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NUMERICAL INVESTIGATION OF AN UNSTEADY FREE CONVECTIVE MHD MICROPOLAR FLUID FLOW OVER A VERTICAL CONE WITH THE EFFECTS OF VARIABLE VISCOSITY AND THERMAL CONDUCTIVITY

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Abstract

The purpose of this study is to present a numerical investigation of the effects of variable viscosity and thermal conductivity on an unsteady laminar free convective MHD micropolar fluid flow along with the combined effects of chemical reaction and heat generation/absorption over a vertical cone. The dimensionless governing partial differential equations are solved by finite differential scheme. The velocity, microrotation, temperature and concentration profiles have been studied for temperature dependent viscosity, thermal conductivity and for other important parameters involved in the problem as well as the skin friction coefficients, Nusselt number and Sherwood number are discussed. Finally the results obtained are shown graphically and in tabulated form and analyzed in detail.

Keywords: Micropolar fluid, viscosity, thermal conductivity, finite difference.

Introduction

Free convective boundary layer flow of a MHD micropolar fluid have been studied widely by many researchers due to its large scale of applications in the field of technology. Heat and mass transfer over a vertical cone with convective flow has various technological applications such as designing of nuclear waste disposal, nuclear reactor cooling systems, geothermal reservoirs etc. [1].

In the recent years, micropolar fluids have been investigated by several researchers due to its industrial and engineering applications like colloids and polymeric suspensions, cervical flows, clean engine lubricants, thrust bearing technologies etc. The initial work of Eringen [2,3,4] in boundary layer theory of micropolar fluid was extended by Peddieson and McNitt [5]. Self-similar solution of two dimensional flow of a micropolar fluid on a semi-infinite plate was discussed by Ahmadi [6]. Hazarika *et al*. [7] studied the effects of variable viscosity and thermal conductivity on steady magnetohydrodynamic flow of a micropolar fluid through a specially characterized horizontal channel. The effects of variable viscosity and thermal conductivity on flow and heat transfer of a stretching surface of rotating micropolar fluid with suction and blowing was analyzed by Borthakur *et al*. [8]. Sulochana *et al*. [9] reported the numerical investigation of magnetohydrodynamic (MHD) radiative flow over a rotating cone in the presence of Soret and chemical reaction and concluded that the variable porosity parameter enhances the heat and mass transfer rate. Hering *et al*. [10] studied the problem of laminar natural convection from a non-isothermal cone. Himasekhar *et al*. [11] analyzed the buoyancy-induced flow and temperature fields around a vertical rotating cone. An analysis to study the flow and heat transfer characteristics for the case of laminar mixed convection along a vertical circular cone was reported by Kumari *et al*. [12]. Wang [13] investigated the boundary layer flow and heat transfer on rotating cones, disks and axisymmetric bodies with concentrated heat sources. The problem of unsteady mixed convection flow from a rotating vertical cone was studied by Takhar *et al*. [14]. Raju *et al*. [15] analyzed the non-Newtonian MHD flow over a cone with thermal radiation and chemical reaction.

Magnetohydrodynamic free convective flow over a non-isothermal vertical cone with Joule heating and viscous dissipation was investigated by Palani *et al*. [16] and found that velocity diminished with the magnetic field and temperature is maximum near the cone surface and tends to zero asymptotically.

Heat and mass transfer on the MHD flow of an unsteady micropolar fluid along a vertical stretching sheet in the presence of induced magnetic field was studied by Sahu *et al*. [17] and solved the problem numerically by Runge- Kutta fourth order shooting technique. Pullepu *et al*. [18] discussed the numerical solutions of free connective flow from a vertical cone with mass transfer under the influence of chemical reaction and heat generation /absorption in the presence of UWT/UWC and solved it by stable finite difference scheme.

The purpose of this paper is to study the numerical investigation of the the effects of temperature dependent viscosity and thermal conductivity on an unsteady free convective flow of a MHD micropolar fluid over a vertical cone with heat and mass transfer.

Viscosity and thermal conductivity are assumed to be the inverse linear functions of temperature following Lai and Kulacki [19]. The governing partial differential equations of motion are solved numerically by finite difference scheme.

1. Mathematical Formulation

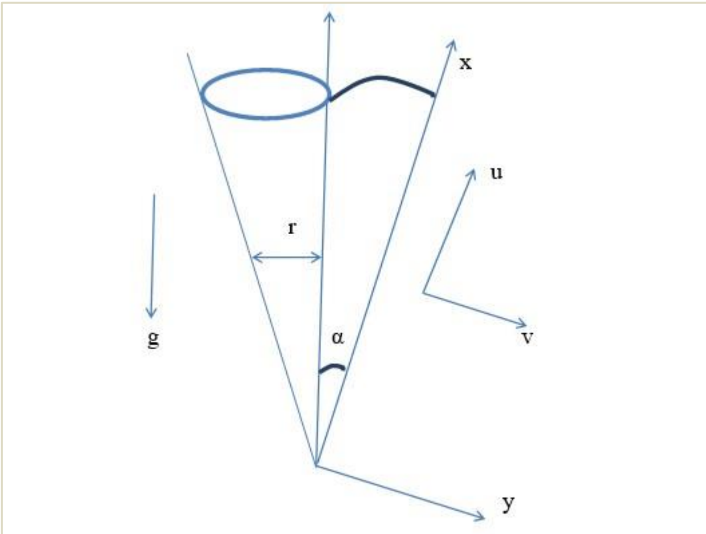

Fig.1: Physical model and co-ordinate system

We consider an axi-symetric unsteady laminar free connective flow of an electrically conducting, viscous incompressible micropolar fluid past a vertical cone. The effects of chemical reaction, heat generation/absorption and viscous dissipation are taken into account. It is assumed that there exists first order chemical reaction between the fluid and the species concentration. The cone surface and the surrounding fluid which is at rest are assumed to be of same temperature and concentration. We consider the co-ordinate system such that x-axis measures the distance along the surface of the cone from the apex (x=0) and y-axis measures the distance normally outwards. Let u and v be the velocity components along x (tangential) and y (radial) direction respectively. Also, let r is the radial distance from the surface element to the axis of symmetry and α is the semi-vertical angle of the cone. A uniform applied magnetic field of strength B_0 is acting parallel to the y-axis. The magnetic Reynolds numbers are assumed to be small enough so that induced magnetic field is negligible. Under the above assumptions, the physical model of the above problem can be given as:

Continuity Equation:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial y}(rv)=0\,. \tag{1}$$

Equation of linear momentum:

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})+\frac{\kappa}{\rho}(\frac{\partial N'}{\partial y}+\frac{\partial^2 u}{\partial y^2})+g\beta(T'-T'_\infty)\cos\alpha+ \\ g\beta_c(C'-C'_\infty)\cos\alpha-\frac{\sigma B_0^2 u}{\rho}\,. \tag{2}$$

Equation of angular momentum:

$$\rho j(\frac{\partial N'}{\partial t'}+u\frac{\partial N'}{\partial x}+v\frac{\partial N'}{\partial y})=-\kappa(2N'+\frac{\partial u}{\partial y})+\gamma\frac{\partial^2 N'}{\partial y^2}\,. \tag{3}$$

Equation of Energy:

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}(\lambda\frac{\partial T'}{\partial y})+\frac{Q_o}{\rho c_p}(T'-T'_\infty)+\frac{\mu+\kappa}{\rho c_p}(\frac{\partial u}{\partial y})^2\,. \tag{4}$$

Equation of Concentration:

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=\frac{\partial}{\partial y}(D_m\frac{\partial C'}{\partial y})-k_1(C'-C'_\infty)\,, \tag{5}$$

where u and v are the components of velocity along x and y- directions respectively, t′ denotes the time factor, ρ is the fluid density, μ is the coefficient of dynamic viscosity, k is the vortex viscosity, g is the acceleration due to gravity, β and β_c are the coefficients of thermal and concentration expansion respectively, σ is the electrical conductivity, N′ is the microrotation component, j is the micro-inertia density, γ is the spin gradient viscosity, T′ is the temperature of the fluid , λ is the thermal conductivity , c_p is the specific heat at the constant pressure, C′ is the concentration of the fluid within the boundary layer, T'_∞ is the temperature of the fluid far away from the surface of the cone, C'_∞ is the is the concentration of the fluid far away from the surface of the cone, $Q_0(x)$ is the heat generation (>0) or absorption (<0) coefficient, k_1 is the chemical reaction parameter and D_m is molecular diffusivity.

The boundary conditions are given as:

$$y = 0 : u = 0,\ v = 0, N' = 0, T' = T'_w, C' = C'_w$$

$$y \to \infty : u \to 0,\ N' \to 0, T' \to T'_\infty, C' \to C'_\infty \tag{6}$$

where T'_w and C'_w are wall temperature and concentration respectively and T'_∞ and C'_∞ are the temperature and concentration of the fluid at infinity respectively.

The non-dimensional variables are defined as:

$$X = \frac{x}{L}, Y = \frac{y}{L}(Gr)^{1/4}, R = \frac{r}{L} \quad \text{where} \quad r = x\sin\alpha,$$

$$V = \frac{vL}{\upsilon_\infty}(Gr)^{-1/4}, U = \frac{uL}{\upsilon_\infty}(Gr)^{-1/2}, t = \frac{\upsilon_\infty t'}{L^2}(Gr)^{1/2},$$

$$N = \frac{L^2}{\upsilon_\infty}(Gr)^{-3/4}N', \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty},$$

$$Gr = \frac{g\beta(T' - T'_\infty)L^3\cos\alpha}{\upsilon_\infty^2}, \Pr = \frac{\upsilon_\infty \rho c_p}{\lambda_\infty},\ S = \frac{Gr^*}{Gr},$$

where, $Gr^* = \dfrac{g\beta_c(C'_w - C'_\infty)L^3\cos\alpha}{\upsilon_\infty^2},\ Q = \dfrac{Q_0 L^2}{c_p\mu}(Gr)^{-1/2}, M = \dfrac{\sigma B_0^2 L^2}{\mu_\infty}(Gr)^{-1/2},$

$$K = \frac{\kappa}{\mu_\infty}, B = \frac{L^2}{j}(Gr)^{-1/2}, G = \frac{\gamma L}{\rho j \upsilon_\infty},\ Ec = \frac{\upsilon_\infty^2}{c_p(T'_\infty - T'_\infty)L^2}Gr,$$

$$S_c = \frac{\upsilon_\infty}{D_m}, K_r = \frac{k_1 L^2}{\upsilon_\infty}(Gr)^{-1/2}, \tag{7}$$

where L is the characteristic length. Also Gr , Gr^*, Pr, S, Q, M, K, B, G, Ec, S_c and K_r denote thermal Grasof number, mass Grasof number, Prandtl number, buoyancy ratio parameter, heat generation/absorption parameter, magnetic parameter, coupling constant parameter, material constant, microrotation parameter, Eckert number, Schmidt number and chemical reaction parameter respectively.

Following Lai and Kulacki [19] , let us assume that,

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}[1+\delta(T'-T'_\infty)] \text{ or } \frac{1}{\mu}=\zeta(T'-T'_r) \text{ where } \zeta=\frac{\delta}{\mu_\infty} \text{ and } T'_r=T'_\infty-\frac{1}{\delta}$$

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}[1+\xi(T'-T'_\infty)] \text{ or } \frac{1}{\lambda}=\varepsilon(T'-T'_c) \text{ where } \varepsilon=\frac{\xi}{\lambda_\infty} \text{ and } T'_c=T_\infty{}'-\frac{1}{\xi} \quad (8)$$

where μ_∞ is the viscosity at infinity, ζ and T'_∞ are constants, $T_r{}'$ is transformed reference temperature, δ and ξ are constants based on thermal property of the fluid. Similarly, λ_∞ is the thermal conductivity at the infinity, ε and $T_c{}'$ are constants and their values depend on the reference state and thermal properties of the fluid.

Using (7), viscosity and thermal conductivity can be expressed as,

$$\upsilon=-\upsilon_\infty\frac{\theta_r}{\theta-\theta_r}, \lambda=-\lambda_\infty\frac{\theta_c}{\theta-\theta_c} \quad (9)$$

where θ_r and θ_c are the dimensionless parameters characterising the influence of viscosity and thermal conductivity respectively and are given by,

$$\theta_r=\frac{T'_r-T'_\infty}{T'_w-T'_\infty}=-\frac{1}{\delta(T'_w-T'_\infty)}, \theta_c=\frac{T'_c-T'_\infty}{T'_w-T'_\infty}=-\frac{1}{\xi(T'_w-T'_\infty)} \quad (10)$$

Equations (1-5) can be written in non-dimensional form by using Eq. (7) and Eq. (9) as follows:

$$\frac{\partial(UR)}{\partial X}+\frac{\partial(VR)}{\partial Y}=0 \quad (11)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\frac{\theta_r}{(\theta-\theta_r)^2}\frac{\partial\theta}{\partial Y}\frac{\partial U}{\partial Y}-\frac{\theta_r}{\theta-\theta_r}\frac{\partial^2 U}{\partial Y^2}+K\left(\frac{\partial N}{\partial Y}+\frac{\partial^2 U}{\partial Y^2}\right)+\theta+S\phi-MU \quad (12)$$

$$\frac{\partial N}{\partial t}+U\frac{\partial N}{\partial X}+V\frac{\partial N}{\partial Y}=-KB\left(2N+\frac{\partial U}{\partial Y}\right)+G\frac{\partial^2 N}{\partial Y^2} \quad (13)$$

$$\frac{\partial\theta}{\partial t}+U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{1}{\Pr}\left[\frac{\theta_c}{(\theta-\theta_c)^2}\left(\frac{\partial\theta}{\partial Y}\right)^2-\frac{\theta_c}{\theta-\theta_c}\frac{\partial^2\theta}{\partial Y^2}\right]+Q\theta+\left(K-\frac{\theta_r}{\theta-\theta_r}\right)Ec\left(\frac{\partial U}{\partial Y}\right)^2 \quad (14)$$

$$\frac{\partial\phi}{\partial t}+U\frac{\partial\phi}{\partial X}+V\frac{\partial\phi}{\partial Y}=\frac{1}{S_c}\left[\frac{\theta_r}{(\theta-\theta_r)^2}\frac{\partial\theta}{\partial Y}\frac{\partial\phi}{\partial Y}-\frac{\theta_r}{\theta-\theta_r}\frac{\partial^2\phi}{\partial Y^2}\right]-K_r\phi \tag{15}$$

The corresponding non-dimensional boundary conditions are:

$$U=0, V=0, N=0, \theta=1, \phi=1 \quad \text{at} \quad Y=0$$

$$U\to 0, N\to 0, \theta\to 0, \phi\to 0 \quad \text{as} \quad Y\to\infty \tag{16}$$

The important physical quantities Skin friction coefficient, Nusselt number and Sherwood number in non-dimensional form are given as:

$c_f=\frac{2\tau_w}{\rho u^2}$ where the shear stress at the surface is given by,

$$\tau_w=\left[(\mu+k)\frac{\partial u}{\partial y}+kN'\right]_{y=0},$$

$Nu=\frac{xq_w}{\lambda_\infty(T_w'-T_\infty')}$, where the heat flux is given by, $q_w=-\lambda\left[\frac{\partial T'}{\partial y}\right]_{y=0}$, and

$Sh=-\frac{Js}{u(C_w'-C_\infty')}$ where $Js=\left[-D_m\frac{\partial C'}{\partial y}\right]_{y=0}$

Therefore,

$c_f=(K-\frac{\theta_r}{1-\theta_r})U^{-2}(Gr)^{-1/4}(\frac{\partial U}{\partial Y})_{Y=0}$, $Nu=\frac{\theta_c}{1-\theta_c}X(Gr)^{1/4}(\frac{\partial\theta}{\partial Y})_{Y=0}$ and

$Sh=-\frac{\theta_r}{1-\theta_r}S_c^{-1}\frac{1}{U}(Gr)^{-1/4}(\frac{\partial\phi}{\partial Y})_{Y=0}$

The ordinary finite difference method is defined as follows:

$$\frac{\partial f}{\partial x}=\frac{f(i+1,j,k)-f(i,j,k)}{\Delta x}$$

$$\frac{\partial f}{\partial y}=\frac{f(i,j+1,k)-f(i,j,k)}{\Delta y}$$

$$\frac{\partial f}{\partial t}=\frac{f(i,j,k+1)-f(i,j,k)}{\Delta t}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{f(i, j+1, k) - 2f(i, j, k) + f(i, j-1, k)}{\Delta y^2} \tag{17}$$

Therefore, the equations (11-15) can be expressed in finite difference method using (16) as follows:

$$V(i, j, k) = \frac{1}{\Delta Y}\left[\frac{U(i+1, j, k) - U(i, j, k)}{\Delta X} + \frac{V(i, j+1, k)}{\Delta Y} + \frac{U(i, j, k)}{X}\right] \tag{18}$$

$$U(i, j, k) = \left(\frac{\theta_r}{(\theta(i, j, k) - \theta_r)^2}\frac{\theta(i, j+1, k) - \theta(i, j, k)}{\Delta Y}\frac{U(i, j+1, k)}{\Delta Y} - \right.$$

$$\frac{\theta_r}{\theta(i, j, k) - \theta_r}\frac{U(i, j+1, k) + U(i, j-1, k)}{\Delta Y^2}$$

$$+K\left[\frac{N(i, j+1, k) - N(i, j, k)}{\Delta Y} + \frac{U(i, j+1, k) - U(i, j-1, k)}{\Delta Y^2}\right]$$

$$\left. + \theta(i, j, k) + S\phi(i, j, k) - \frac{U(i, j, k+1)}{\Delta t} - \frac{V(i, j, k)U(i, j+1, k)}{\Delta Y}\right) /$$

$$\left[-\frac{1}{\Delta t} + \frac{U(i, j+1, k) - U(i, j, k)}{\Delta X} - \frac{V(i, j, k)}{\Delta Y} + \right.$$

$$\left.\frac{\theta_r}{(\theta(i, j, k) - \theta_r)^2}\frac{\theta(i, j+1, k) - \theta(i, j, k)}{\Delta Y^2} - \frac{\theta_r}{\theta(i, j, k) - \theta_r}\frac{2}{\Delta Y^2} + \frac{2K}{\Delta Y^2} + M\right] \tag{19}$$

$$N(i, j, k) = \left(-KB\frac{U(i, j+1, k) - U(i, j, k)}{\Delta Y} + G\frac{N(i, j+1, k) - U(i, j-1, k)}{\Delta Y^2} + \right.$$

$$\left. - \frac{N(i, j, k+1)}{\Delta t} - \frac{U(i, j, k)N(i+1, j, k)}{\Delta X} - \frac{V(i, j, k)N(i, j+1, k)}{\Delta Y}\right)$$

$$/ \left[-\frac{1}{\Delta t} + \frac{U(i, j, k)}{\Delta X} - \frac{V(i, j, k)}{\Delta Y} - 2KB + \frac{2G}{\Delta Y^2}\right] \tag{20}$$

$$\theta(i,j,k) = \left(\frac{1}{\Pr} \left[\frac{\theta_c}{(\theta(i,j,k)-\theta_c)^2} \left(\frac{\theta(i,j+1,k)-\theta(i,j,k)}{\Delta Y}\right)^2 - \frac{\theta_c}{\theta(i,j,k)-\theta_c} \frac{\theta(i,j+1,k)+\theta(i,j-1,k)}{\Delta Y^2} \right] + \left[\left(K - \frac{\theta_r}{\theta(i,j,k)-\theta_r}\right) Ec \left(\frac{U(i,j+1,k)-U(i,j-1,k)}{\Delta Y}\right)^2 \right] - \frac{\theta(i,j,k+1)}{\Delta t} - \frac{U(i,j,k)\theta(i+1,j,k)}{\Delta X} - \frac{V(i,j,k)\theta(i,j+1,k)}{\Delta Y} \right) / \left[-\frac{1}{\Delta t} + \frac{U(i,j,k)}{\Delta X} - \frac{V(i,j,k)}{\Delta Y} - \frac{1}{\Pr} \frac{\theta_c}{\theta(i,j,k)-\theta_c} \frac{2}{\Delta Y^2} - Q \right] \quad (21)$$

$$\phi(i,j,k) = \left(\frac{1}{S_c} \left[\frac{\theta r}{(\theta(i,j,k)-\theta_r)^2} \left(\frac{\theta(i,j+1,k)-\theta(i,j,k)}{\Delta Y}\right)\left(\frac{\phi(i,j+1,k)}{\Delta Y}\right) - \frac{\theta_r}{\theta(i,j,k)-\theta_r} \frac{\phi(i,j+1,k)+\phi(i,j-1,k)}{\Delta Y^2} \right] - \left[\frac{\phi(i,j,k+1)}{\Delta t} + \frac{U(i,j,k)\phi(i+1,j,k)}{\Delta X} + \frac{V(i,j,k)\phi(i,j+1,k)}{\Delta Y} \right] \right) / \left[-\frac{1}{\Delta t} + \frac{U(i,j,k)}{\Delta X} - \frac{V(i,j,k)}{\Delta Y} \frac{1}{S_c} \frac{\theta_r}{(\theta(i,j,k)-\theta_r)^2} \frac{\theta(i,j+1,k)-\theta(i,j,k)}{\Delta Y^2} - \frac{1}{S_c} \frac{\theta_r}{\theta(i,j,k)-\theta_r} \frac{2}{\Delta Y^2} + K_r \right] \quad (22)$$

2. Results And Discussion

The non-linear partial differential equations (11-15) with boundary condition (16) are solved numerically by finite difference method which is based on Gauss Seidal iterative method. To solve the problem, the transformed non- dimensional equations given by (11-15) are again expressed in finite difference equations using Eq. (17) as the equations (18-22). The numerical values of different parameters are taken as θ_c = 2, θ_r =2, S=10, M=.1, Pr=.7, Ec=.001, Q=1, K=.1, G=2, B=2, S_c=.2, K_r=.1 unless otherwise stated. The purpose of this study is to bring out the effects of variable viscosity and thermal conductivity on the governing flow with the combination of the other flow parameters. The numerical computations have been carried out by developing codes for MATLAB and results are presented graphically to get a physical acuity of the problem for the dimensionless velocity profiles, microrotation profiles, temperature profiles and concentration profiles with the variation of different parameters in figures (2-17).

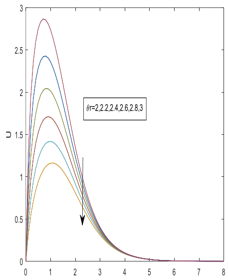

Fig.2: Velocity for various θ_r

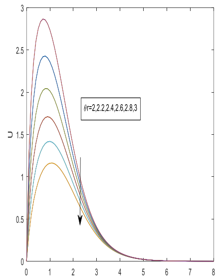

Fig.3: Velocity for various θ_c

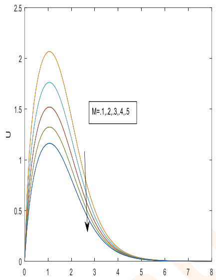

Fig.4: Velocity for various M

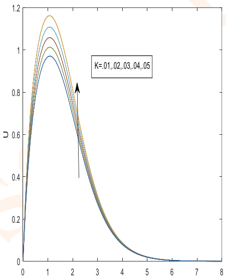

Fig.5: Velocity for various K

Figures 2-5 display the effects of various parameters on velocity profiles. Fig.2 depicts the effect of viscosity variation parameter θ_r on velocity profiles which shows that θ_r retards momentum boundary layer thickness significantly. Due to the enhancement in the magnitude of θ_r, there is an increase in the value of (T_w - T_∞) that reduces the interaction time between the neighboring molecules and the intermolecular forces between the fluid and subsequently, causes an increase in the fluid viscosity which leads to the fluid moving slower. From the fig.3, it is observed that velocity drops with the increase of thermal conductivity parameter θ_c. Physically, due to the increase of thermal conduction within the boundary layer, the heat transposition from region of higher temperature to the region of lower temperature increases, so velocity profiles reduce within the boundary layer. The effects of magnetic parameter M on velocity profiles can be seen in fig.4 which shows that velocity decreases with the increasing values of magnetic parameter. The presence of magnetic field induces a resistive force called Lorentz force, which opposes the velocity field and hence the result. From the fig.5, it is noticed that velocity enhances with the increase of coupling constant parameter K. This is because coupling constant parameter is the ratio of vortex viscosity to the dynamic viscosity. Since velocity is inversely proportional to the dynamic viscosity, hence as K increases momentum boundary layer thickness increases.

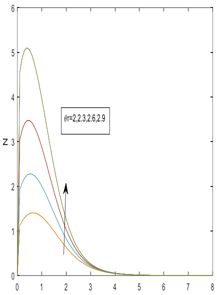

Fig.6: Microrotation for various θ_r.

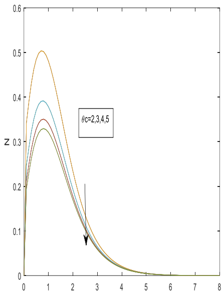

Fig.7: Microrotation for various θ_c.

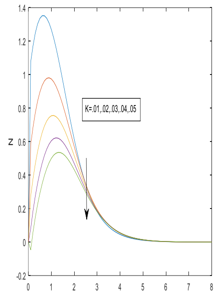

Fig.8: Microrotation for various K

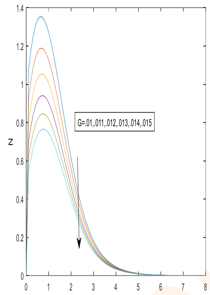

Fig.9: Microrotation for various G

The effects of various parameters on microrotation profiles have been represented by the figures 6-9. From the fig.6, it is found that microrotation profile enhances with the increase of viscosity variation parameter θ_r and it happens due to the elastic property of the micropolar fluid. A significant effect has been observed with the thermal conductivity parameter θ_c on microrotation profiles in the fig.7 which shows that microrotation reduces with the increasing values of thermal conductivity parameter θ_c. From the fig.7, it has been found that microrotation profiles become larger for the increasing values of magnetic parameter M as M increases the Lorentz force increases so that temperature of the fluid enhanced and molecules get released from their bonds holding them and rotation of the fluid elements increase. Microrotation profiles are found to be decreasing effectively for both coupling constant parameter K and microrotation parameter G that can be clearly observed from the figures 8 and 9 respectively.

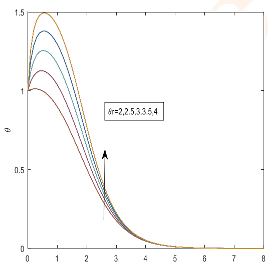

Fig.10: Temperature for various θ_r

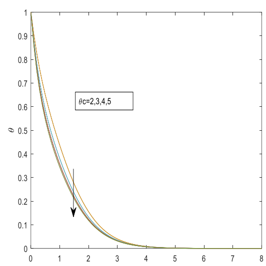

Fig.11: Temperature for various θ_c

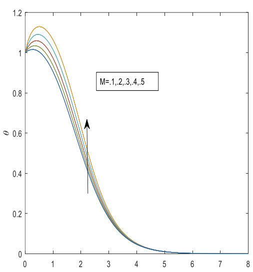

Fig.12: Temperature for various M

Figures 10-12 display the temperature distribution profiles against viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c and magnetic parameter M. Fig.10 yields temperature profiles increase with the increasing values of viscosity variation parameter θ_r. Physically, if viscosity enhances there is an increment of the total viscosity in fluid that makes the fluid more viscous and the convective currents becomes weak and hence temperature rises. From the fig.11, it is observed that thermal boundary layer as well as temperature reduces as the thermal conductivity variation parameter θ_c enhances. It is obvious because thermal conductivity variation parameter is assumed to be the inverse linear function of temperature. Fig.12 shows that temperature profiles becomes larger across the cone with the increasing values of magnetic parameter M. This happens because as M increases, Lorentz force increases and consequently temperature of the fluid increases.

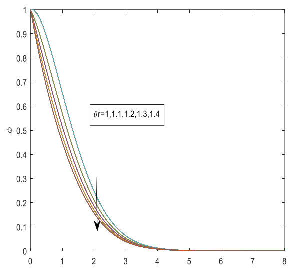

Fig.13: Concentration for various θ_r

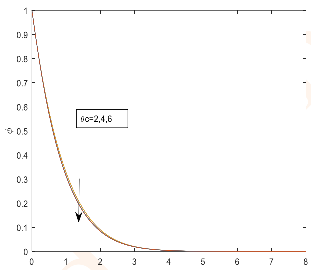

Fig.14: Concentration for various θ_c

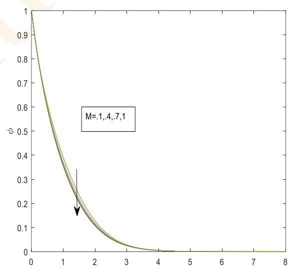

Fig.15: Concentration for various M

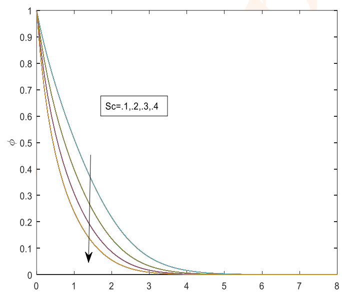

Fig.16: Concentration for various S_c

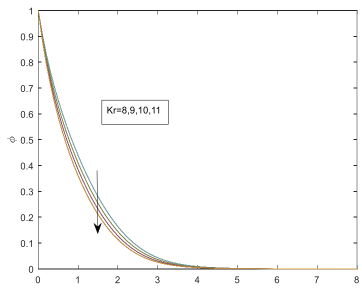

Fig.17: Concentration for various K_r

Figures 13-17 indicate the effects of various parameters on species concentration profiles within the boundary layer. From the figures 13-15, it is observed that species concentration profiles drop with the increasing values of viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c and magnetic parameter M respectively. Due to the rising of viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c and magnetic parameter M, there is an increase in viscous force, convective force and Lorentz force, for which mass diffusivity within the boundary layer decreases and hence concentration is found to be reducing. Fig.16 shows that concentration profile decreases with the increasing values of Schmidt number S_c. Since Schmidt number indicates the relative effectiveness of momentum and mass transport by diffusion in the velocity and species concentration boundary layers and hence concentration decreases as Schmidt number increases causing the decrease of concentration buoyancy effects. The effects of chemical reaction parameter K_r can be observed from the fig.17. It is noticed that species concentration profiles significantly reduce with the enhancement of chemical reaction parameter K_r.

Table:1

M	θ_r	c_f	Nu	Sh
.1	2	0.008601	0.779323	-0.003889
	3	0.036559	0.691197	-0.013994
	4	0.085899	0.640424	-0.030126
.2	2	0.006521	0.784429	-0.003822
	3	0.025748	0.731424	-0.013680
	4	0.058054	0.703857	-0.029361
.3	2	0.005394	0.783883	-0.003779
	3	0.020475	0.746517	-0.013487
	4	0.045225	0.728225	-0.028899

Table: 2

M	θ_c	c_f	Nu	Sh
.1	2	0.008601	0.779323	-0.003889
	3	0.008361	0.643754	-0.003884
	4	0.008265	0.594212	-0.003883
.2	2	0.006521	0.784429	-0.003822
	3	0.006372	0.648787	-0.003817
	4	0.006312	0.599392	-0.003816
.3	2	0.005394	0.783883	-0.003779
	3	0.005288	0.647608	-0.003775
	4	0.005245	0.598021	-0.003773

Table: 3

K_r	S_c	c_f	Nu	Sh
9	.21	0.005394	0.783886	-0.003278
	.22	0.005394	0.783885	-0.003519
	.23	0.005394	0.783884	-0.003766
10	.21	0.005394	0.783884	-0.003473
	.22	0.005394	0.783883	-0.003729
	.23	0.005394	0.783883	-0.003991
11	.21	0.005394	0.783883	-0.003657
	.22	0.005394	0.783882	-0.003927
	.23	0.005394	0.783881	-0.004204

Tables 1-3 display the computed numerical values of skin friction coefficient c_f, Nusselt number Nu and Sherwood number Sh for different values of viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c, magnetic parameter M, Schmidt number S_c and chemical reaction parameter K_r.

From the tables, it is noticed that skin friction coefficient c_f reduces with the increasing values of thermal conductivity variation parameter θ_c, magnetic parameter M; but reverse result is gained for viscosity variation parameter θ_r. The values of Nusselt number Nu are found to be decreasing only with respect to viscosity variation parameter θ_r and for all the other values, it is enhancing as shown in the tables. For the increasing values of viscosity variation parameter θ_r, Schmidt number S_c and chemical reaction parameter K_r, the values of Sherwood number Sh are decreasing significantly; but opposite behavior is observed with thermal conductivity variation parameter θ_c and magnetic parameter M.

3. Conclusion

In this study, a mathematical model has been presented for an unsteady free convective MHD flow of a micropolar fluid over a vertical cone with variable viscosity and thermal conductivity. A parametric analysis is performed to illustrate the influence of variable viscosity and thermal conductivity and other thermo physical parameters on the velocity, microrotation, temperature and concentration profiles and the following observations has been made:

(i) The fluid velocity decreases with the viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c and magnetic parameter M ; but enhances with coupling constant parameter K.

(ii) Microrotation increases with the increasing values of viscosity variation parameter θ_r and magnetic parameter M; but reverse result is obtained for thermal conductivity variation parameter θ_c, coupling constant parameter K and microrotation parameter G.

(iii) Temperature is obtained to be enhancing with respect to viscosity variation parameter θ_r and magnetic parameter M whereas it decreases with thermal conductivity variation parameter θ_c.

(iv) Species concentration becomes smaller for the larger values of viscosity variation parameter θ_r, thermal conductivity variation parameter θ_c, Schmidt number S_c and chemical reaction parameter K_r.

(v) Skin friction coefficient c_f reduces with the increasing values of thermal conductivity variation parameter θ_c and magnetic parameter M.

(vi) Nusselt number Nu is found to be decreasing with respect to viscosity variation parameter θ_r.

(vii) The values of Sherwood number Sh are decreasing significantly for the increasing values of viscosity variation parameter θ_r, Schmidt number S_c and chemical reaction parameter K_r.

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