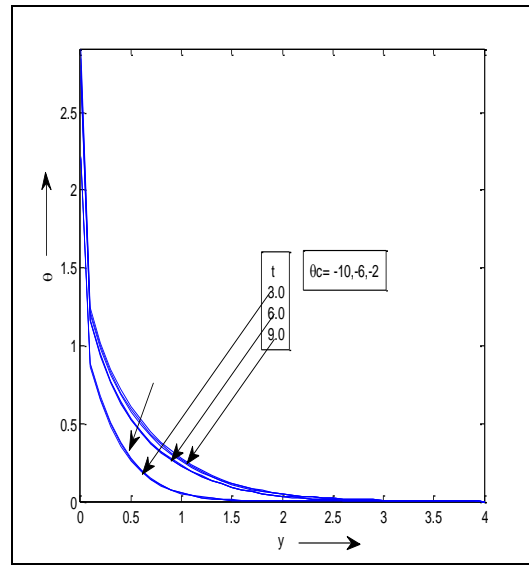
**Fig. 9:** Variation of concentration profile against  $y$  for different values of  $Sc$



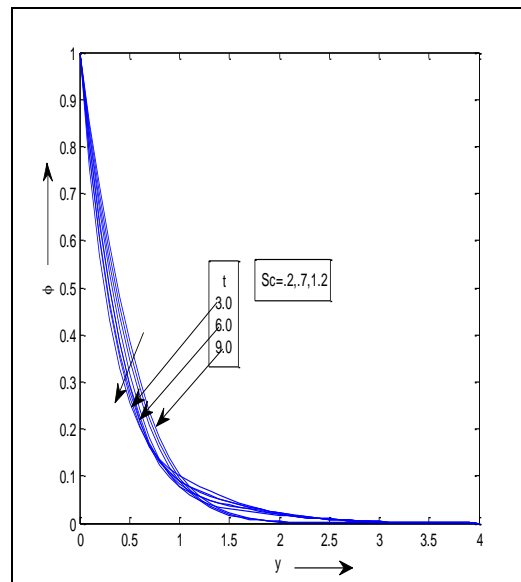
**Fig. 10:** Velocity profile for different values of  $t$  and  $\theta_r$



**Fig. 11:** Velocity profile for different values of  $t$  and  $\theta_c$



**Fig. 12:** Temperature profile for different values of  $t$  and  $\theta_c$



**Fig. 13:-**Concentration profile for different values of  $t$  for  $S_c$

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