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SOFT DIRECTED HYPERGRAPHS AND THEIR AND & OR OPERATIONS

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Abstract

A directed hypergraph is a tool to model some classes of problems arising in Operations Research and Computer Science. They are a natural generalization of directed graphs. Soft set theory is an approach for modelling vagueness and uncertainty. In this paper, we introduce the concept of soft directed hypergraph by applying the idea of a soft set in the directed hypergraph. Using parameterization, a soft directed hypergraph produces a series of descriptions of a relationship described using a directed hypergraph. We introduce the concepts of degrees, soft incidence matrix and soft adjacency matrix associated with soft directed hypergraphs. In addition, we introduce and explore the features of AND and OR operations soft directed hypergraphs.

Keywords: Directed Hypergraph, Soft Directed Hypergraph.

2010 AMS classification: 05C99

1. Introduction

Soft set theory is an approach for modelling vagueness and uncertainty. In 1999, D. Molodtsov [15] defined soft set as follows. Let U be an initial universe set and let E be a set of parameters or attributes with respect to U. Let A be a subset of E. A pair (F, A) is called a soft set (over U) where F is a mapping from A to the power set of U. In other words, a soft set (F, A) over U is a parameterized family of subsets of U. This is a new mathematical tool to deal with uncertainties. Many practical problems can be solved easily with the help of soft set theory rather than some well-known theories viz. fuzzy set theory, probability theory etc. since these theories have certain limitations. The problem with the fuzzy set is that it lacks parameterization tools. Many authors like P.K. Maji, A.R. Roy and R. Biswas [13], [14] have further studied the theory of soft sets and used the theory to solve some decision-making problems.

To provide a parameterized point of view for graphs, the notion of the soft graph is applied. Because of its capacity to cope with the parameterization tool, soft graph theory is a rapidly emerging area in graph theory. R. K. Thumbakara and B. George [20] introduced the concept of the soft graph as follows. Let $G^*= (V, E)$ be a simple graph and A be any nonempty set. Let R be an arbitrary relation between elements of A and elements of V. That is $R \subseteq A\times \mathrm{V}$. A mapping $F: A\rightarrow P(V)$ can be defined as $F(a)= \{y \in V: xRy\}$. The pair (F, A) is a soft set over V. Then (F, A) is said to be a soft graph of G^* if the subgraph induced by $F(a)$ in G^* is a connected subgraph of G^* for all $x \in A$. Also, they studied different concepts in soft graphs [21], [22] and introduced notions like soft semigraphs [8] and soft hypergraphs [7]. In 2015, M. Akram and S. Nawas [1] updated R. K. Thumbakara and B. George's notion of the soft graph as follows. Let $G^*= (V, E)$ be a simple graph and A be any nonempty set. Let R be an arbitrary relation between elements of A and elements of V. That is $R\subseteq A\times V$. A mapping $F: A\rightarrow P(V)$ can be defined as $F(a)= \{y \in V: xRy\}$. Also define a mapping $K: A\rightarrow P(E)$ by $K(a)= \{uv \in E: \{u, v\} \subseteq F(a)\}$. The pair (F, A) is a soft set over V and the pair (K, A) is a soft set over E. Also $(F(a), K(a))$ is a subgraph of G^* for all $a\in A$. Then the 4 -tuple $G = (G^*, F, K, A)$ is called a soft graph if the following conditions are satisfied:

1. $G^*= (V, E)$ is a simple graph,
2. A is a nonempty set of parameters,
3. (F, A) is a soft set over V,
4. (K, A) is a soft set over E,

5. $(F(a), K(a))$ is a subgraph of G^* for all $a \in A$.

If we represent $(F(a), K(a))$ by $H(a)$ then soft graph G is also given by $\{H(a): a \in A\}$. M. Akram and S. Nawas [2] also defined many varieties of soft graphs, such as regular soft graphs, soft trees, and soft bridges, as well as the notions of soft cut vertex, soft cycle and so on. They [3] also introduced the notions of fuzzy soft graphs, strong fuzzy soft graphs, complete fuzzy soft graphs, and regular fuzzy soft graphs and investigated some of their properties. They [4] also described some applications of fuzzy soft graphs. M. Akram and F. Zafar introduced the notions of soft trees [5] and fuzzy soft trees [6]. Further studies on soft graphs were conducted by J.D. Thenge, B.S. Reddy, R.S. Jain [17], [18], [19], N. Sarala, K. Manju [16] and S. Venkatraman, R. Helen [23].

Directed graphs arise naturally in many applications of graph theory. They can be used to analyze and resolve problems with electrical circuits, project timelines, shortest routes, social links, and many other issues. J. Jose, B. George and R.K. Thumbakara [12] introduced the notion of the soft directed graph by applying the concepts of soft set in a directed graph. They also introduced the concepts of indegree, outdegree, degree, adjacency matrix and incidence matrix in soft directed graphs and investigated their properties.

Directed hypergraphs were developed as a generalization of directed graphs. They have a wide range of uses, including representing functional dependency in databases, and-or graphs, Horn formulas in propositional logic, context-free grammars and so on. In this paper, we apply the idea of a soft set in directed hypergraph to introduce the notion of the soft directed hypergraph. We introduce the concepts of degrees, soft incidence matrix, and soft adjacency matrix in soft directed hypergraphs. In addition, we introduce and explore the features of soft directed hypergraphs' AND and OR operations.

2 Preliminaries

2.1 Directed Hypergraphs

Directed hypergraphs are a natural generalization of digraphs, just as ordinary hypergraphs are a natural generalization of graphs. For basic concepts of a directed hypergraph, we refer to [11]. More concepts of directed hypergraphs are discussed in [9], [10]. A *directed hypergraph* $H^* = (V, E)$ consists of a vertex set V and a set of directed hyperedges or hyperarcs $E = \{e = (T(e), H(e)) \mid T(e) \subseteq V \text{ and } H(e) \subseteq V\}$, where $T(e) \neq \phi$ and $H(e) \neq \phi$. The sets $T(e)$ and $H(e)$ are called *tail* and *head* of the hyperarc e, respectively. A directed hypergraph is called *k-uniform* if $|T(e)| =$

$|H(e)| = k$ for all $e \in E$. Two hyperarcs e and e' are said to be *parallel* if $T(e) = T(e')$ and $H(e) = H(e')$. A hyperarc e is said to be a *loop* if $T(e) = H(e)$. A directed hypergraph $H^* = (V, E)$ is called *simple* if it has no parallel hyperarcs and loops. A directed hypergraph is called *trivial* if $|V| = 1$ and $E = \phi$. If the two vertices u and v of H^* are such that $u \in T(e)$ and $v \in H(e)$ then we say that v is *adjacent from* u or u is *adjacent to* v. The *indegree* of a vertex v in H^*, denoted by $d^-(v)$ is the number of hyperarcs that contain v in their head. The *outdegree* of a vertex v in H^*, denoted by $d^+(v)$ is the number of hyperarcs that contain v in their tail. The *degree* of a vertex v in H^*, denoted by $d(v)$ is the sum of outdegree and indegree of v. A directed hypergraph $H'^* = (V', E')$ is a *weak subhypergraph* of the directed hypergraph $H^* = (V, E)$ if $V' \subseteq V$ and E' consists of hyperarcs e' with $T(e') = \{v | v \in T(e) \cap V'\}$ and $H(e') = \{v | v \in H(e) \cap V'\}$ for some $e \in E$. A directed hypergraph $H'^* = (V', E')$ is a *weak induced subhypergraph* of the directed hypergraph $H^* = (V, E)$ if V' and hyperarc set $E' = \{(T(e) \cap V', H(e) \cap V') \mid e \in E \text{ and } T(e) \cap V' \neq \phi$ and $H(e) \cap V' \neq \phi\}$. Consider a directed hypergraph H^* having vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and hyperarc set $E = \{e_1, e_2, \cdots, e_m\}$. Then the *incidence matrix* of H^* is an $m \times n$ matrix $[c_{ij}]$ which is defined as follows:

$$c_{ij} = \begin{cases} 1, if\ v_i \in T(e_j) \\ -1, if\ v_i \in H(e_j) \\ 0, otherwise. \end{cases}$$

2.2 Soft Directed Graphs

J. Jose, B. George and R.K. Thumbakara [12] introduced the notion of the soft directed graph by applying the concepts of soft set in the directed graph as follows. Let $D^* = (V, A)$ be a directed graph having vertex set V and arc set A and let P be a non-empty set. Let a subset R of $P \times V$ be an arbitrary relation from P to V. Define a mapping $J: P \to \mathcal{P}(V)$ by $J(x) = \{u \in V | xRu\}$ where $\mathcal{P}(V)$ denotes the powerset of V. Define another mapping $L: P \to \mathcal{P}(A)$ by $L(x) = \{(u, v) \in A | \{u, v\} \subseteq J(x)\}$ where $\mathcal{P}(A)$ denotes the powerset of E. Then $D = (D^*, J, L, P)$ is called a soft directed graph if it satisfies the following conditions:

1. $D^* = (V, A)$ is a directed graph having vertex set V and arc set A,
2. P is a nonempty set of parameters,
3. (J, P) is a soft set over the vertex set V,
4. (L, P) is a soft set over the arc set A,
5. $(J(x), L(x))$ is a subdigraph of D^* for all $x \in P$.

If we represent $(J(x), L(x))$ by $M(x)$ then the soft directed graph D is also given by $\{M(x): x \in P\}$. Then $M(x)$ corresponding to a parameter x in P is called a *directed* *part* or simply *dipart* of the soft directed graph D.

Let $D = (D^*, J, L, P)$ be a soft directed graph and let $M(x)$ be a dipart of D for some $x \in P$. Let v be a vertex of $M(x)$. Then dipart indegree of v in $M(x)$ denoted by $ideg\ v[M(x)]$ is defined as the number of vertices of $M(x)$ from which v is adjacent. That is, $ideg\ v[M(x)]$ is the number of arcs of $M(x)$ that have v as its head. Similarly, dipart outdegree of v in $M(x)$ denoted by $odeg\ v[M(x)]$ is defined as the number of vertices of $M(x)$ to which v is adjacent. That is, $odeg\ v[M(x)]$ is the number of arcs of $M(x)$ that have v as its tail. The dipart degree of v in $M(x)$ is defined as the sum, $ideg\ v[M(x)] + odeg\ v[M(x)]$ and is denoted by $deg\ v[M(x)]$. Let $M(x)$ be a dipart of the soft directed graph D, for some $x \in P$. Suppose that $M(x)$ contains n vertices, say $v_1, v_2, \cdots, v_n$. That is, $|J(x)| = n$. Then the dipart adjacency matrix of $M(x)$ denoted by $X_{ad}[M(x)]$ is an $n \times n$ matrix $[a_{ij}]$ where

$$a_{ij} = \begin{cases} 1, & if\ there\ is\ an\ arc\ from\ v_i\ to\ v_j\ in\ M(x), \\ 0, & if\ there\ is\ no\ arc\ from\ v_i\ to\ v_j\ in\ M(x). \end{cases}$$

Then the adjacency matrix of the soft directed graph D, denoted by $X_{ad}(D)$ is given by $X_{ad}(D) = \{X_{ad}[M(x)]: x \in P\}$ where $X_{ad}[M(x)]$ denotes the dipart adjacency matrix of $M(x)$.Let $v_1, v_2, \cdots, v_m$ be the m vertices and $a_1, a_2, \cdots, a_n$ be the n arcs of $M(x)$. Then the dipart incidence matrix of $M(x)$ denoted by $X_{in}[M(x)]$ is an $m \times n$ matrix $[c_{ij}]$ where

$$c_{ij} = \begin{cases} 1, & if\ v_i\ is\ the\ initial\ vertex\ (tail) of\ the\ arc\ a_j\ in\ M(x), \\ -1, & if\ v_i\ is\ the\ final\ vertex\ (head) of\ the\ arc\ a_j\ in\ M(x), \\ 0, & if\ v_i\ is\ not\ the\ tail\ or\ the\ head\ of\ the\ arc\ a_j\ in\ M(x). \end{cases}$$

3. Soft Directed Hypergraphs

Definition 3.1 *Let $H^* = (V, E)$ be a simple directed hypergraph with vertex set V and directed hyperedge(hyperarc) set E. Then $e' = (T(e'), H(e'))$ where $T(e')$ and $H(e')$ are nonempty subsets of V, is said to be a subhyperarc of H^* if there exists a hyperarc e in H^* such that $T(e') \subseteq T(e)$ and $H(e') \subseteq H(e)$. We also say that e' is a subhyperarc of e. A hyperarc is a subhyperarc of itself. e' is said to be a proper subhyperarc of e if either $T(e') \subset T(e)$ or $H(e') \subset H(e)$.*

Definition 3.2 *Let $H^* = (V, E)$ be a simple directed hypergraph with vertex set V and directed hyperedge(hyperarc) set E and P be any nonempty set. let E_s be the set of all subhyperarcs of H^*. Let R be an arbitrary relation between elements of P and elements of V. That is $R \subseteq P \times V$. A mapping $X: P \to \mathcal{P}(V)$ can be defined as $X(p) = \{v \in V: pRv\}$ where $\mathcal{P}(V)$ denotes the powerset of V . Also define a mapping $Y: P \to \mathcal{P}(E_s)$ by $Y(p) = \{(T(e) \cap X(p), H(e) \cap X(p)) \mid e \in E$ and $T(e) \cap X(p) \neq \phi$ and $H(e) \cap X(p) \neq \phi\}$ where $\mathcal{P}(E_s)$ denotes the powerset of E_s. The pair (X, P) is* *a soft set over V and the pair (Y,P) is a soft set over E_S. Also $(X(p),Y(p))$ is a weak induced subhypergraph of H^* for all $p \in P$. Then the 4 -tuple $H = (H^*,X,Y,P)$ is called a soft directed hypergraph.*

That is, in a soft directed hypergraph $H = (H^*,X,Y,P)$ the following conditions are satisfied:

1. $H^* = (V,E)$ is a simple directed hypergraph having vertex set V and hyperarc set E,
2. P is a nonempty set of parameters,
3. (X,P) is a soft set over V,
4. (Y,P) is a soft set over E_S,
5. $(X(p),Y(p))$ is a weak induced subhypergraph of H^* for all $p \in P$.

If we represent $(X(p),Y(p))$ by $Z(p)$ then the soft directed hypergraph H is also given by $\{Z(p): p \in P\}$. Then $Z(p)$ corresponding to a parameter p in P is called a *directed hyperpart* or simply *dh-part* of the soft directed hypergraph H.

Definition 3.3 *Let $H^* = (V,E)$ be a simple hypergraph having vertex set V and hyperarc set E. Also let $H_1 = (H^*,X_1,Y_1,P_1)$ and $H_2 = (H^*,X_2,Y_2,P_2)$ be two soft directed hypergraphs of H^* . Then H_2 is a soft weak induced subhypergraph of H_1 if*

1. $P_2 \subseteq P_1$,
2. $Z_2(p) = (X_2(p),Y_2(p))$ is a weak induced subhypergraph of $Z_1(p) = (X_1(p),Y_1(p))$ for all $p \in P_2$.

Example 3.1 Consider a simple directed hypergraph $H^* = (V,E)$ given in Fig. 1.

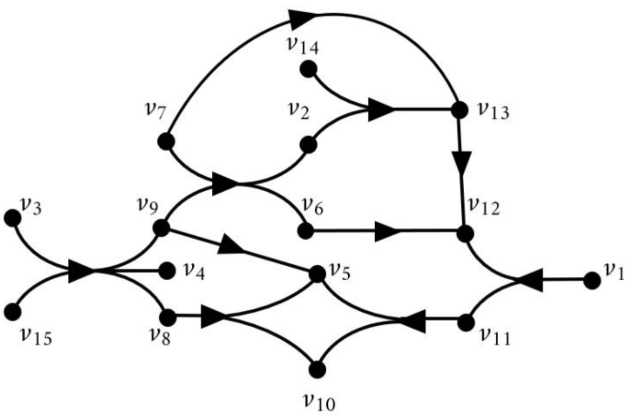


Fig.1: Directed Hypergraph $H^* = (V,E)$

Let $P_1 = \{v_9, v_{13}\} \subseteq V$ be a parameter set. Define a function $X_1: P_1 \to \mathcal{P}(V)$ defined by $X_1(p) = \{v \in V: pRv \Leftrightarrow v = p$ or v is adjacent from p or v is adjacent to $p\}$, for

all $p \in P_1$. That is, $X_1(v_9) = \{v_2, v_3, v_5, v_6, v_9, v_{15}\}$ and $X_1(v_{13}) = \{v_2, v_7, v_{12}, v_{13}, v_{14}\}$. Then (X_1, P_1) is a soft set over V.
Define another function $Y_1: P_1 \to \mathcal{P}(E_s)$ defined by $Y_1(p) = \{(T(e) \cap X_1(p), H(e) \cap X_1(p)) \mid e \in E$ and $T(e) \cap X_1(p) \neq \phi$ and $H(e) \cap X_1(p) \neq \phi\}$.
That is, $Y_1(v_9) = \{(\{v_3, v_{15}\}, \{v_9\}), (\{v_9\}, \{v_2, v_6\}), (\{v_9\}, \{v_5\})\}$ and $Y_1(v_{13}) = \{(\{v_2, v_{14}\}, \{v_{13}\}), (\{v_{13}\}, \{v_{12}\}), (\{v_7\}, \{v_{13}\})\}$. Then (Y_1, P_1) is a soft set over E_s. Also $Z_1(v_9) = (X_1(v_9), Y_1(v_9))$ and $Z_1(v_{13}) = (X_1(v_{13}), Y_1(v_{13}))$ are weak induced subhypergraphs of H^* as shown in Fig. 2.

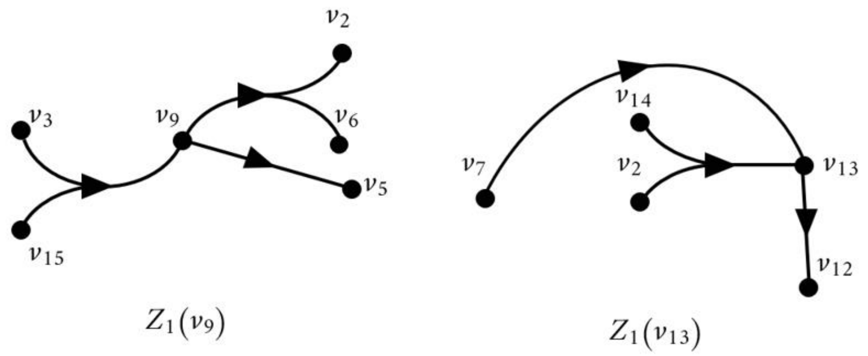


Fig. 2: Soft Directed Hypergraph $H_1 = \{Z_1(v_9), Z_1(v_{13})\}$

Hence $H_1 = \{Z_1(v_9), Z_1(v_{13})\}$ is a soft directed hypergraph of H*.

Let $P_2 = \{v_{13}\} \subseteq V$ be another parameter set. Define a function $X_2: P_2 \to \mathcal{P}(V)$ defined by $X_2(p) = \{v \in V: pRv \Leftrightarrow v = p$ or v is adjacent to $p\}$ for all $p \in P_2$. That is, $X_2(v_{13}) = \{v_2, v_7, v_{13}, v_{14}\}$. Then, (X_2, P_2) is a soft set over V.
Define another function $Y_2: P_2 \to \mathcal{P}(E_s)$ defined by $Y_2(p) = \{(T(e) \cap X_2(p), H(e) \cap X_2(p)) \mid e \in E$ and $T(e) \cap X_2(p) \neq \phi$ and $H(e) \cap X_2(p) \neq \phi\}$.
That is, $Y_2(v_{13}) = \{(\{v_2, v_{14}\}, \{v_{13}\}), (\{v_7\}, \{v_{13}\})\}$. Then, (Y_2, P_2) is a soft set over E_s. Also, $Z_2(v_{13}) = (X_2(v_{13}), Y_2(v_{13}))$ is a weak induced subhypergraph of H^* as shown in Fig. 3.

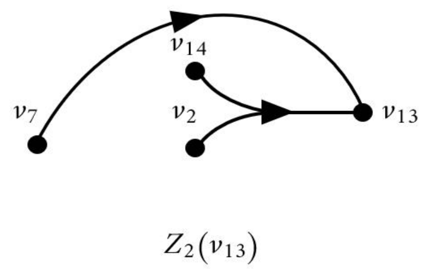


Fig.3: Soft Directed Hypergraph $H_2 = \{Z_2(v_{13})\}$

Hence $H_2 = \{Z_2(v_{13})\}$ is a soft directed hypergraph of H^*.

Here H_2 is a soft weak induced subhypergraph of H_1 since

1. $P_2 \subseteq P_1$,
2. $Z_2(v_{13}) = (X_2(v_{13}), Y_2(v_{13}))$ is a weak induced subhypergraph of $Z_1(v_{13}) = (X_1(v_{13}), Y_1(v_{13}))$.

4 Degrees Associated with Soft Directed Hypergraphs

Definition 4.1 *Let $H = (H^*, X, Y, P)$ be a soft directed hypergraph and let $Z(p)$ be a dh-part of H for some $p \in P$. Let v be any vertex of $Z(p)$. Then dh-part indegree of v in $Z(p)$ denoted by $ideg\ v[Z(p)]$ is defined as the number of hyperarcs in $Z(p)$ that contains v in their head. Similarly, dh-part outdegree of v in $Z(p)$ denoted by $odeg\ v[Z(p)]$ is defined as the number of hyperarcs in $Z(p)$ that contains v in their tail.*

Definition 4.2 *Soft indegree of a vertex v in a soft directed hypergraph H denoted by $ideg\ v$ is defined as $max\{ideg\ v[Z(p)]: p \in P\}$ where $ideg\ v[Z(p)]$ denotes the dh-part indegree of v in $Z(p)$ and $ideg\ v$ is defined for all $v \in \cup_{p \in P} X(p)$. Similarly, soft outdegree of a vertex v in a soft directed hypergraph H denoted by $odeg\ v$ is defined as $max\{odeg\ v[Z(p)]: p \in P\}$ where $odeg\ v[Z(p)]$ denotes the dh-part outdegree of v in $Z(p)$ and $odeg\ v$ is defined for all $v \in \cup_{p \in P} X(p)$.*

Definition 4.3 *Let $H^* = (V, E)$ be a directed hypergraph and $H = (H^*, X, Y, P)$ be a soft directed hypergraph of H^* which is also given by $\{Z(p): p \in P\}$. Let $Z(p)$ be a dh-part of H for some $p \in P$ and v be a vertex of $Z(p)$. Then dh-part degree of v in $Z(p)$ is defined as the sum, $ideg\ v[Z(p)] + odeg\ v[Z(p)]$ and is denoted by $deg\ v[Z(p)]$.*

Definition 4.4 *Soft degree of a vertex v in a soft directed hypergraph H is defined as $max\{deg\ v[Z(p)]: p \in P\}$ and is denoted by $deg\ v$.*

Example 4.1 Consider a simple directed hypergraph $H^* = (V, E)$ given in Fig. 4.
Let $P = \{v_1, v_{13}\} \subseteq V$ be a parameter set. Define a function $X: P \to \mathcal{P}(V)$ defined by $X(p) = \{v \in V: pRv \Leftrightarrow v = p$ or v is adjacent from p or v is adjacent to $p\}$ for all $p \in P$. That is, $X(v_1) = \{v_1, v_3, v_6, v_8, v_{11}\}$ and $X(v_{13}) = \{v_7, v_9, v_{13}, v_{14}\}$.
Then (X, P) is a soft set over V. Define another function $Y: P \to \mathcal{P}(E_s)$ defined by $Y(p) = \{(T(e) \cap X(p), H(e) \cap X(p)) | e \in E$ and $T(e) \cap X(p) \neq \phi$ and $H(e) \cap X(p) \neq \phi\}$.
That is, $Y(v_1) = \{(\{v_6, v_8\}, \{v_1\}), (\{v_1\}, \{v_3, v_{11}\})\}$ and
$Y(v_{13}) = \{(\{v_{14}\}, \{v_{13}\}), (\{v_{13}\}, \{v_7, v_9\})\}$. Then (Y, P) is a soft set over E_s.

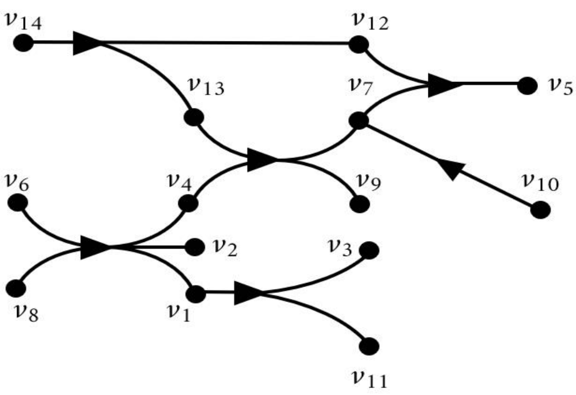


Fig. 4: Directed Hypergraph $H^* = (V, E)$

Also, $Z(v_1) = (X(v_1), Y(v_1))$ and $Z(v_{13}) = (X(v_{13}), Y(v_{13}))$ are weak induced subhypergraphs of H^* as shown in Fig. 5.
Hence, $H = \{Z(v_1), Z(v_{13})\}$ is a soft directed hypergraph of H^*.

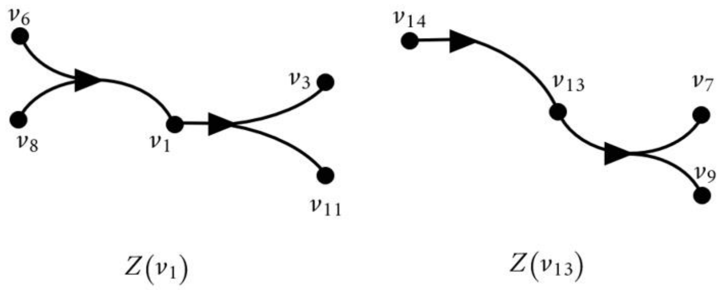


Fig.5: Soft Directed Hypergraph $H = \{Z(v_1), Z(v_{13})\}$

In this soft directed hypergraph H, $ideg\ v_6[Z(v_1)] = 0, ideg\ v_8[Z(v_1)] = 0,$ $ideg\ v_1[Z(v_1)] = 1, ideg\ v_3[Z(v_1)] = 1,\ ideg\ v_{11}[Z(v_1)] = 1,$ $ideg\ v_{14}[Z(v_{13})] = 0, ideg\ v_{13}[Z(v_{13})] = 1, ideg\ v_7[Z(v_{13})] = 1$ and $ideg\ v_9[Z(v_{13})] = 1$. Also $ideg\ v_6 = 0, ideg\ v_8 = 0, ideg\ v_1 = 1,$ $ideg\ v_3 = 1, ideg\ v_{11} = 1, ideg\ v_{14} = 0, ideg\ v_{13} = 1, ideg\ v_7 = 1$ and $ideg\ v_9 = 1$. Similarly, $odeg\ v_6[Z(v_1)] = 1, odeg\ v_8[Z(v_1)] = 1,$ $odeg\ v_1[Z(v_1)] = 1, odeg\ v_3[Z(v_1)] = 0, odeg\ v_{11}[Z(v_1)] = 0,$ $odeg\ v_{14}[Z(v_{13})] = 1, odeg\ v_{13}[Z(v_{13})] = 1, odeg\ v_7[Z(v_{13})] = 0$ and $odeg\ v_9[Z(v_{13})] = 0$. Also $odeg\ v_6 = 1, odeg\ v_8 = 1, odeg\ v_1 = 1,$ $odeg\ v_3 = 0, odeg\ v_{11} = 0, odeg\ v_{14} = 1, odeg\ v_{13} = 1, odeg\ v_7 = 0$ and

$odeg\ v_9 = 0$. Therefore, $\deg v_6[Z(v_1)] = 1, \deg v_8[Z(v_1)] = 1$, $\deg v_1[Z(v_1)] = 2, \deg v_3[Z(v_1)] = 1, \deg v_{11}[Z(v_1)] = 1$, $\deg v_{14}[Z(v_{13})] = 1, \deg v_{13}[Z(v_{13})] = 2, \deg v_7[Z(v_{13})] = 1$ and $\deg v_9[Z(v_{13})] = 1$. Also $\deg v_6 = 1, \deg v_8 = 1, \deg v_1 = 2, deg\ v_3 = 1$, $deg\ v_{11} = 1, deg\ v_{14} = 1, deg\ v_{13} = 2, deg\ v_7 = 1$ and $deg\ v_9 = 1$.

5 Soft Incidence Matrix of a Soft Directed Hypergraph

We define soft incidence matrix of a soft directed hypergraph by extending the definition of incidence matrix in soft directed graphs [12] as follows.

Definition 5.1 *Let $H^* = (V, E)$ be a directed hypergraph having vertex set V and hyperarc set E. Let $H = (H^*, X, Y, P)$ be a soft directed hypergraph of H^* which is also given by $\{Z(p): p \in P\}$. Let $Z(p)$ be a dh-part of the soft directed hypergraph H, for some $p \in P$. Let $v_1, v_2, \cdots, v_m$ be the m vertices and $e_1, e_2, \cdots, e_n$ be the n hyperarcs of $Z(p)$. Then the dh-part incidence matrix of $Z(p)$ denoted by $M_{in}[Z(p)]$ is an $m \times n$ matrix $[c_{ij}]$ where*

$$c_{ij} = \begin{cases} 1; & if\ v_i\ is\ the\ tail\ of\ the\ hyperarc\ e_j\ in\ Z(p), \\ -1; & if\ v_i\ is\ the\ head\ of\ the\ hyperarc\ e_j\ in\ Z(p), \\ 0; & if\ v_i\ is\ not\ the\ tail\ or\ the\ head\ of\ the\ hyperarc\ e_j\ in\ Z(p). \end{cases}$$

Definition 5.2 *Let $H = (H^*, X, Y, P)$ be a soft directed hypergraph of H^* given by $\{Z(p): p \in P\}$. Then the soft incidence matrix of the soft directed hypergraph H, denoted by $M_{in}(H)$ is given by $M_{in}(H) = \{M_{in}[Z(p)]: p \in P\}$ where $M_{in}[Z(p)]$ denotes the dh-part incidence matrix of $Z(p)$.*

Example 5.1 *Consider the directed hypergraph $H^* = (V, E)$ given in Fig. 4 and its soft directed hypergraph $H = \{Z(v_1), Z(v_{13}\}$ given in Fig. 5. We give names to the hyperarcs of H as follows: $e_1 = (\{v_6, v_8\}, \{v_1\}), e_2 = (\{v_1\}, \{v_3, v_{11}\}), e_3 = (\{v_{14}\}, \{v_{13}\})$ and $e_4 = (\{v_{13}\}, \{v_7, v_9\})$. Here $M_{in}(H) = \{M_{in}[Z(p)]: p \in P\} = \{M_{in}[Z(v_1)], M_{in}[Z(v_{13})]\}$ where $M_{in}[Z(v_1)]$ and $M_{in}[Z(v_{13})]$ are as given below.*

$$M_{in}[Z(v_1)] = \begin{array}{cc} \begin{array}{cc} e_1 & e_2 \end{array} & \\ \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 1 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} & \begin{array}{c} v_1 \\ v_3 \\ v_6 \\ v_8 \\ v_{11} \end{array} \end{array}, \quad M_{in}[Z(v_{13})] = \begin{array}{cc} \begin{array}{cc} e_3 & e_4 \end{array} & \\ \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ -1 & 1 \\ 1 & 0 \end{bmatrix} & \begin{array}{c} v_7 \\ v_9 \\ v_{13} \\ v_{14} \end{array} \end{array}.$$

Remark 5.1 *The soft incidence matrix of a soft directed hypergraph* $H = (H^*, X, Y, P) = \{Z(p): p \in P\}$ *has the following properties.*

1. $M_{in}[Z(p)]$ is a matrix of order $m \times n$, if $Z(p)$ contains m vertices and n hyperarcs, $\forall p \in P$.
2. $M_{in}[Z(p)]$ contains only 0, 1 and -1 as entries, $\forall p \in P$.
3. Number of entries equal to 1 in a row of $M_{in}[Z(p)]$ for some $p \in P$ gives the dh-part outdegree of the corresponding vertex v, i.e., $odeg\ v[Z(p)]$. Similarly, number of entries equal to -1 in a row gives the dh-part indegree of the corresponding vertex v, i.e., $ideg\ v[Z(p)]$. Hence number of entries equal to 1 or -1 in a row of $M_{in}[Z(p)]$ gives the dh-part degree of the corresponding vertex v in $Z(p)$.

6 Soft Adjacency Matrix of a Soft Directed Hypergraph

We define the soft adjacency matrix of a soft directed hypergraph by extending the definition of adjacency matrix in soft directed graphs [12] as follows.

Definition 6.1 *Let* $H^* = (V, E)$ *be a directed hypergraph having vertex set* V *and hyperarc set* E. *Let* $H = (H^*, X, Y, P)$ *be a soft directed hypergraph of* H^* *which is also given by* $\{Z(p): p \in P\}$. *Let* $Z(p)$ *be a* dh*-part of the soft directed hypergraph* H, *for some* $p \in P$. *Let* $v_1, v_2, \cdots, v_m$ *be the* m *vertices in* $Z(p)$. *Then the* dh*-part adjacency matrix of* $Z(p)$ *denoted by* $M_{ad}[Z(p)]$ *is an* $m \times m$ *matrix* $[a_{ij}]$ *where*

$$a_{ij} = \begin{cases} 1; & if\ v_i\ is\ adjacent\ to\ v_j\ in\ Z(p), \\ 0; & if\ not. \end{cases}$$

Definition 6.2 *Let* $H = (H^*, X, Y, P)$ *be a soft directed hypergraph of* H^* *given by* $\{Z(p): p \in P\}$. *Then the adjacency matrix of the soft directed hypergraph* H, *denoted by* $M_{ad}(H)$ *is given by* $M_{ad}(H) = \{M_{ad}[Z(p)]: p \in P\}$ *where* $M_{ad}[Z(p)]$ *denotes the* dh*-part adjacency matrix of* $Z(p)$.

Example 6.1 *Consider the directed hypergraph* $H^* = (V, E)$ *given in Fig. 4 and its soft directed hypergraph* $H = \{Z(v_1), Z(v_{13})\}$ *given in Fig. 5. Here* $M_{ad}(H) = \{M_{ad}[Z(p)]: p \in P\} = \{M_{ad}[Z(v_1)], M_{ad}[Z(v_{13})]\}$ *where* $M_{ad}[Z(v_1)]$ *and* $M_{ad}[Z(v_{13})]$ *are as given below.*

$$M_{ad}[Z(v_1)] = \begin{array}{c} \begin{array}{ccccc} v_1 & v_3 & v_6 & v_8 & v_{11} \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_1 \\ v_3 \\ v_6 \\ v_8 \\ v_{11} \end{array},$$

$$M_{ad}[Z(v_{13})] = \begin{array}{c} \begin{array}{cccc} v_7 & v_9 & v_{13} & v_{14} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_7 \\ v_9 \\ v_{13} \\ v_{14} \end{array}.$$

Remark 6.1 *The soft adjacency matrix of a soft directed hypergraph* $H = (H^*, X, Y, P) = \{Z(p): p \in P\}$ *has the following properties.*

1. $M_{ad}[Z(p)]$ is a square matrix of order m, if $Z(p)$ contains m vertices, $\forall p \in P$.
2. $M_{ad}[Z(p)]$ contains only 0 and 1 as its entries.
3. In $M_{ad}[Z(p)], p \in P$ diagonal entries are 0.

7 AND Operation on Soft Directed Hypergraphs

Definition 7.1 *Let* $H^* = (V, E)$ *be a simple directed hypergraph having vertex set* V *and hyperarc set* E. *Also let* $H_1 = (H^*, X_1, Y_1, P_1)$ *and* $H_2 = (H^*, X_2, Y_2, P_2)$ *be two soft directed hypergraphs of* H^* *such that* $X_1(p) \cap X_2(q) \neq \phi$ *for* $(p, q) \in P_1 \times P_2$. *Then AND operation on* H_1 *and* H_2 *denoted by* $H_1 \wedge H_2$ *is defined as* $H_1 \wedge H_2 = H = (H^*, X, Y, P)$, *where* $P = P_1 \times P_2$ *and for all* $(p, q) \in P = P_1 \times P_2$, $X(p, q) = X_1(p) \cap X_2(q)$ *and* $Y(p, q) = \{(T(e) \cap X(p, q), H(e) \cap X(p, q)) \mid e \in E$ *and* $T(e) \cap X(p, q) \neq \phi$ *and* $H(e) \cap X(p, q) \neq \phi\}$. *If* $Z(p, q) = (X(p, q), Y(p, q))$, $\forall (p, q) \in P = P_1 \times P_2$, *then* $H_1 \wedge H_2 = \{Z(p, q): (p, q) \in P\}$.

Example 7.1 *Consider a simple directed hypergraph* $H^* = (V, E)$ *given in Fig. 6.*

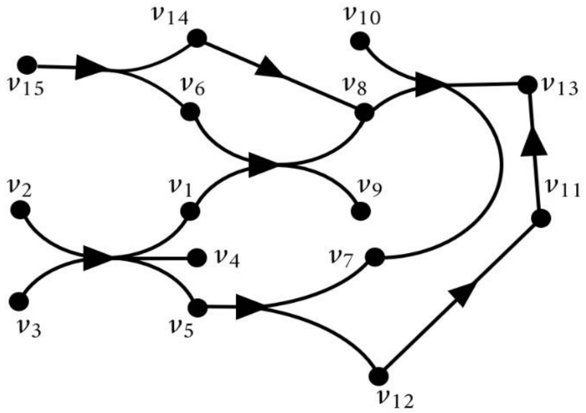


Fig.6: Directed Hypergraph $H^* = (V, E)$

Let $P_1 = \{v_1, v_{13}\} \subseteq V$ be a parameter set. Define a function $X_1: P_1 \to \mathcal{P}(V)$ defined by $X_1(p) = \{v \in V: pRv \Leftrightarrow v = p$ or v is adjacent from p or v is adjacent to $p\}$, for all $p \in P_1$. That is, $X_1(v_1) = \{v_1, v_2, v_3, v_8, v_9\}$ and $X_1(v_{13}) = \{v_8, v_{10}, v_{11}, v_{13}\}$. Then (X_1, P_1) is a soft set over V.
Define another function $Y_1: P_1 \to \mathcal{P}(E_s)$ defined by $Y_1(p) = \{(T(e) \cap X_1(p), H(e) \cap X_1(p)) \mid e \in E$ and $T(e) \cap X_1(p) \neq \phi$ and $H(e) \cap X_1(p) \neq \phi\}$.
That is, $Y_1(v_1) = \{(\{v_2, v_3\}, \{v_1\}), (\{v_1\}, \{v_8, v_9\})\}$ and
$Y_1(v_{13}) = \{(\{v_8, v_{10}\}, \{v_{13}\}), (\{v_{11}\}, \{v_{13}\})\}$. Then (Y_1, P_1) is a soft set over E_s. Also $Z_1(v_1) = (X_1(v_1), Y_1(v_1))$ and $Z_1(v_{13}) = (X_1(v_{13}), Y_1(v_{13}))$ are weak induced subhypergraphs of H^* as shown in Fig. 7.

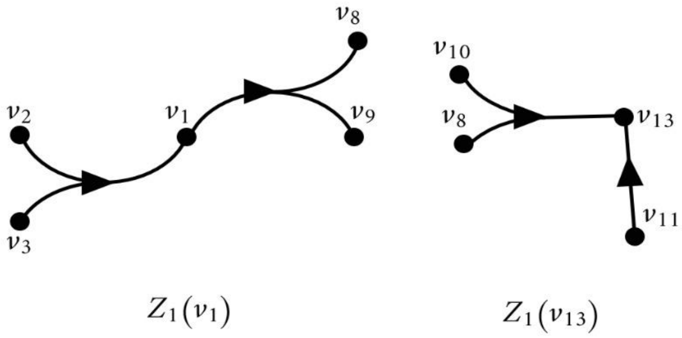


Fig.7: Soft Directed Hypergraph $H_1 = \{Z_1(v_1), Z_1(v_{13})\}$

Hence $H_1 = \{Z_1(v_1), Z_1(v_{13})\}$ is a soft directed hypergraph of H^*.

Let $P_2 = \{v_8\} \subseteq V$ be another parameter set. Define a function $X_2: P_2 \to \mathcal{P}(V)$ defined by $X_2(p) = \{v \in V: pRv \Leftrightarrow v = p$ or v is adjacent to p or v is adjacent from $p\}$, for all $p \in P_2$. That is, $X_2(v_8) = \{v_1, v_6, v_7, v_8, v_{13}, v_{14}\}$. Then (X_2, P_2) is a soft set over V.

Define another function $Y_2: P_2 \to \mathcal{P}(E_s)$ defined by $Y_2(p) = \{(T(e) \cap X_2(p), H(e) \cap X_2(p)) \mid e \in E$ and $T(e) \cap X_2(p) \neq \phi$ and $H(e) \cap X_2(p) \neq \phi\}$.
That is, $Y_2(v_8) = \{(\{v_1, v_6\}, \{v_8\}), (\{v_{14}\}, \{v_8\}), (\{v_8\}, \{v_7, v_{13}\}\}$. Then (Y_2, P_2) is a soft set over E_s. Also $Z_2(v_8) = (X_2(v_8), Y_2(v_8))$ is a weak induced subhypergraph of H^* as shown in Fig. 8.

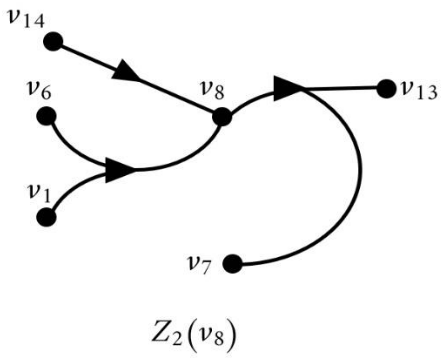


Fig.8: Soft Directed Hypergraph $H_2 = \{Z_2(v_8)\}$

Hence $H_2 = \{Z_2(v_8)\}$ is a soft directed hypergraph of H^*.

Thus we get two soft directed hypergraphs H_1 and H_2 of the directed hypergraph H^*. Then $H_1 \wedge H_2 = (H^*, X, Y, P)$, where $P = P_1 \times P_2 = \{(v_1, v_8), (v_{13}, v_8)\}$, $X(v_1, v_8) = X_1(v_1) \cap X_2(v_8) = \{v_1, v_8\}$, $Y(v_1, v_8) = \{(\{v_1\}, \{v_8\})\}$, $X(v_{13}, v_8) = X_1(v_{13}) \cap X_2(v_8) = \{v_8, v_{13}\}$ and $Y(v_{13}, v_8) = \{(\{v_8\}, \{v_{13}\})\}$. Here P is the parameter set, (X, P) is a soft set over V and (Y, P) is a soft set over E_s. Also $Z(v_1, v_8) = (X(v_1, v_8), Y(v_1, v_8))$ and $Z(v_{13}, v_8) = (X(v_{13}, v_8), Y(v_{13}, v_8))$ are weak induced subhypergraphs of H^*. Hence, $H_1 \wedge H_2 = \{Z(v_1, v_8), Z(v_{13}, v_8)\}$ is a soft directed hypergraph of H^* and is given in Fig. 9.

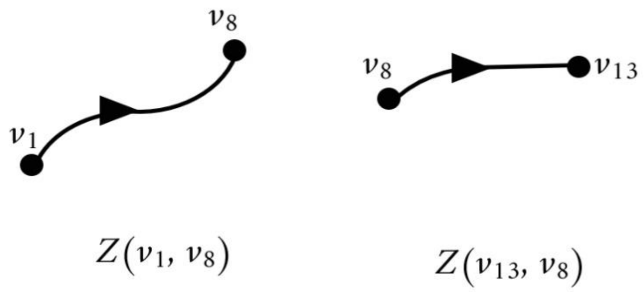


Fig.9: $H_1 \wedge H_2 = \{Z(v_1, v_8), Z(v_{13}, v_8)\}$

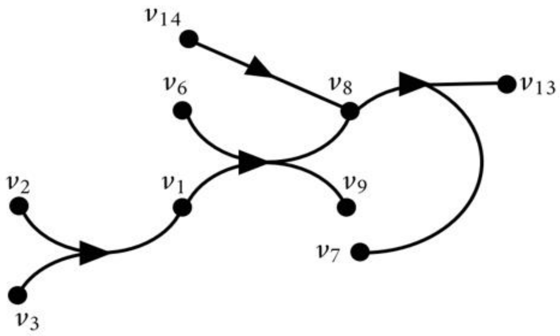
Theorem 7.1 *Let $H^* = (V, E)$ be a simple directed hypergraph and $H_1 = (H^*, X_1, Y_1, P_1)$ and $H_2 = (H^*, X_2, Y_2, P_2)$ be two soft directed hypergraphs of H^* such* *that $X_1(p) \cap X_2(q) \neq \phi$ for $(p,q) \in P_1 \times P_2$. Then $H_1 \wedge H_2$ is also a soft directed hypergraph of H^*.*

Proof. AND operation on H_1 and H_2 is defined as $H_1 \wedge H_2 = H = (H^*, X, Y, P)$, where the parameter set $P = P_1 \times P_2$ and for all $(p,q) \in P = P_1 \times P_2$, $X(p,q) = X_1(p) \cap X_2(q)$ and $Y(p,q) = \{(T(e) \cap X(p,q), H(e) \cap X(p,q)) \mid e \in E$ and $T(e) \cap X(p,q) \neq \phi$ and $H(e) \cap X(p,q) \neq \phi\}$. Clearly X is a mapping from P to $\mathcal{P}(V)$ and Y is a mapping from P to $\mathcal{P}(E_s)$. So (X,P) is a soft set over V and (Y,P) is a soft set over E_s. When $(p,q) \in P = P_1 \times P_2$, the corresponding dh-part of $H_1 \wedge H_2$ is $Z(p,q) = (X(p,q), Y(p,q))$, where $X(p,q) = X_1(p) \cap X_2(q)$ and $Y(p,q) = \{(T(e) \cap X(p,q), H(e) \cap X(p,q)) \mid e \in E$ and $T(e) \cap X(p,q) \neq \phi$ and $H(e) \cap X(p,q) \neq \phi\}$. Here $X_1(p) \cap X_2(q) \subseteq V$ and each hyperarc in $Y(p,q)$ is a subhyperarc of a hyperarc in H^*. So $Z(p,q)$ is a weak induced subhypergraph of H^* for every $(p,q) \in P$. That is, $H = H_1 \wedge H_2$ is a soft directed hypergraph of H^* since $H_1 \wedge H_2$ can be expressed as (H^*, X, Y, P) and it satisfies all the conditions for a soft directed hypergraph.

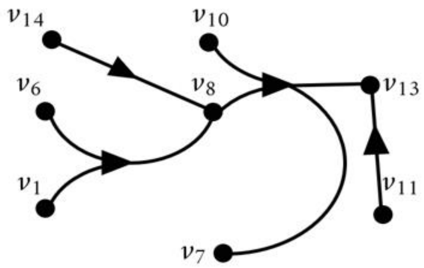
8 OR Operation on Soft Directed Hypergraphs

Definition 8.1 *Let $H^* = (V,E)$ be a simple directed hypergraph having vertex set V and hyperarc set E. Also let $H_1 = (H^*, X_1, Y_1, P_1)$ and $H_2 = (H^*, X_2, Y_2, P_2)$ be two soft directed hypergraphs of H^*. Then OR operation on H_1 and H_2 denoted by $H_1 \vee H_2$ is defined as $H_1 \vee H_2 = H = (H^*, X, Y, P)$, where $P = P_1 \times P_2$ and for all $(p,q) \in P = P_1 \times P_2$, $X(p,q) = X_1(p) \cup X_2(q)$ and $Y(p,q) = \{(T(e) \cap X(p,q), H(e) \cap X(p,q)) \mid e \in E$ and $T(e) \cap X(p,q) \neq \phi$ and $H(e) \cap X(p,q) \neq \phi\}$. If $Z(p,q) = (X(p,q), Y(p,q)), \forall (p,q) \in P = P_1 \times P_2$, then $H_1 \vee H_2 = \{Z(p,q): (p,q) \in P\}$.*

Example 8.1 *Consider the simple directed hypergraph H^* given in Fig. 6 and its soft directed hypergraphs H_1 and H_2 given in Fig. 7 and Fig. 8 respectively. Then $H_1 \vee H_2 = (H^*, X, Y, P)$, where $P = P_1 \times P_2 = \{(v_1, v_8), (v_{13}, v_8)\}$, $X(v_1, v_8) = X_1(v_1) \cup X_2(v_8) = \{v_1, v_2, v_3, v_6, v_7, v_8, v_9, v_{13}, v_{14}\}$,*
$Y(v_1, v_8) = \{(\{v_2, v_3\}, \{v_1\}), (\{v_1, v_6\}, \{v_8, v_9\}), (\{v_{14}\}, \{v_8\}), (\{v_8\}, \{v_7, v_{13}\})\}$,
$X(v_{13}, v_8) = X_1(v_{13}) \cup X_2(v_8) = \{v_1, v_6, v_7, v_8, v_{10}, v_{11}, v_{13}, v_{14}\}$ and $Y(v_{13}, v_8) = \{(\{v_1, v_6\}, \{v_8\}), (\{v_{14}\}, \{v_8\}), (\{v_8, v_{10}\}, \{v_7, v_{13}\}), (\{v_{11}\}, \{v_{13}\})\}$. Here P is the parameter set, (X,P) is a soft set over V and (Y,P) is a soft set over E_s. Also $Z(v_1, v_8) = (X(v_1, v_8), Y(v_1, v_8))$ and $Z(v_{13}, v_8) = (X(v_{13}, v_8), Y(v_{13}, v_8))$ are weak induced subhypergraphs of H^*. Hence $H_1 \vee H_2 = \{Z(v_1, v_8), Z(v_{13}, v_8)\}$ is a soft directed hypergraph of H^* and is given in Fig. 10.



$Z(v_1, v_8)$



$Z(v_{13}, v_8)$

Fig.10: $H_1 \vee H_2 = \{Z(v_1, v_8), Z(v_{13}, v_8)\}$

Theorem 8.1 *Let $H^* = (V, E)$ be a simple directed hypergraph and $H_1 = (H^*, X_1, Y_1, P_1)$ and $H_2 = (H^*, X_2, Y_2, P_2)$ be two soft directed hypergraphs of H^*. Then $H_1 \vee H_2$ is also a soft directed hypergraph of H^*.*

Proof. OR operation on H_1 and H_2 is defined as $H_1 \vee H_2 = H = (H^*, X, Y, P)$, where the parameter set $P = P_1 \times P_2$ and for all $(p, q) \in P = P_1 \times P_2$, $X(p, q) = X_1(p) \cup X_2(q)$ and $Y(p, q) = \{(T(e) \cap X(p, q), H(e) \cap X(p, q)) \mid e \in E$ and $T(e) \cap X(p, q) \neq \phi$ and $H(e) \cap X(p, q) \neq \phi\}$. Clearly X is a mapping from P to $\mathcal{P}(V)$ and Y is a mapping from P to $\mathcal{P}(E_s)$. So (X, P) is a soft set over V and (Y, P) is a soft set over E_s. When $(p, q) \in P = P_1 \times P_2$, the corresponding dh-part of $H_1 \vee H_2$ is $Z(p, q) = (X(p, q), Y(p, q))$, where $X(p, q) = X_1(p) \cup X_2(q)$ and $Y(p, q) = \{(T(e) \cap X(p, q), H(e) \cap X(p, q)) \mid e \in E$ and $T(e) \cap X(p, q) \neq \phi$ and $H(e) \cap X(p, q) \neq \phi\}$. Here $X_1(p) \cup X_2(q) \subseteq V$ and each hyperarc in $Y(p, q)$ is a subhyperarc of a hyperarc in H^*. So $Z(p, q)$ is a weak induced subhypergraph of H^* for every $(p, q) \in P$. That is, $H = H_1 \vee H_2$ is a soft directed hypergraph of H^* since $H_1 \vee H_2$ can be expressed as (H^*, X, Y, P) and it satisfies all the conditions for a soft directed hypergraph.

Theorem 8.2 *Let $H^* = (V, E)$ be a simple directed hypergraph and $H_1 = (H^*, X_1, Y_1, P_1)$ and $H_2 = (H^*, X_2, Y_2, P_2)$ be two soft directed hypergraphs of H^* such that $X_1(p) \cap X_2(q) \neq \phi$ for $(p, q) \in P_1 \times P_2$. Then $H_1 \wedge H_2$ is a soft weak induced subhypergraph of $H_1 \vee H_2$.*

Proof. By theorems 7.1 and 8.1, we have $H_1 \wedge H_2$ and $H_1 \vee H_2$ are soft directed hypergraphs of H^*. Assume that $H_1 \wedge H_2 = H_\wedge = (H^*, X_\wedge, Y_\wedge, P_\wedge)$ and $H_1 \vee H_2 = H_\vee = (H^*, X_\vee, Y_\vee, P_\vee)$. By the definitions of AND and OR operations, the parameter sets of $H_\wedge$ and $H_\vee$ are respectively $P_\wedge = P_1 \times P_2$ and $P_\vee = P_1 \times P_2$. Clearly $P_\wedge \subseteq P_\vee$. For $(p, q) \in P_\wedge = P_1 \times P_2$, the corresponding dh-part $Z_\wedge(p, q)$ of $H_\wedge$ is $(X_\wedge(p, q), Y_\wedge(p, q))$, where $X_\wedge(p, q) = X_1(p) \cap X_2(q)$ and $Y_\wedge(p, q) = \{(T(e) \cap X_\wedge(p, q), H(e) \cap X_\wedge(p, q)) \mid e \in E$ and $T(e) \cap X_\wedge(p, q) \neq \phi$ and $H(e) \cap X_\wedge(p, q) \neq \phi\}$. Also For $(p, q) \in P_\vee = P_1 \times P_2$, the corresponding dh-part $Z_\vee(p, q)$ of $H_\vee$ is $(X_\vee(p, q), Y_\vee(p, q))$, where $X_\vee(p, q) = X_1(p) \cup X_2(q)$ and $Y_\vee(p, q) = \{(T(e) \cap X_\vee(p, q), H(e) \cap X_\vee(p, q)) \mid e \in E$ and $T(e) \cap X_\vee(p, q) \neq \phi$ and $H(e) \cap X_\vee(p, q) \neq \phi\}$. Clearly $X_\wedge(p, q) \subseteq X_\vee(p, q)$ since $X_1(p) \cap X_2(q) \subseteq X_1(p) \cup X_2(q)$. Also each hyperarc in $Y_\wedge(p, q)$ is a subhyperarc of a hyperarc in $Y_\vee(p, q)$. So $Z_\wedge(p, q)$ is a weak induced subhypergraph of $Z_\vee(p, q)$ for every $(p, q) \in P_\wedge = P_1 \times P_2$. That is $H_1 \wedge H_2$ is a soft weak induced subhypergraph of $H_1 \vee H_2$ since the following conditions are satisfied:

1. $P_\wedge \subseteq P_\vee$,
2. $Z_\wedge(p, q) = (X_\wedge(p), Y_\wedge(q))$ is a weak induced subhypergraph of $Z_\vee(p) = (X_\vee(p), Y_\vee(q))$ for all $(p, q) \in P_\wedge = P_1 \times P_2$.

9. Conclusion

Soft directed hypergraph was introduced by applying the concept of soft set in the directed hypergraph. Using parameterization, a soft directed hypergraph produces a series of descriptions of a complicated relation described using a directed hypergraph. We introduced the notions of degrees, soft incidence matrix, and soft adjacency matrix related to soft directed hypergraphs. We also presented and explored the features of soft directed hypergraphs' AND and OR operations.

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