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An Inventory Model for Deteriorating Items with Two Components of Demand Rate and Fuzzy Type Backlogging Rate

Sushil Kumar & U S Rajput
Department of Mathematics & Astronomy,
University of Lucknow-Lucknow U.P. (India)
E-mail: sushilmath4444@gmail.com
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Abstract

In the present paper, an attempt has been made to develop an inventory model for deteriorating items with two components of demand rate and fuzzy type backlogging rate. Assuming shortages and allowing partially backlogged. The unsatisfied demand is backlogged and depend a function of waiting time. The problem of perishability or deterioration plays an important role in the field of inventory control and management. The aim of this study is to optimize the cycle length and the total profit. The applicability of the developed model is shown by a numerical example to notice the behavior of key parameters of the model.

Keywords: Inventory, Deterioration, Partial-backlogging and Two Components Demand
AMS Subject Classification: MSC2010 (90B05)

1. INTRODUCTION

Nowadays, in any business scenario inventory management plays an important role. Inventory planners have a strategy, how much of the inventory will be produced and supplied based on the market demand. The market demand of a product varies up to a certain period of time and later it becomes constant. The inventory products/items can be classified in two categories namely deterioration and obsolescence. One kind of products has wastage, damage, deterioration or decay and other does not have these but can be replaced by the upcoming new and better products in the market. Perishable inventory is a small portion of the total inventory and includes fashionable products (Readymade Garments and Toys), life care vaccines, electronic items (Smart Phone's, LEd's LCD's, AC'S), digital products (Computer's, Chip's, Camera's), and periodicals (News Paper's, Magazine's, Bakery products, Medicines).

In this paper, we are concerned with the basic factors emerging from the earlier studies. First, we propose the idea of absolute values in object recognition problems. For this we improve the formulas for computation to an appropriate order of expansion so that the results give the approximate value very near to the exact values.

In the existing scenario, researchers have paid their interest in the development of inventory control and its uses. Some scholars in this area are Aggarwal et al. [1] developed an inventory model for coordinating ordering, pricing and advertisement policy for an advance sales system. Chang and Dye [2] constructed an EOQ (economic order quantity) model for deteriorating items with time varying demand and partial backlogging. Chung et al. [3] presented a note on EOQ models for perishable items and stock dependent selling rate. Dye [4] focused on joint pricing and ordering policy for an inventory model of deteriorating items with partial backlogging. Dye and Ouyang [5] worked an EOQ model for perishable items with time varying partial backlogging and stock dependent selling rate. Giri and Chaudhuri [6] analyzed a deterministic inventory model for perishable items with stock dependent demand rate and non linear holding cost. Giri et al. [7] studied an inventory model for perishable items with stock dependent demand rate. Goyal and Giri [8] worked on a production inventory model with time varying production, deterioration and demand rate. Goswami and Chaudhuri [9] constructed an EOQ model for deteriorating items with linear trend in demand and allowing shortages. Hargia [10] considered an EOQ model for perishable items and time varying demand. Khatri and Gothi [11] analyzed an EPQ (economic production quantity) model with exponential demand rate and constant amelioration. They were also considered different deterioration rates and shortages in their inventory model. Lee and Wu constructed two inventory models [12] and [13]. In the model [12] they were studied an EOQ model for weibull deteriorates items with shortages and time varying power pattern based demand. And in the model [13] they were presented a note on EOQ model for perishable items with exponential distribution type deterioration and time dependent demand and shortages. Lin et al. [14] worked on an EOQ model for deteriorating items with time varying demand and shortages. Nezhad et al. [15] focused on a periodic and continuous review inventory model for deteriorating items with fuzzy type inventory costs. Padmanabhan and Vrat [16] analyzed an EOQ model for perishable items with stock dependent selling rate. Papachristos and Skouri were developed two inventory models [17] and [18]. In the model [17] they were presented an optimal replenishment policy for deteriorating items with time varying demand and exponential type backlogging rate. And in the model [18] they were considered the quantity discount, pricing and time dependent partial backlogging. Roy [19] constructed a fuzzy inventory model of deteriorating items and price dependent demand rate. Shah and Shukla [20] proposed an inventory model of deteriorating items with waiting time dependent partial backlogging. Shah and Chakrabarty [21] studied a fuzzy EOQ model of perishable items and time dependent demand with partial backlogging. Sen and Saha [22] analyzed an inventory model for deteriorating items with negative exponential demand, probabilistic deterioration and fuzzy lead time under partial backlogging. Uddin et al. [23] worked on a production inventory model with level dependent demand allowing few defective items. Wang [24] discussed an optimal lot sizing policy for an inventory model of deteriorating items with time varying demand and allowing shortages. Wu [25] proposed an EOQ model for weibull deteriorating items with time varying demand and partial backlogging. Zadeh [26] introduced fuzzy sets in inventory modeling.

2. Some Definitions

Fuzzy Set- A fuzzy set $\tilde{A}$ on the given universal set X is a set of ordered pairs,

$$\tilde{A} = \left\{ \left(x, \quad \lambda_{\tilde{A}}(x) \right) : \quad x \in X \quad \right\}$$

Where $\lambda_{\tilde{A}} : \; X \to [0, 1]$ is called the membership function.

Fuzzy Number – A fuzzy number $\tilde{A}$ is a fuzzy set on the real line if its membership function $\lambda_{\tilde{A}}$ has the following properties,

1- $\lambda_{\tilde{A}}(x)$ is upper semi continuous.

2- $\lambda_{\tilde{A}}(x) = 0$, outside some interval $[a_1, a_4]$.

3- $\exists$ real numbers $a_2 \,\&\, a_3$, $\quad a_1 \le a_2 \le a_3 \le a_4$ such that $\lambda_{\tilde{A}}(x)$ is increasing on $[a_1, a_2]$, decreasing on $[a_3, a_4]$, and $\lambda_{\tilde{A}}(x) = 1$, for each x in $[a_2, a_3]$.

Triangular Fuzzy Number- A triangular fuzzy number (TFN) is specified by the triplet (a_1, a_2, a_3) where $a_1 < a_2 < a_3$ and is defined by a continuous membership function $\lambda_{\tilde{A}} : \; X \to [0, 1]$, as follows,

$$\lambda_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

Signed Distance- If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number then the signed distance of $\tilde{A}$ is defined as $d(\tilde{A}, 0) = \dfrac{(a+2b+c)}{4}$.

Centroid Method- The centroid method on the triangular fuzzy number $\tilde{A} = (a, b, c)$ is defined as $C\left(\tilde{A}\right) = \dfrac{a+b+c}{3}$.

3. Mathematical Notations and Assumptions

Corresponding to the developed model, we consider the following assumptions and notations

1. The demand rate is $R(t) = \begin{cases} \alpha e^{-\beta t}, & 0 \le t \le T_1 \\ \alpha, & T_1 \le t \le T \end{cases}$

   Where α and β are constants.
2. θ is the deterioration parameter.
3. δ is the backlogging parameter.
4. $\tilde{\delta}$ is the backlogging parameter in fuzzy nature.
5. o_C is the ordering cost per order.
6. h_c is the holding cost per unit per cycle.

7. s_c is the shortage cost per unit per cycle.
8. p_c is the purchase cost per unit.
9. s_P is the selling price per unit.
10. op_C is the opportunity cost.
11. T is the replenishment cycle length.
12. $I(t)$ is the inventory level at any time t in $[0,T]$.
13. T_1 is the time at which shortage starts.
14. $TP(T_1,T)$ is the total profit per unit time.
15. $\tilde{TP}(T_1,T)$ is the total profit per unit time in fuzzy nature.
16. The replenishment rate is infinite.
17. The lead time is zero.
18. There is no repair or replacement of the deteriorated items.

4. Mathematical Derivation of the Model

Graphically, it has been observed that an inventory system contains the maximum inventory level Q in the beginning of the cycle. The inventory level decreases due to both demand and deterioration in the interval $[0,T_1]$ and it becomes zero at $t = T_1$. The unsatisfied demand in the shortage interval $[T_1,T]$ is backlogged at a rate of $\delta(T-t)$. At any time t in $[0,T]$ the inventory level is given by the following differential equations

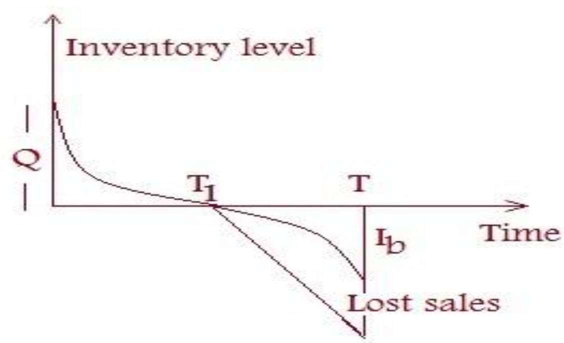

Figure 1, Inventory Model

$$\frac{dI}{dt}+\theta I=-\alpha e^{-\beta t}, \qquad 0\le t\le T_1 \tag{1}$$

with condition $I(T_1)=0$

$$\frac{dI}{dt}=-\alpha\delta(T-t), \qquad T_1\le t\le T \tag{2}$$

with condition $I(T_1)=0$

The solutions of equations (1) and (2) are given by the equations (3) and (4) respectively.

$$I=\left(\frac{\alpha}{\theta-\beta}\right)[\beta t+(\theta-\beta)T_1-\frac{\beta^2}{2}t^2+(\theta^2+\beta^2-2\beta\theta)T_1^2] \tag{3}$$

$$I=\left(\frac{\alpha\delta}{2}\right)[2TT_1-2Tt-T_1^2+t^2] \tag{4}$$

In the cycle, the ordering cost is

$$O_C=o_C \tag{5}$$

In the cycle, the holding cost is

$$H_C = h_c \int_0^{T_1} I(t)dt$$

or

$$H_C = \left(\frac{\alpha h_c}{\theta - \beta}\right)[\theta T_1^2 - \frac{\beta}{2} T_1^2 + \frac{(6\theta^2 + 5\beta^2 - 12\beta\theta)}{6} T_1^3] \tag{6}$$

In the cycle, the shortage cost is

$$S_C = -s_c \int_{T_1}^{T} I(t)dt$$

or

$$S_C = -s_c \int_{T_1}^{T} \left(\frac{\alpha\delta}{2}\right)[2TT_1 - 2Tt - T_1^2 + t^2]dt$$

or

$$S_C = -\left(\frac{\alpha\delta s_c}{6}\right)[6T^2 T_1 - 3TT_1^2 - 4T_1^3 - 2T^3] \tag{7}$$

The maximum ordered quantity Q^* is given by

$$Q^* = Q + q \tag{8}$$

Here Q is the initial order quantity and q is the back ordered quantity.

The initial order quantity Q is obtained by putting $t = 0$ in equation (3)

$$Q = \alpha[T_1 + (\theta - \beta)T_1^2] \tag{9}$$

The back-ordered quantity q is obtained by putting $t = T$ in equation (4)

$$q = \left(\frac{\alpha\delta}{2}\right)[2TT_1 - T_1^2 - T^2] \tag{10}$$

Using the values of Q and q, which are given by the equations (9) and (10), in equation (8) we obtain

$$Q^* = \left(\frac{\alpha}{2}\right)[2T_1 + 2(\theta - \beta)T_1^2 - \delta(2TT_1 - T_1^2 - T^2)] \tag{11}$$

In the cycle, the purchasing cost is

$$P_C = p_c Q^*$$

or

$$P_C = \left(\frac{\alpha p_c}{2}\right)[2T_1 + 2(\theta-\beta)T_1^2 - \delta(2TT_1 - T_1^2 - T^2)] \qquad (12)$$

In the cycle, the opportunity cost is

$$OP_C = op_C \int_{T_1}^{T} \alpha[1-\delta(T-t)]dt$$

or

$$OP_C = \left(\frac{\alpha.op_C}{2}\right)\left[2T - 2T_1 - \delta(T^2 + T_1^2 - 2TT_1)\right] \qquad (13)$$

In the cycle, the sales revenue is

$$S_R = s_p\left[\int_0^{T_1} \alpha\, e^{-\beta t} dt + \int_{T_1}^{T} \alpha\delta(T-t)dt\right]$$

or

$$S_R = \left(\frac{\alpha s_p}{2}\right)\left[2T_1 - \beta T_1^2 + \delta(T^2 + T_1^2 - 2T\,T_1)\right] \qquad (14)$$

In the cycle, the total profit is

$$TP(T_1,T) = \frac{1}{T}[S_R - (O_C + H_C + S_C + P_C + OP_C)]$$

$$TP(T_1,T) = \frac{1}{T}\Big[-o_C + \alpha(s_p + op_C - p_C)T_1 - \alpha op_C T + \frac{\alpha}{2}\Big\{(\delta-\beta)s_p - \frac{(2\theta-\beta)h_C}{2(\theta-\beta)} - 2(\theta-\beta)$$

$$+\delta(op_C - p_C)\Big\}T_1^2 + \frac{\alpha\delta(s_p + op_C - p_C)}{2}T^2 - \alpha\delta(s_p + op_C - p_C)TT_1 - \frac{\alpha}{6}\{4\delta\, s_C$$

$$+\frac{h_c(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\}T_1^3-\frac{\alpha\delta s_c}{3}T^3+\alpha\delta s_c T^2 T_1-\frac{\alpha\delta s_c}{2}TT_1^2\Big] \qquad (15)$$

The total profit $TP(T_1,T)$ will be necessarily maximum, if

$$\frac{\partial TP(T_1,T)}{\partial T_1}=0$$

$$\frac{\partial TP(T_1,T)}{\partial T}=0$$

On solving these equations, we find the optimum values of T_1 and T, for which the profit is maximum.

For optimality of total profit, the sufficient conditions are

$$\left(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TP(T_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TP(T_1,T)}{\partial T_1\partial T}\right)^2>0$$

$$\left(\frac{\partial^2 TP(T_1,T)}{\partial T_1^2}\right)<0$$

at the optimum values of T_1 and T.

From the equation (15), we get

$$\frac{\partial TP(T_1,T)}{\partial T_1}=\frac{1}{T}\Big[\alpha(s_p+op_C-p_C)+\alpha\Big\{(\delta-\beta)s_p-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)p_C+\delta(op_C-p_C)\Big\}T_1$$

$$-\alpha\delta(s_p+op_C-p_C)T-\frac{\alpha}{2}\left\{4\delta s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)h_C}{(\theta-\beta)}\right\}T_1^2+\alpha\delta s_C T^2-\alpha\delta s_C TT_1\Big]$$

$$(16)$$

$$\frac{\partial TP(T_1,T)}{\partial T}=\frac{1}{T}\Big[-\alpha op_C+\alpha\delta(s_p+op_C-p_C)T-\alpha\delta(s_p+op_C-p_C)T_1-\alpha\delta s_C T+2\alpha\delta s_C TT_1-\frac{\alpha\delta s_C}{2}T_1^2\Big]$$

$$-\frac{1}{T^2}\Big[-o_C+\alpha(s_p+op_C-p_C)T_1+\frac{\alpha}{2}\Big\{(\delta-\beta)s_p-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)p_C$$

$$+\frac{(6\theta^2+5\beta^2-12\beta\theta)h_c}{(\theta-\beta)}\}T_1^2-\frac{\alpha\delta s_c}{3}T^3+\alpha\delta s_c T^2 T_1-\frac{\alpha\delta s_c}{2}TT_1^2\Big] \qquad (17)$$

$$\frac{\partial^2 TP(T_1,T)}{\partial T \partial T_1} = \frac{1}{T}\left[2\alpha\delta\, s_C T - \alpha\delta\, s_C T_1\right] - \frac{1}{T^2}\Big[\alpha(s_p + op_C - p_C) + \alpha\Big\{(\delta-\beta)s_p - \frac{(2\theta-\beta)h_C}{2(\theta-\beta)} - 2(\theta-\beta)p_C$$

$$+\delta(op_C - p_C)\Big\}T_1 - \alpha\delta(s_p + op_C - p_C)T - \frac{\alpha}{2}\left\{4\delta\, s_C + \frac{(6\theta^2 + 5\beta^2 - 12\beta\theta)h_C}{(\theta-\beta)}\right\}T_1^2$$

$$+\alpha\delta\, s_c T^2 - \alpha\delta\, s_c T T_1\Big] \qquad (18)$$

$$\frac{\partial^2 TP(T_1,T)}{\partial T^2} = \frac{1}{T}\left[\alpha\delta(s_p + op_C - p_C) - \alpha\delta\, s_C + 2\alpha\delta\, s_C T_1\right] - \frac{1}{T^2}\Big[-2\alpha op_C + 2\alpha\delta(s_p + op_C - p_C)T$$

$$-2\alpha\delta(s_p + op_C - p_C)T_1 - \alpha\delta\, s_C T + 4\alpha\delta\, s_C T T_1 - \alpha\delta\, s_C T_1^2 - \alpha\delta\, s_C T^2\Big] + \frac{2}{T^3}\Big[-o_C$$

$$+\alpha(s_p + op_C - p_C)T_1 - \alpha op_C T + \frac{\alpha}{2}\Big\{(\delta-\beta)s_p - \frac{(2\theta-\beta)h_C}{2(\theta-\beta)} - 2(\theta-\beta)p_C + \delta(op_C - p_C)\Big\}T_1^2$$

$$+\frac{\alpha\delta(s_p + op_C - p_C)}{2}T^2 - \alpha\delta(s_p + op_C - p_C)TT_1 - \frac{\alpha}{6}\left\{4\delta\, s_C + \frac{(6\theta^2 + 5\beta^2 - 12\beta\theta)h_C}{(\theta-\beta)}\right\}T_1^3$$

$$-\frac{\alpha\delta\, s_c}{3}T^3 + \alpha\delta\, s_c T^2 T_1 - \frac{\alpha\delta\, s_c}{2}TT_1^2\Big] \qquad (19)$$

$$\frac{\partial^2 TP(T_1,T)}{\partial T_1^2} = \frac{1}{T}\Big[\alpha(s_p + op_C - p_C) + \alpha\Big\{(\delta-\beta)s_p - \frac{(2\theta-\beta)h_C}{2(\theta-\beta)} - 2(\theta-\beta)p_C + \delta(op_C - p_C)\Big\}$$

$$-\alpha\left\{4\delta\, s_C + \frac{(6\theta^2 + 5\beta^2 - 12\beta\theta)h_C}{(\theta-\beta)}\right\}T_1 - \alpha\delta\, s_C T\Big] \qquad (20)$$

5. Mathematical Model in Fuzzy Environment

The super market is filled with uncertainty. Uncertainty deals with fuzzy set theory. Let us consider the uncertainty in backlogging parameter. Let us treats the backlogging parameter $\delta = (\delta_1, \delta_2, \delta_3)$ is a triangular fuzzy number.

In the cycle, the total profit in fuzzy sense is

$$\tilde{TP}(T_1,T)=\frac{1}{T}\Bigg[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha\, op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}\, s_P-\beta\, s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}(op_C-p_C)\right\}T_1^2$$

$$+\frac{\alpha\,\tilde{\delta}(s_P+op_C-p_C)}{2}T^2-\alpha\,\tilde{\delta}(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}\, s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$-\frac{\alpha\,\tilde{\delta}\, s_C}{3}T^3+\alpha\,\tilde{\delta}\, s_C T^2 T_1-\frac{\alpha\,\tilde{\delta}\, s_C}{2}TT_1^2\Bigg] \tag{21}$$

Now, we defuzzify the total profit by two methods namely Signed distance method and centroid method.

Signed Distance Method-

By this method, the total profit is

$$\tilde{TP}(T_1,T)=\frac{1}{4}\left[\tilde{TP}_1(T_1,T)+2\tilde{TP}_2(T_1,T)+\tilde{TP}_3(T_1,T)\right] \tag{22}$$

Where,

$$\tilde{TP}_1(T_1,T)=\frac{1}{T}\Bigg[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha\, op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}_1\, s_P-\beta\, s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}_1(op_C-p_C)\right\}T_1^2$$

$$+\frac{\alpha\,\tilde{\delta}_1(s_P+op_C-p_C)}{2}T^2-\alpha\,\tilde{\delta}_1(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}_1\, s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$-\frac{\alpha\,\tilde{\delta}_1\, s_C}{3}T^3+\alpha\,\tilde{\delta}_1\, s_C T^2 T_1-\frac{\alpha\,\tilde{\delta}_1\, s_C}{2}TT_1^2\Bigg]$$

$$\tilde{TP}_2(T_1,T)=\frac{1}{T}\Bigg[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha\,op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}_2\,s_P-\beta\,s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}_2(op_C-p_C)\right\}T_1^2$$

$$+\frac{\alpha\,\tilde{\delta}_2(s_P+op_C-p_C)}{2}T^2-\alpha\,\tilde{\delta}_2(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}_2\,s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$-\frac{\alpha\,\tilde{\delta}_2\,s_C}{3}T^3+\alpha\,\tilde{\delta}_2\,s_C T^2T_1-\frac{\alpha\,\tilde{\delta}_2\,s_C}{2}TT_1^2\Bigg]$$

$$\tilde{TP}_3(T_1,T)=\frac{1}{T}\Bigg[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha\,op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}_3\,s_P-\beta\,s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}_3(op_C-p_C)\right\}T_1^2$$

$$+\frac{\alpha\,\tilde{\delta}_3(s_P+op_C-p_C)}{2}T^2-\alpha\,\tilde{\delta}_3(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}_3\,s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$-\frac{\alpha\,\tilde{\delta}_3\,s_C}{3}T^3+\alpha\,\tilde{\delta}_3\,s_C T^2T_1-\frac{\alpha\,\tilde{\delta}_3\,s_C}{2}TT_1^2\Bigg]$$

Putting the values of $\tilde{TP}_1(T_1,T),\tilde{TP}_2(T_1,T)$ and $\tilde{TP}_3(T_1,T)$ in equation (22), we obtain

$$\tilde{TP}(T_1,T)=\frac{1}{4T}\Bigg[-4o_C+4\alpha(s_p+op_C-p_C)T_1-4\alpha op_C T+\frac{\alpha}{2}\Big\{(\delta_1+2\delta_2+\delta_3)s_P-4\beta\,s_P-\frac{2(2\theta-\beta)h_C}{(\theta-\beta)}$$

$$-8(\theta-\beta)+(\delta_1+2\delta_2+\delta_3)(op_C-p_C\Big\}T_1^2+\frac{\alpha(\delta_1+2\delta_2+\delta_3)(s_P+op_C-p_C)}{2}T^2$$

$$-\alpha(\delta_1+2\delta_2+\delta_3)(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4(\delta_1+2\delta_2+\delta_3)s_C+\frac{4(6\theta^2+5\beta^2-12\beta\theta)h_C}{(\theta-\beta)}\right\}T_1^3$$

$$-\frac{\alpha(\delta_1+2\delta_2+\delta_3)s_C}{3}T^3+\alpha(\delta_1+2\delta_2+\delta_3)s_C T^2T_1-\frac{\alpha(\delta_1+2\delta_2+\delta_3)s_C}{2}TT_1^2\Bigg]\quad(23)$$

For optimality of total profit, the necessary conditions are

$\dfrac{\partial \tilde{TP}(T_1,T)}{\partial T_1}=0$ and $\dfrac{\partial \tilde{TP}(T_1,T)}{\partial T}=0$, on solving these equations, we find the optimum values of T_1 and T for which $\tilde{TP}(T_1,T)$ is maximum. And the sufficient conditions for $\tilde{TP}(T_1,T)$ to be optimum are

$$\left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T_1 \partial T}\right)^2>0 \text{ and } \left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T_1^2}\right)<0.$$

Centroid Method-

By this method, the total profit in fuzzy sense is

$$\tilde{TP}(T_1,T)=\frac{1}{3}\left[\tilde{TP}_1(T_1,T)+\tilde{TP}_2(T_1,T)+\tilde{TP}_3(T_1,T)\right] \tag{24}$$

Where,

$$\tilde{TP}_1(T_1,T)=\frac{1}{T}\left[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha\, op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}_1 s_P-\beta s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}_1(op_C-p_C)\right\}T_1^2\right.$$

$$+\frac{\alpha\tilde{\delta}_1(s_P+op_C-p_C)}{2}T^2-\alpha\tilde{\delta}_1(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}_1 s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$\left.-\frac{\alpha\tilde{\delta}_1 s_C}{3}T^3+\alpha\tilde{\delta}_1 s_C T^2 T_1-\frac{\alpha\tilde{\delta}_1 s_C}{2}TT_1^2\right]$$

$$\tilde{TP}_2(T_1,T)=\frac{1}{T}\left[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha\, op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}_2 s_P-\beta s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}_2(op_C-p_C)\right\}T_1^2\right.$$

$$+\frac{\alpha\tilde{\delta}_2(s_P+op_C-p_C)}{2}T^2-\alpha\tilde{\delta}_2(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}_2 s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$\left.-\frac{\alpha\tilde{\delta}_2 s_C}{3}T^3+\alpha\tilde{\delta}_2 s_C T^2T_1-\frac{\alpha\tilde{\delta}_2 s_C}{2}TT_1^2\right]$$

$$\tilde{TP}_3(T_1,T)=\frac{1}{T}\left[-o_C+\alpha(s_P+op_C-p_C)T_1-\alpha op_C T+\frac{\alpha}{2}\left\{\tilde{\delta}_3 s_P-\beta s_P-\frac{(2\theta-\beta)h_C}{2(\theta-\beta)}-2(\theta-\beta)+\tilde{\delta}_3(op_C-p_C)\right\}T_1^2\right.$$

$$+\frac{\alpha\tilde{\delta}_3(s_P+op_C-p_C)}{2}T^2-\alpha\tilde{\delta}_3(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4\tilde{\delta}_3 s_C+\frac{(6\theta^2+5\beta^2-12\beta\theta)}{(\theta-\beta)}\right\}T_1^3$$

$$\left.-\frac{\alpha\tilde{\delta}_3 s_C}{3}T^3+\alpha\tilde{\delta}_3 s_C T^2T_1-\frac{\alpha\tilde{\delta}_3 s_C}{2}TT_1^2\right]$$

Putting the values of $\tilde{TP}_1(T_1,T),\tilde{TP}_2(T_1,T)$ and $\tilde{TP}_3(T_1,T)$ in equation (24), we obtain

$$\tilde{TP}(T_1,T)=\frac{1}{3T}\left[-3o_C+3\alpha(s_p+op_C-p_C)T_1-3\alpha op_C T+\frac{\alpha}{2}\left\{(\delta_1+\delta_2+\delta_3)s_P-3\beta s_P-\frac{3(2\theta-\beta)h_C}{2(\theta-\beta)}\right.\right.$$

$$\left.-6(\theta-\beta)+(\delta_1+\delta_2+\delta_3)(op_C-p_C)\right\}T_1^2+\frac{\alpha(\delta_1+\delta_2+\delta_3)(s_P+op_C-p_C)}{2}T^2$$

$$-\alpha(\delta_1+\delta_2+\delta_3)(s_P+op_C-p_C)TT_1-\frac{\alpha}{6}\left\{4(\delta_1+\delta_2+\delta_3)s_C+\frac{3(6\theta^2+5\beta^2-12\beta\theta)h_C}{(\theta-\beta)}\right\}T_1^3$$

$$\left.-\frac{\alpha(\delta_1+\delta_2+\delta_3)s_C}{3}T^3+\alpha(\delta_1+\delta_2+\delta_3)s_C T^2T_1-\frac{\alpha(\delta_1+\delta_2+\delta_3)s_C}{2}TT_1^2\right] \quad (25)$$

The necessary conditions for optimality of $\tilde{TP}(T_1,T)$ are

$\frac{\partial \tilde{TP}(T_1,T)}{\partial T_1}=0$ and $\frac{\partial \tilde{TP}(T_1,T)}{\partial T}=0$, on solving these equations, we find the optimum values of T_1 and T for which $\tilde{TP}(T_1,T)$ is maximum. The sufficient conditions for the optimality of $\tilde{TP}(T_1,T)$ are

$$\left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T_1 \partial T}\right)^2>0 \text{ and } \left(\frac{\partial^2 \tilde{TP}(T_1,T)}{\partial T_1^2}\right)<0.$$

6. Numerical Example for the Model and Analysis of Key Parameters

Let us consider the following data for parameters in the appropriate units

$o_C=50,\ \alpha=100,\ \beta=0.2,\ \delta=0.3,\ \theta=0.01,\ h_c=0.5,\ s_c=0.8,\ p_c=5,\ op_C=10,\ s_p=25$

Table1, variation in total profit w.r.to θ

θ	T_1	T	TP
0.01	2.61379	5.09892	877.846681
0.05	2.49777	4.90547	833.553155
0.10	2.33928	4.65984	785.207217
0.15	2.24021	4.51429	760.143898

Here in table 1, we see that as we increase θ, the values of T_1, T and TP are decreased. The reason is that the demand is decreased.

Table 2, variation in total profit w.r.to δ

δ	T_1	T	TP

0.3	2.61379	5.09882	877.8466812
0.6	1.79443	3.05483	1032.237473
0.9	1.46563	2.32939	1135.726552
1.2	1.26543	1.92540	1202.664381

From the table 2, we observe that as we increase δ , the values of *TP* are increased, but the values of T_1, and T are decreased. The reason is that either selling price or demand is increased.

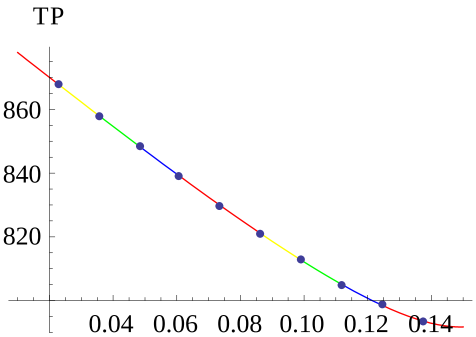

Figure 2 variation in *TP* with θ

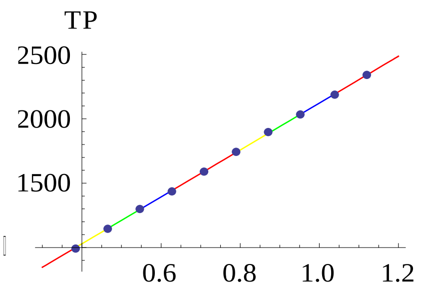

Figure 3 variation in *TP* with δ

Table 3, variation in total profit w.r.to α

α	T_1	T	TP
100	2.61379	5.09892	877.8466812
120	2.49498	4.90449	1002.165362
140	2.49299	4.90379	1170.778112
160	2.49149	4.90326	1339.388488

Here in this table 3, we observe that as we increase α , the values of TP are increased, but the values of T_1, and$T$ are decreased. The reason is that the selling price is increased.

Table 4, variation in total profit w.r.to β

β	T_1	T	TP
0.2	2.61379	5.09892	877.8466812
0.4	1.95641	4.02233	629.2720885
0.6	1.46932	3.33734	478.7705170
0.8	1.10953	2.96012	368.9478805

From this table 4, we observe that as we increase β, the values of T_1, T and TP are decreased. The reason is that either the selling price is increased or the maximum units are backlogged.

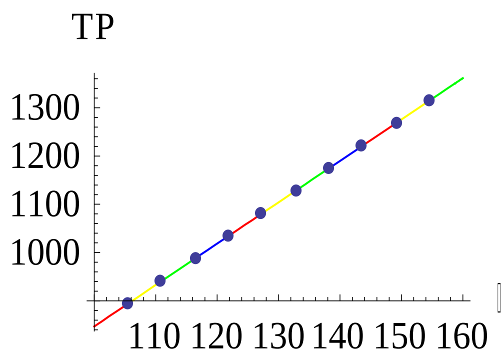

Figure 3 variation in TP with α

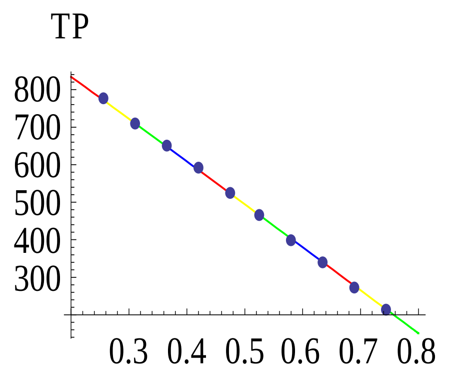

Figure 4 variation in TP with β

Table 5, variation in $\tilde{TP}$ by signed distance method

$\delta = (\delta_1, \delta_2, \delta_3)$	T_1	T	$\tilde{TP}$
(0.3, 0.6, 0.9)	0.1523	1.8618	585.8609

(0.4, 0.8, 1.2)	0.1293	1.3998	591.2570
(0.5, 1.0, 1.5)	0.1122	1.1238	593.7679

From table 5, we see that as we increase the triangular fuzzy number δ, the total profit $\tilde{TP}$ is increased, and the values of T_1 and T are decreased.

Table 6, variation in $\tilde{TP}$ by centroid method

$\delta=(\delta_1,\delta_2,\delta_3)$	T_1	T	$\tilde{TP}$
(0.3, 0.6, 0.9)	0.1521	1.8599	584.3192
(0.4, 0.8, 1.2)	0.1291	1.3986	590.0022
(0.5, 1.0, 1.5)	0.1119	1.2228	715.2179

From table 6, we see that as we increase the triangular fuzzy number δ, the total profit $\tilde{TP}$ is increased, and the values of T_1 and T are decreased.

These are the sufficient conditions for total profit

$$\frac{\partial^2 TP}{\partial T_1^2}=-650.03772,\qquad \frac{\partial^2 TP}{\partial T^2}=-119.95140,\qquad \frac{\partial^2 TP}{\partial T_1\partial T}=-181.01161$$

And

$$\left(\frac{\partial^2 TP}{\partial T_1^2}\right)\left(\frac{\partial^2 TP}{\partial T^2}\right)-\left(\frac{\partial^2 TP}{\partial T_1\partial T}\right)^2=45207.73111$$

7. CONCLUSION

In this paper model, a profit maximization model is developed for obtaining the optimum cycle. Following this, the model is also considered in fuzzy environment. To illustrate the model, numerical analysis is also performed. The results suggests that in the prescribed crisp model the total profit is deeply impacted by the parameters δ, α and β in comparison of parameter θ. Also in case of fuzzy environment, the centroid method is more significant than the signed distance method. The reason is that, during a cycle of minimum length the backlogging of products will increase the profit of any business organization or the firm. Backlogging is beneficial for the firm to accelerate the sales for obtaining more profit by achieving greater sales revenue. Therefore, the periodical products such as seasonal garments, vegetables, milk, bakery products and newspapers/magazines etc. becomes necessarily to be sold in the market as the cycle length decreases.

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