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ON NEUTROSOPHIC IDEAL OF BN-ALGEBRAS

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Abstract

The purpose of this paper is to present the notion of a neutrosophic ideal of BN-algebra. We further investigate some of the related properties with illustrations. Moreover, we obtain some equivalent conditions for a neutrosophic ideal. Finally, we discuss the homomorphism between neutrosophic ideals and two neutrosophic sets.

Key Words: *BN-algebra; Ideal; Neutrosophic Set; Neutrosophic ideals; Homomorphism.*

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1. Introduction

Zadeh [15] first proposed the concept of a fuzzy set in 1965. Atanassov [2] proposed the concept of intuitionistic fuzzy sets in 1986. Gau and Bueurer [4] developed the concept of ambiguous sets in 1993. The concept of neutrosophic sets was first developed by Samarandache [10] in 1998. BCI and BCK algebras were first introduced in 1966 by Imai, Iseki, and Tanaka [13]. Hu and Li [5] proposed the concept of BCH-algebras in 1983. The concept of fuzzy ideals in BN-algebras was presented by Andrzej Walendziak and Grzegorz Dymek [1] in 2014.Young Bae Jun and Eun Hwan Roh [13] introduced the concept of MBJ- neutrosophic ideals of BCK/BCI-algebras. Mehmet Ali Ozturk and Young Bae Jun [8] introduced the concept of neutrosophic ideals in BCK/BCI-algebras based on neutrosophic points. A. A. Salama and H. A. Elagamy [9] introduced the concept of neutrosophic fuzzy ideal. In this article, we define a neutrosophic ideal in BN-algebra and look into some of its properties. We give BN-algebraic examples of neutrosophic ideals.

2. Preliminaries

We review some fundamental definitions of BN-algebra, ideals, properties of ideals, and the concept of neutrosophic sets in this section because they are relevant to our primary findings.

Definition 2.1[1] An algebra $(A, \odot, 0)$ of type (2,0) is called BN-algebra, if the following are satisfied for all $\alpha, \beta, \gamma \in A$,

(i) $(\alpha \odot \alpha) = 0$

(ii) $(\alpha \odot 0) = \alpha$

(iii) $(\alpha \odot \beta) \odot \gamma = (0 \odot \gamma) \odot (\beta \odot \alpha)$.

Proposition 2.2[1] Let $(A, \odot, 0)$ be a BN-algebra, then the following are satisfied for all $\alpha, \beta \in A$,

(i) $0 \odot (0 \odot \alpha) = \alpha$

(ii) $0 \odot (\alpha \odot \beta) = \beta \odot \alpha$

(iii) $\beta \odot \alpha = (0 \odot \alpha) \odot (0 \odot \beta)$

(iv) $\alpha \odot \beta = 0 \Rightarrow \beta \odot \alpha = 0$

(v) $0 \odot \alpha = 0 \odot \beta \Longrightarrow \alpha = \beta$.

Definition 2.3[1] A subset I of a BN-algebra $(A, \odot, 0)$ is called an ideal of A, if it is satisfied the following for all $\alpha, \beta \in A$,

(i) $0 \in I$

(ii) $\alpha \odot \beta \in I$ and $\beta \in I$ imply $\alpha \in I$.

Proposition 2.4[1] Let I be an ideal of BN-algebra $(A, \odot, 0)$, if $\alpha \leq \beta$ and $\beta \in I$ then $\alpha \in I$.

Definition 2.5[1] Let $(A, \odot, 0_1)$ and $(B, \odot, 0_2)$ be the BN-algebras. A mapping $f: A \rightarrow B$ is called homomorphism from A to B, if $f(\alpha \odot \beta) = f(\alpha) \odot f(\beta)$ for all $\alpha, \beta \in A$,

Definition 2.6[1] Let $(A, \odot, 0_1)$ and $(B, \odot, 0_2)$ be the BN-algebras, and $f: A \rightarrow B$ be a a homomorphism. Then the kernel of f is defined as $Kerf = \{\alpha \in A: f(\alpha) = 0_2\}$ of A.

Proposition 2.7[1] Let $f: A \rightarrow B$ be a homomorphism from BN-algebra A into BN-algebra B. Then $Ker\ f$ is an ideal of B.

Definition 2.8[1] Let $(A, \odot, 0)$ be a BN-algebra. We define a binary relation $'\leq'$ on A by $\alpha \leq \beta$ if and only if $\alpha \odot \beta = 0$ for all $\alpha, \beta \in A$,

Note. It is easily verified that for all $\alpha \in A$, if $\alpha \leq 0$, then $\alpha = 0$.

Definition 2.9[11] Let A be the discourse universe. A truth membership function T_N, an indeterminacy membership function I_N, and a falsity membership function F_N characterise a neutrosophic set N in A, where T_N, I_N, and F_N are real standard elements of [0, 1]. It can be written as $N = \{(\alpha, T_N(\alpha), I_N(\alpha), F_N(\alpha)) / \alpha \in A\}$ $T_N, I_N, F_N \in]0, 1[$. There is no restriction on the sum of $T_N(\alpha), I_N(\alpha)$ and $F_N(\alpha)$, and so $0^- \leq T_N(\alpha) + I_N(\alpha) + F_N(\alpha) \leq 3^+$.

Definition 2.10[11] The complement of a neutrosophic set N is denoted by $C(N)$ and is defined by $T_{c(N)}(\alpha)$ $=\{1^+\}-T_N(\alpha)$, $I_{c(N)}(\alpha)$ $=\{1^+\}-I_N(\alpha)$, $F_{c(N)}(\alpha)$ $=\{1^+\}-F_N(\alpha)$.

Definition 2.11[11] Another neutrosophic set N, indicated by, contains a neutrosophic set M that is $M \subseteq N$ if and only if $inf\, T_M(\alpha) \leq inf\, T_N(\alpha)$, $sup\, T_M(\alpha) \leq sup\, T_N(\alpha)$, $inf\, I_M(\alpha) \geq inf\, I_N(\alpha)$, $sup I_M(\alpha) \geq sup\, I_N(\alpha)$, $inf F_M(\alpha) \geq inf F_N(\alpha)$ and $sup\, F_M(\alpha) \geq sup\, F_N(\alpha)$ for all $\alpha \in N$.

Definition 2.12[12] A neutrosophic set W is formed by the union of two neutrosophic sets U and V. That is $W = U \cup V$, whose truth-membership, indeterminacy-membership and falsity-membership functions are related to those of U and V by $T_W(\alpha) = \max\{T_U(\alpha), T_V(\alpha)\}$, $I_W(\alpha) = max\{I_U(\alpha), I_V(\alpha)\}$, $F_W(\alpha) = min\{F_U(\alpha), F_V(\alpha)\}$ for all $\alpha \in U$.

Definition 2.13[12] A neutrosophic set W is formed by the intersection of two neutrosophic sets U and V, written as $W = U \cap V$, whose truth-membership, indeterminacy-membership, and falsity-membership are connected to those of U and V by $T_W(\alpha) = \min\{T_U(\alpha), T_V(\alpha)\}$, $I_W = min\{I_U(\alpha), I_V(\alpha)\}$, $F_W(\alpha) = max\{F_U(\alpha), F_V(\alpha)\}$ for all $\alpha \in U$.

Definition 2.14[6] Let A be any neutrosophic set. If for any $\rho \in [0, 3]$, then an neutrosophic level set of ρ is defined by $L(A, \rho\}$ $=\{\alpha \in U: T_N(\alpha) \geq \rho, I_N(\alpha) \geq \rho, F_N(\alpha) \leq \rho\}$.

3. Neutrosophic ideal

This section introduces the idea of a neutrosophic ideal of BN-algebra and explores some of its key characteristics.

Definition 3.1 A neutrosophic set $N = \{T_{N,} I_N, F_N\}$ of BN-algebra $(A, \odot, 0)$, if N is called an neutrosophic ideal of BN-algebra $(A, \odot, 0)$, if it satisfies the following for all $\alpha, \beta \in A$,

(i) $T_N(0) \geq T_N(\alpha), I_N(0) \geq I_N(\alpha), F_N(0) \leq F_N(\alpha)$

(ii) $T_N(\alpha) \geq min\,\{T_N(\alpha \odot \beta), T_N(\beta)\}$; $I_N(\alpha) \geq min\,\{I_N(\alpha \odot \beta), I_N(\beta)\}$
$F_N(\alpha) \leq max\,\{F_N(\alpha \odot \beta), F_N(\beta)\}$.

Note. Throughout this paper we considered N is a neutrosophic set.

Example 3.2 Consider a set $A = \{0, a, b, c, 1\}$. Define a binary operation $'\odot'$ on A given by the following Table 3.1, and neutrosophic set define by the Table 3.2 as shown below

$\odot$	0	a	b	c	1
0	0	a	c	c	1
a	a	0	a	c	a
b	b	a	0	c	b
c	c	c	c	0	c
1	1	a	b	c	0

Table 3.1: $'\odot'$ **Operation**

A / N	0	a	b	c	1
$T_N(x)$	0.9	0.5	0.5	0.4	0.5
$I_N(x)$	0.8	0.4	0.4	0.4	0.4
$F_N(x)$	0.1	0.4	0.4	0.5	0.4

Table 3.2: Neutrosophic set

It is easily verified that N is a neutrosophic ideal of A, and that it satisfies the conditions (i) and (ii) of definition 3.1.

Example 3.3 Consider a set $A = \{0, a, b, c, 1\}$. Define a binary operation $'\odot'$ on A given by the following Table 3.3, and neutrosophic set define by the Table 3.4 as shown below:

$\odot$	0	a	b	c	1
0	0	a	c	c	1
a	a	0	a	c	a
b	b	a	0	c	b
c	c	c	c	0	c
1	1	a	b	c	0

Table 3.3: $'\odot'$ Operation

A \ N	0	a	b	c	1
$T_N(x)$	0.3	0.5	0.6	0.6	0.8
$I_N(x)$	0.4	0.3	0.8	0.5	0.4
$F_N(x)$	0.7	0.5	0.4	0.7	0.5

Table 3.4: Neutrosophic set

Here, $N$is not neutrosophic ideal of A, since the following condition does not satisfied $F_N(0) \leq F_N(\alpha)$.

Proposition 3.4 Let N be an neutrosophic ideal of the BN-algebra $(A, \odot, 0)$. Then T_N and I_N are order preserving, and F_N is order reversing.

Proof: Let N be a neutrosophic ideal of BN-algebra $(A, \odot, 0)$

If $\alpha \leq \beta$, then $\alpha \odot \beta = 0$ for all $\alpha, \beta \in A$,

From (ii) of proposition 2.2, we have $\beta \odot \alpha = 0$.

Thus, from (i) and (ii) of definition 3.1, we have

$$T_N(\beta) \geq min\,\{T_N\,(\beta \odot \alpha), T_N(\alpha)\} = min\,\{T_N\,(0), T_N(\alpha)\}$$

$$T_N(\beta) \geq T_N(\alpha) \text{ for all } \alpha, \beta \in A,$$

$$I_N(\beta) \geq \min\{I_N\,(\beta \odot \alpha), I_N(\alpha)\}$$

$$= min\,\{I_N\,(0), I_N(\alpha)\}$$

$$I_N(\beta) \geq I_N(\alpha)\} \text{ for all } \alpha, \beta \in A,$$

$$F_N(\beta) \leq max\,\{F_N(\beta \odot \alpha), F_N(\alpha)\}$$

$$= max\,\{F_N\,(0), F_N(\alpha)\}$$

$$F_N(\beta) \leq F_N(\alpha)\} \text{ for all } \alpha, \beta \in A, \blacksquare$$

Proposition 3.5 An neutrosophic set N on the BN-algebra $(A, \odot, 0)$ is an neutrosophic ideal of A if and only if it satisfies the following for all $\alpha, \beta, \gamma \in A$,

(i) $T_N\,(0) \geq T_N\,(\alpha)$, $I_N(0) \geq I_N(\alpha)$, $F_N(0) \leq F_N(\alpha)$

(ii) If $(\gamma \odot \beta) \odot \alpha = 0$. Then $T_N(\gamma) \geq min\{T_N(\alpha), T_N(\beta)\}$;

$I_N(\gamma) \geq min\{I_N(\alpha), I_N(\beta)\}$;

$F_N(\gamma) \leq max\{F_N(\alpha), F_N(\beta)\}$.

Proof: Let N be an neutrosophic ideal of BN-algebra $(A, \odot, 0)$.

Then from (i) of definition 3.1,

we have $T_N(0) \geq T_N(\alpha), I_N(0) \geq I_N(\alpha)$,

$F_N(0) \leq F_N(\alpha)$ for all $\alpha \in A$.

If $(\alpha \odot \beta) \odot \gamma = 0$ and N be an neutrosophic ideal of BN-algebra $(A, \odot, 0)$.

Then from (ii) of definition 3.1,

we get $T_N(\gamma \odot \beta) \geq min\{T_N((\gamma \odot \beta) \odot \alpha), T_N(\alpha)\}$ for all $\alpha, \beta, \gamma \in A$,

$= min\{T_N(0), T_N(\alpha)\}$

$= T_N(\alpha)$ [from (i) of definition 3.1]

And $T_N(\gamma) \geq min\{T_N(\gamma \odot \beta), T_N(\beta)\}$ for all $\alpha, \beta, \gamma \in A$,

[from (ii) of definition 3.1]

Thus, $T_N(\gamma) \geq min\{T_N(\alpha), T_N(\beta)\}$

$I_N(\gamma \odot \beta) \geq min\{I_N((\gamma \odot \beta) \odot \alpha), I_N(\alpha)\}$ for all $\alpha, \beta, \gamma \in A$,

$= min\{I_N(0), I_N(\alpha)\}$

$= I_N(\alpha)$ [from (i) of definition 3.1]

And $I_N(\gamma) \geq \min\{I_N(\gamma \odot \beta), I_N(\beta)\}$ for all $\alpha, \beta, \gamma \in A$, [from (ii) of definition 3.1]

Thus, $I_N(\gamma) \geq min\{I_N(\alpha), I_N(\beta)\}$

$F_N(\gamma \odot \beta) \leq max\{F_N((\gamma \odot \beta) \odot \alpha), F_N(\alpha)\}$

$= max\{F_N(0), F_N(\alpha)\}$

$= F_N(\alpha)$ [from (i) of definition 3.1]

And $F_N(\gamma) \leq max\{F_N(\gamma \odot \beta), F_N(\beta)\}$ for all $\alpha, \beta, \gamma \in A$,

[from (ii) of definition 3.1]

Thus $F_N(\gamma) \leq max\{F_N(\alpha), F_N(\beta)\}$

Conversely, let N satisfies (i) and (ii) in BN-algebra $(A, \odot, 0)$.

Then from (i) of definition 2.1, we have $(\alpha \odot \beta) \odot (\alpha \odot \beta) = 0$

From (ii) of definition 3.1 we have $T_N(\alpha) \geq min\{T_N(\alpha \odot \beta), T_N(\beta)\}$,

$$I_N(\alpha) \geq min\,\{I_N(\alpha \odot \beta), I_N(\beta)\},$$
$$F_N(\alpha) \leq max\,\{F_N(\alpha \odot \beta), F_N(\beta)\}$$

for all $\alpha, \beta, \gamma \in A$.

Hence, N satisfies (ii) of definition 3.1.

Therefore, N is a neutrosophic ideal of BN-algebra A. ■

Theorem 3.6 Let N be a neutrosophic set of the BN-algebra $(A, \odot, 0)$, then N is an neutrosophic ideal of A if and only if a neutrosophic level subset $L(A, \alpha) = \{\alpha \in U/T_N(\alpha) \geq \rho, I_N(\alpha) \geq \rho, F_N(\alpha) \leq \rho\}$ is either empty or an ideal of the BN-algebra A for all $\rho \in [0, 1]$.

Proof: Let N be an neutrosophic ideal of BN-algebra $(A, \odot, 0)$.

Let $\rho \in [0,1]$ and $L(A, \rho) \neq \varphi$.

Then, $N(\alpha_0) \geq \rho$ for some $\alpha_0 \in A$.

$N(0) \geq N(\alpha)$, we have $0 \in L(A, \rho)$.

Let $\alpha, \beta \in A$ such that $\alpha \odot \beta, \beta \in L(A, \rho)$, then $N(\alpha \odot \beta) \geq \rho$ and $N(\beta) \geq \rho$.

From (ii) of definition 3.1, we have $N(\alpha) \geq min\{N(\alpha \odot \beta), N(\beta)\} \geq \alpha$

So that, $u \in L(A, \rho)$, $L(A, \rho)$ is an ideal of $(A, \odot, 0)$.

Conversely, if for each $\rho \in]0,3[$, $L(A, \rho) = \varphi$ or $L(A, \rho)$is an ideal of

$(A, \odot, 0)$

If $N(0) \geq N(\alpha)$ is not valid. Then there exist $\alpha_0 \in A$ such that

$N(0) < N(\alpha_0) = \delta$

Then, $L(A, \delta) \neq \varphi$ and $N(0) \geq \delta$. This is contradiction, and $N(0) \geq N(\alpha)$ is valid.

Now, if (ii) of definition 3.1 does not satisfy, then there exist $a, b \in A$ such that $N(a) < min\{N(a \odot b), N(b)\}$.

Take $\sigma = \frac{1}{2}(N(a) + min\{N(a \odot b), N(b)\})$.

Then, we have $N(a) < \sigma < min\{N(a \odot b), N(b)\}) \leq N(a \odot b)$ and $\beta < N(b)$.

Thus, $a \odot b$, $b \in L(A, \sigma)$, but $a \notin L(A, \sigma)$.

This is not possible, and N is not a neutrosophic ideal of $(A, \odot, 0)$. ■

Corollary 3.7 If N is an neutrosophic ideal of the BN-algebra $(A, \odot, 0)$, then the set $A_N(\alpha) = \{ \alpha \in A: N(\alpha) = N(0)\}$ is an ideal of $(A, \odot, 0)$.

Theorem 3.8 Let $(A, \odot, 0)$ and $(B, \odot, 0)$ be the BN-algebras. Let $f: A \rightarrow B$ be a homomorphism, and N is an neutrosophic ideal of B. Then $f^{-1}(N)$ is an neutrosophic ideal of A.

Proof: Let $f: A \rightarrow B$ be a homomorphism from BN-algebra $(A, \odot, 0)$ to $(B, \odot, 0)$

Let $\alpha \in A$. Since $f(\alpha) \in B$ and N is a neutrosophic ideal of B.

Then $N(0) \geq N(f(\alpha))$ [from (i) of definition 3.1]

$N(0) \geq f^{-1}(N(\alpha))$

But, $N(0) = N(f(0))$

$N(0) = f^{-1}(N(0))$

$N(0) \geq f^{-1}((N(\alpha))$

It satisfies (i) of definition 3.1. That is, $N(0) \geq N(\alpha)$.

Now, let $\alpha, v \in A$. Since, N is a neutrosophic ideal of B

Therefore, we obtain $N(f(\alpha)) \geq min\{ N\{f(\alpha)\odot f(\beta)\}, N\{f(\beta)\}$

$\geq min\{N\{f(\alpha\odot\beta)\}, N\{f(\beta)\}$ [from definition 2.5]

$f^{-1}((N(\alpha)) \geq min\{f^{-1}(N(\alpha\odot\beta), f^{-1}(N(\beta))\}$.

It satisfies (ii) of definition 3.1

Therefore, $f^{-1}(N)$ neutrosophic ideal of A. ■

Proposition 3.9: Let $(A, \odot, 0_1)$ and $(B, \odot, 0_2)$ be the BN-algebras. If $f: A \rightarrow B$ be a homomorphism from A to B, and N is neutrosophic ideal of A. Then, if N is constant on $Kerf = f^{-1}(0)$ and $f^{-1}\{f(N)\} = N$.

Proof: Let $f: A \rightarrow B$ is a homomorphism from BN-algebra $(A, \odot, 0_1)$ to $(B, \odot, 0_2)$

Define $f(\alpha) = \beta$ for all $\alpha \in A$ and $\beta \in B$, then $f^{-1}[f(N(\alpha))] = f(N)f(\alpha) = f(N)\,\beta$

Thus $min\{N(a)/a \in f^{-1}(\beta)$, we have $f(\alpha) = f(a)$

Therefore, $f(a \odot \alpha) = 0$. That is, $a \odot \alpha \in Ker\, f, N(a \odot \alpha) = N(0)$

Hence, $N(a) \geq min\ \{N(a \odot \alpha), N(\alpha)\ \}$

$$\geq min\ \{N(0), N(\alpha)\}$$

$$= N(\alpha) \tag{3.1}$$

Similarly, we have $N(\alpha) \geq N(a)$ (3.2)

From (3.1) and (3.2), $N(\alpha) = N(a)$. ■

4. Conclusion

In this study, we have introduced the notion of a neutrosophic ideal in BN-algebra. Additionally, by employing appropriate examples, we were able to derive some properties. In addition, we investigated homomorphism and the kernel in BN-algebra. Finally, we developed the necessary and sufficient conditions for the neutrosophic ideal through neutrosophic level subsets.

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